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Conflict Avoidance: 0-1 linear models for Conflict Detection & Resolution

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Abstract The Conflict Detection and Resolution Problem for Air Traffic Flow Management consists of deciding the best strategy for airborne aircraft so that there is guarantee that no conflict takes place, i.e., all aircraft maintain the minimum safety distance at every time instant. Two integer linear optimization models for conflict avoidance between any number of aircraft in the airspace are proposed, the first being a pure 0-1 linear which avoids conflicts by means of altitude changes, and the second a mixed 0-1 linear whose strategy is based on altitude and speed changes. Several objective functions are established. Due to the small elapsed time that is required for solving both problems, the approach can be used in real time by using state-of-the-art mixed integer linear optimization software.
Keywords: Air Traffic Flow Management, Conflict Avoidance, Mixed 0-1 Linear Optimization, Pure 0-1 Linear Optimization, Conflict Detection and Resolution.

1 Introduction. Brief state-of-the-art

Air traffic in Europe and the USA has undergone an astonishing growth during recent years, and a further 50% increase is expected by 2018 over the traffic in 1999, see [2]. In this scenario, the aim of Air Traffic Flow Management consists of extending the airspace allowing the so called "Free Flight", where the pilots and the airlines are able to decide freely the flight plan, keeping in touch with the air traffic controller. To maintain safety the air flow, the Conflict Detection and Resolution Problem (CDR) or Conflict Avoidance Problem (CA) is currently attracting the interest of air transportation service providers and has been studied extensively.

Unfortunately, the CDR has proven to be a hard problem to solve. To give some idea, the way in which to represent the actual trajectory of an aircraft is by means of a dynamic model that has to take into account, as an example, the following relationships: speed of the aircraft will depend on the wind direction and altitude on which it flies (such that the higher a aircraft flies, the lesser the air is around it and thus it needs

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to go faster to maintain its position); acceleration depends on the speed (e.g., at lower speeds, a plane can reach higher acceleration ratios) and altitude, and so on. Notice that the aircraft is losing mass throughout the flight as fuel burns, and this influences the speed and acceleration of the aircraft (and, viceversa, the speed influences the consumption of fuel and thus the mass loss), etc. Good introductions to flight dynamics modelization can be found in [5, 10, 25]. Finally, CDR has to deal with the simultaneous trajectories of (possibly) many aircraft. Moreover, we must bear in mind that given the intended trajectories, captured in the flight plans, some uncertainty regarding the actual trajectories of the aircraft is unavoidable, which makes CDR harder to solve. Trying to address all these issues within a mathematical optimization model would lead today to an unmanageable problem (in terms of computing effort, i.e., elapsed time and memory requirements).

Different methods have appeared in the literature. What follows is a brief state-of-the-art on the subject. Magister (2002) [16] presents two different models: The first applies to conflict detection. The second is related to conflict resolution to solving the conflict by lowering one of the two aircraft that are taken into consideration in the conflict. In addition, the same author [17] describes the conflict resolution problem in great detail and makes a quantitative analysis of avoidance procedures.

One of the most recent works, see Jardin (2005) [14], presents some algorithms for strategic conflict detection, based on the use of a 4-dimensional space and time grid to represent the airspace. This approach to compute conflict detection was previously introduced by Jardin (2003) [12, 13], where he uses a 3-dimensional grid (two horizontal spatial dimensions and time). Prandini and Hu (2008) [22] present a stochastic approximation scheme to estimate the probability that a single aircraft will enter a forbidden area of the airspace within a finite time horizon. A numerical algorithm is also proposed for computing an estimate of the probability that the aircraft might enter an unsafe region of the airspace or come too close to another aircraft. Hu, Pradini and Sastry (2005) [9] introduce a model of a two-aircraft encounter with a random field term to address correlation of the wind perturbations to the aircraft motions. Based on this model, they estimate the probability of conflict by using a Markov chain approximation scheme. The same authors [8] study the problem that consists of evaluating whether the flight plan assigned to an aircraft is safe. They introduce a kinematic model of the aircraft motion in a three dimensional wind field with spatially correlated random perturbations.

