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On modeling the air traffic control coordination in the collision avoidance problem by mixed integer linear optimization

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Abstract A mixed integer linear optimization (MILO) model is presented for providing a cooperative system between Air Traffic Controllers (ATC) who manage the airspace for aircraft conflict detection and resolution. The model considers the main features of the object modeling of an important, crucial extension so named Coordinated Velocity and Altitude Changes (CVAC) of the VAC model that we have presented elsewhere. It allows the aircraft to ascend or descend one or more altitude levels. The new model is so tight that a state-of-the-art MILO solver provides the solution in a very affordable computing time even for large-scale instances. It is worth to pointing out that only in a few pilot cases of the testbed, the software engine needs to use the branch-and-cut phase of the solver.

Keywords Air Traffic Management (ATM) · collision avoidance · mixed integer linear optimization

1 Introduction

Currently the efforts for solving many different problems involved in Air Traffic Management (ATM) are being increased due to the growing demand in the last years. A specific and important problem devoted to better management of the airspace is the Collision Avoidance. This problem consists of modifying a given

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a set of aircraft configurations if necessary, in order to give a new configuration such that the possible conflicts among the aircraft are avoided, being the loss of the minimum safety distance a conflict situation.

Eurocontrol published a paper [7] that presents the Medium-Term Conflict Detection (MTCD) problem in detail. The purpose is providing a system able to detect and notify the possible conflicts to an Air Traffic Controller (ATC). The system will assume that data and trajectories are given where some uncertainty can exist.

Different works have been published in the literature. In Kuchar and Yang (2000) [9] and in Martín-Campo (2010) [10] an extensive state-of-the-art is presented. Pallottino et al. (2002) present two different linear optimization models for solving the collision avoidance problem. The first one solves conflict situations by using velocity maneuvers and the second one by using heading angle changes. Their geometric construction is important in the model we are presenting in this paper since it helps to detect and solve the possible conflict situations. Dell'Ollmo and Lulli (2003) [6] propose a model that can be solved even for large-scale problems by using a combination of exact and heuristic methodologies. Christodoulou and Kodaxakis [5] propose a Mixed Integer Non Linear Optimization (MINLO) approach to solve the conflict detection and resolution problem by velocity changes and heading angle control by using a standard optimization engine. In Alonso-Ayuso et al. (2011) [1] we propose the Velocity and Altitude Changes model (VAC) based on Mixed Integer Linear Optimization (MILO), solving the conflicts in small elapsed time. Two linear optimization models that solve the problem by using altitude changes the first one and altitude and velocity changes combined together in the second one are proposed in Alonso-Ayuso et al. (2011) [3]. Cafieri and Durand (2012) [4] also propose a MINLO model based on velocity regulation and considering different times for doing the velocity changes.

The VAC model in [1] solves the conflicts by using velocity and altitude maneuvers but it does not consider those cases in which one or more aircraft are changing their altitude level. This can produce one conflict situation at least, mainly if the aircraft changes more than one altitude level what is normal in real-life. Then, several conflict situations may occur. This model does not also consider the conflict situation that may occur when other aircraft are approaching to the aerial sector. But potential conflicts should also be avoided if the information about the aircraft close to the frontier is taken into account. Then, we present a model so-called Coordinated Velocity and Altitude Changes (CVAC) that takes into account the previous features in the air traffic considered in the VAC model plus the necessary coordination between the ATCs responsible for aircraft conflict detection and resolution of close enough aerial sectors. The CVAC model is a MILO with 0–1 and discrete variables (which increases the difficulty in the solution of the problem). In spite of that the model is so tight that the number of the branch-and-cut nodes of the MILO solver is so small that the related computing time allows to use the tool in real time.

The main new characteristics of the MILO CVAC model that we propose in this paper are as follows: (1) it solves the potential conflict situations in an airspace divided in aerial sectors in small elapsed time; (2) it considers that an aircraft can change more than one altitude level and the horizontal velocity will be smaller; (3) it gives simple instructions to the ATC responsible for the control of the aerial sector under consideration as well as to the adjacent other ones in order to coordinate the aircraft maneuvers among the different aerial sectors and detecting possible conflict situations in advance; (4)The computing resolution by a MILO solver does not need to use its branch-and-cut phase but for a few simulation pilot cases of the broad testbed used for assessing the validation of the new approach on solving this crucial problem in real-time.

This reminder of the paper is organized as follows. Section 2 presents the general features of the problem to consider. Section 3 introduces the main characteristics of the CVAC model. Section 4 presents its mathematical formulation. Section 5 reports the main computational results. And, finally, Section 6 concludes.

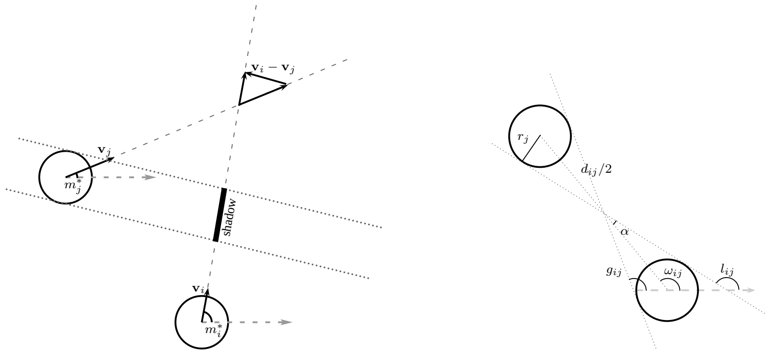
2 Problem statement

One important problem in Air Traffic Management is the conflict Collision Avoidance. The aim is to provide a new configuration (as similar as possible to the initial one) for every aircraft such that all the possible conflicts situations are avoided. Notice that a conflict situation occurs if two or more aircraft violate the safety conditions that they have to keep. The starting point is included by the configuration, velocity, altitude level, angle of motion, etc. of each aircraft under consideration.

It is worth to pointing out that in real-life the airspace is divided in aerial sectors and, each one is managed by one ATC, that gives instructions to the aircraft pilots under his/her own responsibility. Then, coordinating the different ATCs is important for an efficient management of the airspace. One important feature of the problem that is not usually considered in the existing literature about Collision Avoidance consists of the coordination among the ATCs. An specific case happens when an aircraft out of the aerial sector under responsibility of an ATC is approaching and there could be a conflict situation with another aircraft that is being monitoring by a different ATC member. So, a coordinating system is required.

In Alonso-Ayuso et al. (2011) [1] a MILO model was introduced for solving conflict situations in small elapsed time in a given aerial sector (being the portion of airspace managed by an ATC). This model was so-named Velocity and Altitude Changes (VAC) model. However, it is necessary to extend the VAC model by including coordination between ATCs monitoring close enough aerial sectors. This coordination will provide more efficient management of Air Traffic, detecting in advance the conflicts and avoiding dangerous conflict situations close to the frontier. The MILO model for optimizing that coordination is so named CVAC.

Another important feature to take into account is the situation when an aircraft is climbing or descending one or more altitude levels. In these cases the aircraft could be involved in a conflict situation in the levels that is taking up. The CVAC model also includes this important feature. The different horizontal velocities are also taken into account since if an aircraft is climbing or descending some altitude levels, the horizontal velocity will be reduced up to the moment in which the aircraft reach the expected level.



(a) Geometric construction for conflict avoidance constraints.

(b) Angles for the conflict avoidance constraints.

Fig. 1 Geometric Construction for the VC problem.

3 Modeling objects for CVAC

The model presented in this paper is based on the geometric construction that is depicted in Fig. 1, see [1,2,11].

The main idea of the model proposed in [11] is based on the construction of the relative velocity vector $\mathbf{v}_i - \mathbf{v}_j$ between two aircraft, say i and j, where v_i and v_j are the velocity vectors. Depending on their tangent and the tangent of the angles g_{ij} and l_{ij} (see Fig. 1) a conflict situation can be detected. Therefore, no conflict occurs between aircraft i and j if one of the next expressions is satisfied:

$$\frac{(v_i + q_i)\sin(m_i^*) - (v_j + q_j)\sin(m_j^*)}{(v_i + q_i)\cos(m_i^*) - (v_j + q_j)\cos(m_j^*)} \geqslant \tan(l_{ij}) \tag{1a}$$

$$\frac{(v_i + q_i)\sin(m_i^*) - (v_j + q_j)\sin(m_j^*)}{(v_i + q_i)\cos(m_i^*) - (v_j + q_j)\cos(m_j^*)} \leqslant \tan(g_{ij}), \tag{1b}$$

where m^* and $v + q$ denote the angle of motion and the optimal velocity for the conflict avoidance, respectively. (Notice that q is the necessary velocity variation to avoid the conflict situation). The nonlinearity in the previous expressions can be easily transformed in linearity by taking into account four different cases modeled by using the 0-1 variables δ_{ij}^n, for $n = 1, 2, 3, 4$ (see [1,11]).

The aim of the CVAC model consists of coordinating the ATCs when facing special conflict situations that take place close to the aerial sector. It also assumes that if an aircraft is climbing or descending, then it can take more than one altitude level and all possible conflicts can be taken into account.

3.1 The Velocity and Altitude Changes (VAC) model

For completeness, we present the VAC model fully described in [1]. Let us consider an aerial sector and a given set of aircraft $\mathcal{F}$ flying in a specific aerial sector divided in a set $\mathcal{Z}$ of levels as well as their initial flight plans.

Parameters

For all $f \in \mathcal{F}$:

x_f, y_f, the position (abscissa and ordinate) of aircraft f.
v_f, z_f, current horizontal velocity and altitude level of aircraft f, respectively.
v^*_f, z^*_f, m^*_f, initial velocity, flight level and angle of motion configuration for aircraft f, respectively.
$\underline{v_f}, \overline{v_f}$, minimum and maximum velocity for aircraft f, respectively.
r_f, safety radius for each aircraft f, usually 2.5 nautical miles (nm).
n^v_f, n^a_f number of changes in velocity and altitude in the sector for aircraft f until the new execution, respectively.
$c^{q^+}_f, c^{q^-}_f, c^j_f, c^v_f, c^a_f, c^{\hat{v}}_f$, costs for positive and negative velocity changes, number of levels changing, velocity and altitude changing for aircraft f, respectively.

Variables

For all $f \in \mathcal{F}$:

q_f, velocity variation for aircraft f. This variable is real, and we divide it in two nonnegative variables, say, q^+_f and q^-_f, such that $q_f = q^+_f - q^-_f$ as it is standard in optimization, where q^+_f and q^-_f are the positive and negative velocity variation for aircraft f.
a_f, 0-1 variable that takes value 1 if aircraft f changes its velocity at the end of the current execution and, otherwise, it is zero.
b_f, 0-1 variable that takes value 1 if aircraft f changes its altitude at the end of the current execution and, otherwise, it is zero.
ρ_f, nonnegative integer variable that shows the number of levels that the aircraft f ascends or descends.

For all $f \in \mathcal{F}$ and $z \in \mathcal{Z}$:

ν^z_f, 0-1 variable that takes value 1 if aircraft f is at altitude level z at the end of the current execution and, otherwise, it is zero.