Kuchar and Yang (2000) [15] present a survey of CDR modeling methods. The Traffic alert and Collision Avoidance System (TCAS), which is an implementation of the Airborne Collision Avoidance System mandated by the International Civil Aviation Organization, searches through a set of potential climb or descent manoeuvres and selects the least-aggressive one that still provides adequate protection; see [24]. Pannequin et al. (2007) [21] present an approach to the problem with severe weather conditions by using a Nonlinear Model Predictive Control scheme. Christodoulou and Costoulakis (2004) [3] propose a Mixed Integer Nonlinear Programming (MINLP) model for solving the conflict problem. It allows for speed changes and heading angle control optimization to be solved by using standard optimization software, but it may require, once again, more computing effort than may be affordable. A MINLP model proposed by Christodoulou and Kodaxakis (2006) [4], with linear objective function and nonlinear constraints only allows speed changes as manoeuvres. Treleaven (2007) [26] assumes that aircraft travel at the same altitude and at the same speed, using only horizontal manoeuvres for conflict resolution; two, three and multiple intersecting flows are considered.

Obstacle avoidance by using the linearized constrained Uninhabited Aerial Vehicle (UAV) dynamic has been modeled by Richards and How (2002) [23]. Centralized Model Predictive Control has been widely developed for constrained systems and has been applied to the co-operative control of multiple vehicles.

Pallottino, Feron and Bicchi (2002) [20] propose two mixed integer models for CDR, one allows speed changes and the other one allows angle changes, both on the same plane. These models are based on a geometric approach. The second model assumes that the speed is the same for all aircraft, such that each one can manoeuvre only once with an instantaneous heading angle deviation that can be positive (left turn), negative (right turn) or null (no deviation). It does not consider returning to the original route, nor does it explain how the aircraft, after a manoeuvre, reaches its destination. The mixed 0-1 linear model presented in Alonso-Ayuso, Escudero and Martin-Campo (2010) [1] is inspired in [20], whose first model (speed changing) is extended to permit aircraft to change both their speed and altitude levels, such infeasible situations caused by the speed and "head to head" conflict are avoided. Moreover, all aircraft will be forced to return to the initial configuration when conflict situations are resolved and, finally, a pathological case unresolved in [20] is avoided. However, these two approaches are intended to be executed repeatedly, each execution within a short time horizon. The trajectories are assumed to be linear over a horizontal plane (even though flight level changes are allowed), which could be problematic since they rely on direction angles. Notice that projecting the trajectories onto a plane could appreciably change the actual angles, which makes these models suitable only for small airspace regions in the short term.

Hu, Pradini and Sastry (2002) [6] study the multi-aircraft encounters in a three dimensional environment and propose an algorithm for solving the two aircraft nonlinear optimization problem. For more than two aircraft, they consider what is called two-legged manoeuvres approach, such that a manoeuvre consists of two stages, moving at a constant speed and through a straight line during both stages. The original optimization problem is then reduced to a finite dimensional convex optimization problem with linearly approximated conflict-free constraints on the waypoints and a quadratic objective function. Path flightability is taken into account by introducing an upper bound on the speed and turning angle constraints, which can be expressed by using second order cone expressions. So, the optimization problem becomes a Second Order Cone Programming (SOCP) one. However, the assumptions on which the proposal are based (namely, every aircraft departs and arrives at the same time, all aircraft move linearly except for one heading angle change in the two-legged manoeuvre, etc.) force to apply the model recursively, which could make it unaffordable as an option in most practical cases, due to the non-linearity of its constraints and objective function. In [7], the same authors study the problem as above, although constrained to the plane, proposing a randomized convex optimization algorithm to find numerically the optimal multi-legged manoeuvres (with an arbitrary number of stages).

Mao, Feron and Billimoria (2001) [18] set out geometric constructions to solve the problem, including aircraft one-by-one until representing the total number of aircraft, considering the previous aircraft as obstacles and making a sequential process. Mao et al. (2005) [19] tackle the problem using instantaneous heading changes as manoeuvres between two aircraft. The approach extends the results of the previous work in which the manoeuvres that have been considered are not physically realistic.