For all $i, j \in \mathcal{F} : i < j$ and the common feasible altitude leves for aircraft i and j, and $n = 1, \ldots, 5$:

δ^n_{ijz} auxiliary 0-1 variables to model or-constraint types.

The model is as follows, see [1] for the details,

$$\min w_1 \sum_{f \in \mathcal{F}} \left(\frac{c^{q^+}_f q^+_f}{\overline{v}_f - \underline{v}_f} + \frac{c^{q^-}_f q^-_f}{\overline{v}_f - \underline{v}_f} \right) + w_2 \sum_{f \in \mathcal{F}} c^j_f \rho_f + w_3 \sum_{f \in \mathcal{F}} (c^v_f n^v_f a_f + c^a_f n^a_f b_f) \quad (2)$$

subject to:

$$\underline{v_f} \leqslant v_f + q_f \leqslant \overline{v_f} \quad \forall f \in \mathcal{F} \quad (3)$$

$\forall i, j \in \mathcal{F} : i < j, \forall z \in \mathcal{Z}^i \cap \mathcal{Z}^j$:

$$(v_i + q_i)\big(\cos(m_i^*)(1 - pc_{ij}) - \sin(m_i^*)pc_{ij}\big) - \\ (v_j + q_j)\big(\cos(m_j^*)(1 - pc_{ij}) - \sin(m_j^*)pc_{ij}\big) \leqslant (\overline{v_i} + \overline{v_j})(1 - \delta^1_{ijz}) \quad (4)$$

$$-(v_i + q_i)\big(h_i(1 - pc_{ij}) + \hat{h}_i pc_{ij}\big) + (v_j + q_j)\big(h_j(1 - pc_{ij}) + \hat{h}_j pc_{ij}\big) \\ \leqslant \big((\overline{v_i}|h_i| + \overline{v_j}|h_j|)(1 - pc_{ij}) + (\overline{v_i}|\hat{h}_i| + \overline{v_j}|\hat{h}_j|)pc_{ij}\big)(1 - \delta^1_{ijz}) \quad (5)$$

$$(v_i + q_i)\big(\cos(m_i^*)(1 - pc_{ij}) - \sin(m_i^*)pc_{ij}\big) - \\ (v_j + q_j)\big(\cos(m_j^*)(1 - pc_{ij}) - \sin(m_j^*)pc_{ij}\big) \leqslant (\overline{v_i} + \overline{v_j})(1 - \delta^2_{ijz}) \quad (6)$$

$$(v_i + q_i)\big(k_i(1 - pc_{ij}) + \hat{k}_i pc_{ij}\big) - (v_j + q_j)\big(k_j(1 - pc_{ij}) + \hat{k}_j pc_{ij}\big) \\ \leqslant \big((\overline{v_i}|k_i| + \overline{v_j}|k_j|)(1 - pc_{ij}) + (\overline{v_i}|\hat{k}_i| + \overline{v_j}|\hat{k}_j|)pc_{ij}\big)(1 - \delta^2_{ijz}) \quad (7)$$

$$-(v_i + q_i)\big(\cos(m_i^*)(1 - pc_{ij}) - \sin(m_i^*)pc_{ij}\big) + \\ (v_j + q_j)\big(\cos(m_j^*)(1 - pc_{ij}) - \sin(m_j^*)pc_{ij}\big) \leqslant (\overline{v_i} + \overline{v_j})(1 - \delta^3_{ijz}) \quad (8)$$

$$(v_i + q_i)\big(h_i(1 - pc_{ij}) + \hat{h}_i pc_{ij}\big) - (v_j + q_j)\big(h_j(1 - pc_{ij} + \hat{h}_j pc_{ij})\big) \\ \leqslant \big((\overline{v_i}|h_i| + \overline{v_j}|h_j|)(1 - pc_{ij}) + (\overline{v_i}|\hat{h}_i| + \overline{v_j}|\hat{h}_j|)pc_{ij}\big)(1 - \delta^3_{ijz}) \quad (9)$$

$$-(v_i + q_i)\big(\cos(m_i^*)(1 - pc_{ij}) - \sin(m_i^*)pc_{ij}\big) + \\ (v_j + q_j)\big(\cos(m_j^*)(1 - pc_{ij}) - \sin(m_j^*)pc_{ij}\big) \leqslant (\overline{v_i} + \overline{v_j})(1 - \delta^4_{ijz}) \quad (10)$$

$$-(v_i + q_i)\big(k_i(1 - pc_{ij}) + \hat{k}_i pc_{ij}\big) + (v_j + q_j)\big(k_j(1 - pc_{ij}) + \hat{k}_j pc_{ij}\big) \\ \leqslant \big((\overline{v_i}|k_i| + \overline{v_j}|k_j|)(1 - pc_{ij}) + (\overline{v_i}|\hat{k}_i| + \overline{v_j}|\hat{k}_j|)pc_{ij}\big)(1 - \delta^4_{ijz}) \quad (11)$$

$$\delta^1_{ijz} + \delta^2_{ijz} + \delta^3_{ijz} + \delta^4_{ijz} + \delta^5_{ijz} = 1 - p_{ij} \quad (12)$$

$\forall i, j \in \mathcal{F} : i < j, \forall z \in \mathcal{Z}^i \cap \mathcal{Z}^j$:

$$\delta^5_{ijz} = 1 \qquad \text{if } hth_{ij} + sc_{ij} \geqslant 1 \quad (13)$$

$$\nu^z_i + \nu^z_j \geqslant \delta^5_{ijz} - 1 \quad (14)$$

$$\nu^z_i + \nu^z_j \leqslant 2 - \delta^5_{ijz} \quad (15)$$

$\forall f \in \mathcal{F}:$

$$\sum_{z \in \mathcal{Z}^f} \nu_f^z = 1 \tag{16}$$

$$-\varepsilon(1-a_f)+\varepsilon \leqslant q_f^+ + q_f^- \tag{17}$$

$$q_f^+ + q_f^- \leqslant (\overline{v_f} - \underline{v_f})a_f \tag{18}$$

$$1-\nu_f^{z_f} = b_f \tag{19}$$

$$\sum_{z \in \mathcal{Z}^f} z\nu_f^z - z_f^* \leqslant \rho_f \tag{20}$$

$$z_f^* - \sum_{z \in \mathcal{Z}^f} z\nu_f^z \leqslant \rho_f \tag{21}$$

$\forall f \in \mathcal{F}:$

$$q_f \in \mathbb{R} \tag{22}$$

$$q_f^+, q_f^- \in \mathbb{R}^+ \tag{23}$$

$$\rho_f \in \mathbb{Z}^+ \tag{24}$$

$\forall f, i, j \in \mathcal{F} : i < j, \forall z \in \mathcal{Z}:$

$$\nu_f^z, a_f, b_f, \delta_{ijz}^1, \delta_{ijz}^2, \delta_{ijz}^3, \delta_{ijz}^4, \delta_{ijz}^5 \in \{0,1\}. \tag{25}$$

The objective function (2) to minimize considers the following terms: Velocity variations, number of altitude levels that an aircraft climbs or descends and number of maneuvers done for each aircraft. Constraints (3) give the velocity bounds. Constraints (4)-(12) detect conflicts that can be solved with velocity changes. Constraints (13)-(15) consider the altitude changes in the model. Constraints (16) force aircrafts to fly at one and only one altitude level. Constraints (17)-(19) update the number of changes in velocity and altitude, respectively. Constraints (20)-(21) define the number of altitude levels that an aircraft ascends or descends. The type of the variales in the model are given in (22)-(25).

3.2 Coordinating cases

For avoiding the conflict situations that can take place close to the aerial sector, a ring around it must be considered. Then, the following parameter can define if an aircraft is inside or outside the aerial sector under the supervision of a given ATC.

$$os_f = \begin{cases} 1, & \text{if aircraft } f \text{ is out of the aerial sector} \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

Notice that a conflict situation will be detected by the proposed model even if it occurs out the aerial sector. So, three different cases can occur (see Fig. 2):

- Case 1: If $\exists i < j \in \mathcal{F} : os_i = os_j = 0$. It is the case of the aircraft denoted by a circle in Fig. 2, where both os parameters are equal to zero, and, then, the model VAC works efficiently and the same for the CVAC model (see below).

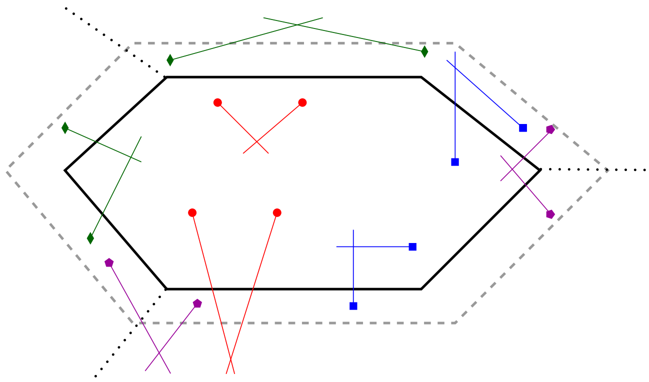


Fig. 2 Cases on the border

- Case 2: If $\exists i < j \in \mathcal{F} : os_i = 1$ and $os_j = 0$ or vice-versa. It is the case of the aircraft denoted by a square in Fig. 2, where one of them is out of the aerial sector, so model CVAC will provide the optimal new configuration for the aircraft to help the ATCs under consideration.
- Case 3: If $\exists i < j \in \mathcal{F} : os_i = os_j = 1$. It is the case of the aircraft denoted by a rhombus or a pentagon in Fig. 2, where both os parameters are equal to one. There are two situations that can occur in this case between the two aircraft involved in the conflict under consideration:
  1. The two aircraft share the same aerial sector but they are not in the sector under consideration. It is the case of the aircraft denoted by a rhombus in Fig. 2, such that the ATC of the corresponding aerial sector should solve the conflict since the two aircraft are under his/her supervision. The model CVAC works efficiently providing to each ATC with the optimal new configuration.
  2. The two aircraft belong to different aerial sectors different to the sector under consideration. It is the case of the aircraft denoted by a pentagon in Fig. 2, such that the model CVAC can help to the ATCs that are controlling the different aerial sectors. In this case the ATCs, should warn each other that there is a conflict between aircraft out of its area of responsibility.

3.3 Aircraft changing their altitude levels

Another important point that is not considered in the VAC model is the situation where an aircraft is changing more than one altitude level in a given maneuver. This can involve conflict situations in those altitude levels and they must be avoided, something that is to be done in the CVAC model. For this purpose the following parameter is required for knowing the number of altitude levels that the aircraft is taking up at the moment of the maneuver,

$$ca_f = \begin{cases} +a & \text{if the aircraft } f \text{ is changing its altitude climbing and is taking up} \\ & a \text{ altitude levels (the initial one included)} \\ -a & \text{if the aircraft } f \text{ is changing its altitude descending and is taking} \\ & \text{down } a \text{ altitude levels (the initial one included)} \\ 1 & \text{otherwise, i.e., the aircraft } f \text{ is not changing its altitude level.} \end{cases}$$

Notice that at the aircraft maneuver (i.e., changing its altitude level), the velocity variation (i.e., q_f^+ and q_f^-), the scheme that prevents its change in velocity or altitude level (i.e., a_f and b_f, respectively) and the number of levels that the aircraft has to climb of descend (i.e., ρ_f) must be fixed to zero, that is

$$q_f^+, q_f^-, a_f, b_f, \rho_f = 0 \quad \text{if } ca \neq 1.$$

Additionally, the corresponding variable ν_f^z takes the value 0 if aircraft f is not taken up the altitude level z; otherwise, it takes the value 1, i.e.,

$$\nu_f^z = 1, \qquad \text{if } ca_f \neq 1 \text{ and aircraft } f \text{ is taking up level } z$$

On the other hand, the following constraints must appended to the model,

$$\nu_f^{z_f^* + (n-1)\mathrm{sign}(ca_f)} = 1 \quad \text{if } ca_f \neq 1 \quad \forall f \in \mathcal{F}, \forall n = 1, \ldots, |ca_f|,$$

since the variables ν are fixed to 1 for those altitude levels that the aircraft takes up. Notice that if $\mathrm{sign}(ca_f)$ is positive then the aircraft is climbing and, negative if the aircraft is descending.

It is important to point out that if we do not allow the aircraft to do maneuvers while they are changing their altitude level, a conflict situation could occur. In these cases, the model will be infeasible and, then, a warning must alert the ATC in order to avoid this situation when the aircraft have finished the maneuvers. This should not happen if the model is solved continuously, since in the previous resolution every conflict situation must be solved. But it is important to consider this anomalous case as a warning.

Notice that if an aircraft is climbing or descending, the horizontal velocity to consider is smaller than the aircraft velocity. In this paper we consider the horizontal velocity by projecting the aircraft velocity vector taking into account the angle of climbing or descending φ_f for each aircraft $f \in \mathcal{F}$ which is normally known. Then, the horizontal velocity for each aircraft f can be expressed

$$v_f = \begin{cases} v_f \cos(\varphi_f) & \text{if the aircraft } f \text{ is climbing or descending} \\ v_f & \text{otherwise.} \end{cases}$$

Once this change is done, the same has to be done for $\overline{v}_f$ and $\underline{v}_f$ for all $f \in \mathcal{F}$.

The cases in which an aircraft is climbing or descending several levels simultaneously are allowed in the CVAC model. If an aircraft is climbing or descending, it takes up more than one level and, then, constraint (16) has to be replaced by the following one,

$$\sum_{n \in \mathcal{Z}^f} \nu_f^n = |ca_f| \quad \forall f \in \mathcal{F}.$$

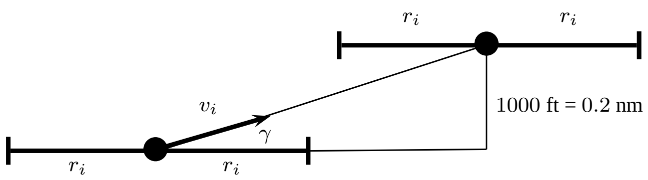


Fig. 3 Geometric construction for calculating parameter e_{ij}

Furthermore, the variable b_f has to be fixed to 0 in those cases when an aircraft is changing its altitude level. The same occurs with variables a_f and ρ_f. Consequently, the constraints (17)-(21), that force variables a_f and b_f to be 0 or 1, has to be inactive for those aircraft that are taking up more than one level. These assumptions are taken into account in the CVAC model for technical difficulties in real problems. However, the model could allow velocity maneuvers but assuming the horizontal velocity while the aircraft is changing its altitude level.

Finally, it is worth to considering the parameter e_{ij} in order to decide taking or not taking the pair of aircraft i and j in the conflict detection and resolution constraints. This parameter is used as a bound, such that if the distance between the aircraft i and j is bigger than this bound, then the two aircraft will not be considered in the conflict detection and resolution constraints and, otherwise, they will be. It is useful for avoiding changes in possible conflicts between two aircraft that only coincide in one altitude level for a short time while one of them is changing its altitude level. When two aircraft are far each other, it can be used for the reduction of model dimensions and, consequently the computing time. It is defined as follows,

$$e_{ij} = \begin{cases} \cos\left(\frac{0.2}{|\sin\gamma|}\right) + r_i + v_j\left(\frac{0.2v_i}{|\sin\gamma|}\right) + r_j & \text{if } ca_i \neq 1 \text{ or } ca_j \neq 1 \\ e & \text{otherwise,} \end{cases} \tag{27}$$

where the first part of the expression gives the minimum distance for not having a conflict while one of the aircraft i and j is climbing or descending. If the climbing or descending angle is known (γ), the necessary distance to cover by the aircraft to get the new altitude level is $\frac{0.2}{|\sin\gamma|}$ (see Fig. 3). (Notice that $1000ft \approx 0.2nm$). The required time to cover this distance is $\frac{0.2v_i}{|\sin\gamma|}$. The horizontal distance covered by the other aircraft j in the covered time by aircraft i to get the new altitude level is $v_j\frac{0.2v_i}{|\sin\gamma|}$. The other terms in expression (27) are the two safety radii r_i and r_j for aircraft i and j, respectively.

Parameter e can be chosen by the ATC but if e is bigger than the possible maximum distance in the aerial sector, all pair of aircraft will be considered, except those aircraft where at least one of them is changing its altitude level. In order to taking or not taking into consideration a pair of aircraft in the conflict detection and resolution constraints, the parameter p_{ij} is used, such that its value is 0 if the pair of aircraft has to be considered and 1, otherwise. This parameter forces the rhs constraint (12) equal to 0 and, then, it invalidates the conflict resolution constraints (14)-(11) in model VAC.

4 The Coordinated Velocity and Altitude Changes (CVAC) model

The aim of the model consists of solving the possible conflict situations among a set of aircraft. This model takes into account if an aircraft is climbing or descending and, in consequence, taking more than one altitude level. Furthermore, this model coordinates the maneuvers in those aircraft out of a given ATC responsibility if he/she detects some conflict situations. The objective function is the same as the function to minimize in model VAC (2). The constraints are as follows,

Constraints (3) – (15), (22) – (25)

$\forall f \in \mathcal{F}$:

$$\sum_{n \in \mathcal{Z}^f} \nu_f^n = |ca_f| \tag{28}$$

$\forall f \in \mathcal{F},$ such that $ca_f = 1$:

$$-\varepsilon(1 - a_f) + \varepsilon \leqslant q_f^+ + q_f^- \tag{29}$$

$$q_f^+ + q_f^- \leqslant (\overline{v_f} - \underline{v_f})a_f \tag{30}$$

$$1 - \nu_f^{z_f} = b_f \tag{31}$$

$$\sum_{z \in \mathcal{Z}^f} z\nu_f^z - z_f \leqslant \rho_f \tag{32}$$

$$z_f - \sum_{z \in \mathcal{Z}^f} z\nu_f^z \leqslant \rho_f. \tag{33}$$

Constraints (28) force each aircraft to take as many altitude levels as needed. In those cases in which an aircraft is changing its altitude level, there are as many variables ν_f^n equal to one as altitude levels the aircraft is taking, otherwise, only one variable is equal to 1. Constraints (31)-(33) are the same ones as in the VAC model, but they are only activated in the model if $ca_f = 1$, i.e., if an aircraft is not changing its altitude level. Those constraints count the number of velocity and altitude level changes for each aircraft as well as the number of altitude levels that it changes.

5 Computational results

The model CVAC can be optimized by using any MILO state-of-the-art optimization engine, in our case we use CPLEX v12.1 [8], default options in the following HW/SW platform: Intel Core 2 Duo P8400, 2.26 GHz, 2-GB random access memory (RAM), Microsoft Windows 7 Professional OS.

5.1 Illustrative instance

For testing the validity of the model, the instance depicted in Fig. 4(a) is used where a square aerial sector is surrounded by four different aerial ones. A ring around the aerial sector under consideration is drawn and all the aircraft in that ring are considered in the model. Four different altitude levels are used for resolving conflict situation and some aircraft have different altitude levels. Notice that those aircraft are denoted in the figure with two color geometric figures where the small and the big ones denote the departure and arrival altitude levels respectively. In the cases in which an aircraft is changing its altitude level, the velocity is taken such that we force a conflict situation in order to prove that the aircraft changing altitude does not allow to change its velocity, such that it forces to the other aircraft involved in the conflict to change it (see for instance aircraft 4, 5 and 6 or 26, 27 and 28).

Aircraft 1, 2, 3 have a multiple conflict situation that can be solved by velocity changes. They are under the responsibility of the same ATC in the current aerial sector (see Case 1 in Section 3.2). Aircraft 4, 5, 6 have a multiple conflict but aircraft 4 is not able to change neither altitude nor velocity. Notice that the pair of aircraft 29-30 is similar. Aircraft 7 and 8 have a conflict situation solved by a velocity change. Notice that they are in the same aerial sector and then, the corresponding ATC knows the conflict and will solve it (see Case 3.1 in Section 3.2). Aircraft 9 and 10 have a conflict situation solved by an altitude level change. They are under the responsibility of different ATCs, and then, the corresponding ATCs have to be advised by the ATC of the aerial sector under consideration (see Case 3.2 in Section 3.2). Notice that the same situation happens with pairs of aircraft 11-12, 13-14 and 15-16. Aircraft 17 and 18 have a conflict situation solved by a velocity change. They are in different aerial sectors and the ATC of the corresponding one will advice the other one to force aircraft 17 to change the velocity (see Case 2 in Section 3.2). This is the same case as pairs of aircraft 19-20, 21-22, and the multiple conflict among aircraft 23-24-25. Notice that the multiple conflict among aircraft 26-27-28 is solved rightly, as well as among aircraft 31-32-33-34 in which aircraft 32 is taking the four levels and among 35-36-37.

The main results of the CVAC model are as follows, see the solution in Fig. 4(b),

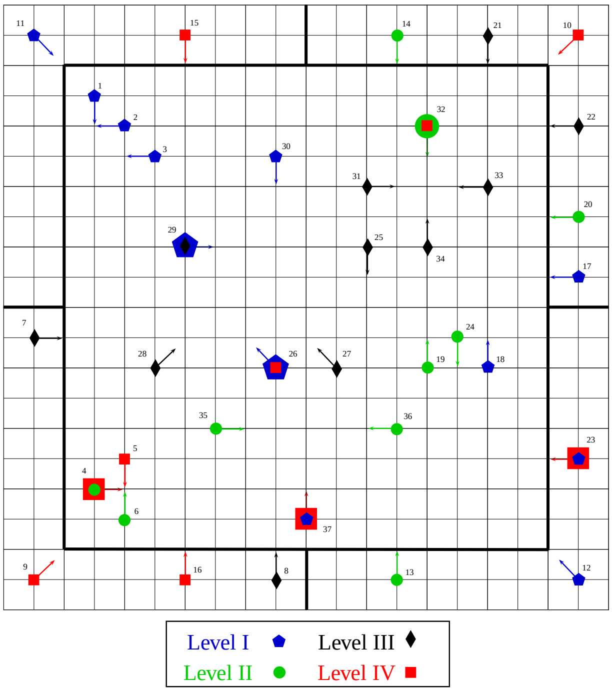
- The computing time is 15.66 secs.
- There have been 16 and 8 conflict situations and they are solved by velocity and altitude changes, respectively.
- There are 231, 330, 25 and 80 cases of types 1, 2, 3.1 and 3.2, respectively (see Section 3.2), and, then, the corresponding ATCs have to be coordinated as explained in Section 3.2.
- As a consequence, every conflict situation is solved by coordinating the work of the ATCs.

5.2 Computational experience for large-scale instances

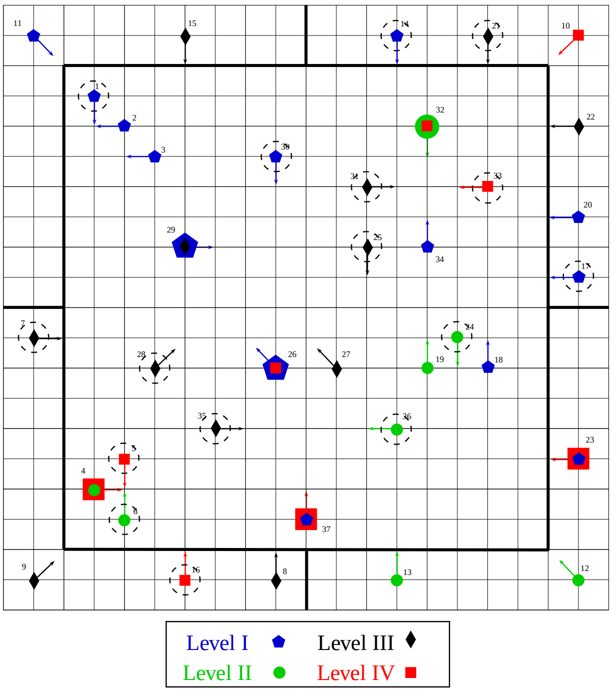
We report the main results for a testbed of 25 random instances for each of the 21 pilot cases under consideration that differ in the number of aircraft and considered altitude levels. In total, 3675 simulations are performed. The following technical elements are taken into account for the experiment:

- The situation in the form of the aerial sector depicted in Fig. 4(a) is taken as support.
- The positions, velocities, angle of motion are uniformly generated.

Table 1 shows the dimensions (in average) for each pilot case and Table 2 shows the main results. The headings are as follows:



(a) Initial situation.



(b) Final situation CVAC solution.

Fig. 4 Illustrative instance for testing the CVAC model

Table 1 CVAC model dimensions of the testbed.

Case	m	n_l	n_i	n_{0-1}	den	m^*	n^*	den^*
C020-05	10569.6	60.0	20.0	4890.0	0.09	854.0	539.9	0.73
C020-07	14754.8	60.0	20.0	6830.0	0.07	1314.3	880.3	0.46
C020-10	21025.0	60.0	20.0	9740.0	0.05	1801.0	1212.0	0.34
C025-05	16642.0	75.0	25.0	7675.0	0.06	1062.4	661.8	0.57
C025-07	23241.8	75.0	25.0	10725.0	0.04	1647.0	972.0	0.41
C025-10	33135.4	75.0	25.0	15300.0	0.03	1278.0	852.0	0.45
C030-05	24098.6	90.0	30.0	11085.0	0.04	1579.0	958.1	0.40
C030-07	33668.4	90.0	30.0	15495.0	0.03	2134.2	1285.4	0.31
C030-10	48025.0	90.0	30.0	22110.0	0.02	2987.0	1890.6	0.22
C035-05	32917.0	105.0	35.0	15120.0	0.03	1797.5	1070.8	0.35
C035-07	46004.6	105.0	35.0	21140.0	0.02	2102.5	1222.1	0.33
C035-10	65649.6	105.0	35.0	30170.0	0.02	3788.8	2310.2	0.18
C040-05	43128.6	120.0	40.0	19780.0	0.02	2075.9	1273.9	0.30
C040-07	60304.6	120.0	40.0	27660.0	0.02	4262.3	2513.5	0.16
C040-10	86046.8	120.0	40.0	39480.0	0.01	5991.0	3598.5	0.12
C045-05	54697.4	135.0	45.0	25065.0	0.02	2610.7	1594.5	0.23
C045-07	76505.8	135.0	45.0	35055.0	0.01	4770.1	2853.9	0.14
C045-10	109141.2	135.0	45.0	50040.0	0.01	4620.3	2787.8	0.14
C050-05	67645.0	150.0	50.0	30975.0	0.01	3013.2	1814.7	0.20
C050-07	94596.0	150.0	50.0	43325.0	0.01	3939.3	2384.5	0.16
C050-10	135017.2	150.0	50.0	61850.0	0.01	5329.6	3208.1	0.12

- Case: Pilot case configuration where CAAA-ZZ denotes the number of aircraft (AAA) and altitude levels (ZZ)
- m: Number of constraints
- n_l: Number of continuous variables
- n_i: Number of integer variables
- n_{0-1}: Number of $0-1$ variables
- den: Constraint matrix density (%)
- m^*, n^*, and den^*: Number of constraints, variables, and constraint matrix density (%), respectively, after CPLEX preprocessing
- z_{lp}: LP relaxation solution value
- z_s: Strong LP relaxation solution value after CPLEX preprocessing
- z_{ip}: Original problem solution value
- GAP_{lp}: $(z_{ip} - z_{lp}/z_{ip})$%
- GAP_s: $(z_{ip} - z_s/z_{ip})$%
- n_b: Number of branchings
- nn: Number of CPLEX branch-and-cut nodes
- t_{lp}: Time (in seconds) to obtain the z_{lp} value
- t_s: Time (in seconds) to obtain the z_s value
- t_{ip}: Time (in seconds) to obtain the z_{ip} value;
- nc: Total number of cuts performed by CPLEX.

The first observation that can be made in Table 1 is the strength of the CPLEX preprocessing by comparing the columns m and m^*, and n and n^*. However, the tight model that results has still big dimensions. Remind that the integer variables in each pilot case are ρ_f and they vary from 0 to the number of levels used in the aerial sector (for instance, notice that for cases C020-07, the most unfavorable variation rank of the integer variables is from 0 to 7 and, it depends on the level in which the aircraft is flying).

Table 2 CVAC model computational results of the testbed.

Case	z_{lp}	z_s	z_{ip}	GAP_{lp}	GAP_s	nb	nn	t_{lp}	t_s	t_{ip}	nc
C020-05	0.0821	4.2403	4.3345	43.06 98.11 100.00	0.00 2.17 18.47	6	48.50	0.04	0.69	0.75	21.4
C020-07	2.8800	2.9600	2.9606	0.00 2.72 100.00	0.00 0.02 0.70	0	0.00	0.05	1.11	1.02	1.0
C020-10	0.0000	1.3253	1.3902	100.00 100.00 100.00	4.67 4.67 4.67	0	0.00	0.07	1.24	1.46	117.0
C025-05	0.8455	5.0547	5.1490	63.61 83.58 100.00	0.00 1.83 7.28	6	6.50	0.06	1.05	1.01	58.8
C025-07	1.6424	4.8203	4.8456	38.33 66.11 100.00	0.00 0.52 4.37	1	22.00	0.08	1.66	1.64	81.0
C025-10	0.0000	1.7491	1.7672	0.00 100.00 100.00	0.00 1.03 1.05	0	0.00	0.09	0.98	0.88	13.8
C030-05	0.6963	6.2340	6.3637	18.81 89.06 100.00	0.00 2.04 18.05	8	142.13	0.08	1.37	1.30	110.2
C030-07	0.0000	6.8055	6.9474	100.00 100.00 100.00	0.00 2.04 4.13	9	46.00	0.10	1.56	1.36	103.8
C030-10	0.8800	4.0624	4.1812	65.27 78.95 100.00	0.00 2.84 6.07	0	0.00	0.14	1.52	1.14	27.0
C035-05	2.6932	9.6895	9.9749	37.65 73.00 100.00	0.00 2.86 10.02	8	101.00	0.10	1.71	1.74	116.1
C035-07	1.4400	4.2351	4.7442	59.45 69.65 100.00	0.00 10.73 42.69	13	15.00	0.13	1.11	1.43	107.6
C035-10	0.4400	5.3608	5.4679	91.25 91.95 100.00	0.00 1.96 7.75	23	40.09	0.17	1.97	1.96	268.6
C040-05	4.4078	14.4140	14.7690	49.17 70.15 100.00	0.00 2.40 12.26	17	122.94	0.13	1.38	1.83	163.1
C040-07	2.7600	6.7062	6.7500	57.57 59.11 100.00	0.00 0.65 19.45	1	547.00	0.18	2.27	1.92	53.6
C040-10	0.3657	5.2626	5.4651	55.47 93.31 100.00	0.00 3.71 9.76	8	24.00	0.24	2.94	2.61	220.7
C045-05	5.4069	17.1351	17.5949	48.04 69.27 96.08	0.02 2.61 13.99	20	210.60	0.16	2.56	2.95	240.1
C045-07	1.7626	11.9864	12.0977	46.13 85.43 100.00	0.00 0.92 2.87	9	182.89	0.22	2.65	2.56	323.7
C045-10	2.0819	7.7827	8.0513	38.64 74.14 100.00	0.00 3.34 52.80	12	20.17	0.28	3.43	3.41	336.5
C050-05	6.0764	16.0928	16.7397	37.93 63.70 93.82	0.00 3.86 18.66	15	279.40	0.18	2.54	3.65	208.7
C050-07	2.0025	12.0479	12.2295	52.71 83.63 100.00	0.06 1.49 9.75	8	120.13	0.25	2.58	3.03	183.8
C050-10	1.7527	9.5476	9.7165	36.41 81.96 100.00	0.00 1.74 20.21	7	136.86	0.33	5.07	4.26	194.6

Table 2 also reports the minimum, average and maximum GAP_{lp} and GAP_s. Notice the very small GAP_s for all the instances. It shows the tightness of the model plus the CPLEX cuts. Notice also that *nb* is very small, in fact it is only different from zero in 171 of the 3675 simulations, then, the elapsed time is very astonishing small as well.

6 Conclusions

A model so called CVAC based on MILO have been presented in order to avoid conflict situations in the airspace between a given number of aircraft. It provides a useful tool for coordinating the ATCs in different aerial sectors when conflict situations take place close to the frontier of them. It also takes into account that an aircraft can climb or descend several altitude levels. Finally, an extensive computational experience has been reported for assessing the usefulness of the model. As a result it can be observed the tightness of the model such that the computing time is so small even for large-scale instances that the tool can be very useful in the real-life work of ATCs.

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