The main contributions of our work are as follows:

1. A new point of view has been adopted, so that it does not tackle the CDR problem by directly modeling the physical laws under which the aircraft have to fly. On the contrary, the approach requires some simple parameters which constraint the variations of a given flight plan in order to avoid conflicts, so that such laws are implicitly taken into account. Additionally, only linear models are required which can be computed in very small elapsed time.
2. We propose a scheme for conflict detection that would allow to decide if a certain manoeuvre for conflict avoidance should be applied or else all flight plans can be left as they are. Then, it would help to reduce the dimensions of the models aimed at finding such manoeuvres.
3. Two novel optimization models are proposed. The first one is a pure 0-1 linear model, whose aim consists of changing flight levels (i.e., forcing the aircraft to climb or descend in order to avoid conflicts). The second model is a mixed 0-1 linear one that solves the problem by changing aircraft flight levels and speed. Both models are very tight and, then, require very small elapsed time for solving even large-scale instances, so, the can be used in real time for realistic conflict detection and resolution problems.
4. We assume the aircraft flying on any kind of surface (particularly, a geoid), hence their trajectories are not restricted to be linear. So, the given flight plans may be either the rigid ones with fixed beacon points, the future freely decided flight plan in the context of ‘"Free Flight", or straight-line extrapolation of the current speed vector as in [1,20]. Speed is not restricted to be constant as it is the case in many of the approaches found in the literature. Additionally, our approach is specially suited for being used in long term time horizons as well in wider airspace regions than the preceding ones.

Based on our computational experience reported in Section 5, we can point out that our first model is tighter than the second one (and, then, it requires smaller computational effort), so, it allows to consider wider aerial zones with a higher set of aircraft and a longer time horizon than the second model. Nevertheless, this other model is quite efficient, according to the computational experience to report below. On the other hand, the first model has the drawback of only allowing flight level changes, a manoeuvre that may not be the preferred choice for many pilots and airlines, since these changes could cause annoyance to passengers and crew. Nevertheless, it will not be necessary in most real-life cases to accumulate many of such flight level changes and, so, this model will be useful and applicable in most practical situations. Further more, it may be the preferred manoeuvre, as opposed to speed changes, since the latter may imply greater fuel consumption and more risks than the former. To summarize, the models we propose are both efficient and useful in most real-life situations, the second being more comprehensive than the first one.

The remainder of the paper is organized as follows: Section 2 technically introduces the problem and some notation. Section 3 presents the first model, its preprocessing and its pure 0-1 formulation. Section 4 presents the second model, with some new elements, its preprocessing and its mixed 0-1 formulation. Section 5 reports the computational results for two testbeds of realistic airborne aircraft conflict instances. And, finally, section 6 concludes and outlines future work.

2 Problem description

A conflict is an event in which two or more aircraft are within an unsafe distance from one another at a given instant. The minimum safety distance is typically 5 nm (nautical miles) of horizontal distance between aircraft outside the TRACON (Terminal Radar Approach Control) and 3 nm inside the TRACON, or at least 1000 feet of vertical separation (the current en-route separation standard at lower altitudes).

Let us consider a set of aircraft $\mathcal{F}$. For each flight $f \in \mathcal{F}$, a dynamic trajectory model is required to project the states into the future in order to predict whether a conflict would occur. This projection may be based solely on current state information (e.g., a straight-line extrapolation of the current speed vector) or may be based on additional procedural information such as a flight plan. In both situations there is generally some uncertainty in estimating the future trajectory. It is represented via a finite sequence of waypoints, $\mathcal{W}_f$. A waypoint is a reference point in the physical space that consists of a tupla with latitudinal and longitudinal coordinates, generally with respect to a reference geoid. At each waypoint, we also know the scheduled speed for moving to the next waypoint. Let also define $\mathcal{W}_f^{'}$ and $\mathcal{W}_f^{-}$ as the the sets of all waypoints to transverse by flight f.

So, let us assume that the route path for each aircraft is broken down into segments (not necessarily with equal size), altitude (flight level) and speed through each one of these segments, such that the number of waypoints for every aircraft is sufficiently representative of the route. Thus, the distance between two given consecutive waypoints (i.e., the length of a segment) should be less than 5nm (according to the current en-route separation standard at lower altitudes), so, 2nm can be a reasonable distance.

Additionally, let $\mathcal{L}_i^f = \{\underline{z}_i^f, \underline{z}_i^f + 1, \ldots, \overline{z}_i^f\}$ denote the set of the allowed flight levels for aircraft f to traverse its ith waypoint, for $f \in \mathcal{F}$, $i \in \mathcal{W}_f$. In order to prevent infeasible flight level changes, let us define $\overline{V}_i^f$ ($\underline{V}_i^f$) as the max (min) number of flight levels that aircraft f is allowed to climb or descend from its ith waypoint to the next one, for $f \in \mathcal{F}$, $i \in \mathcal{W}_f^-$. Let also define t_i^f and z_i^f as the scheduled time and altitude of aircraft f while traversing its ith waypoint of its route, for $f \in \mathcal{F}$, $i \in \mathcal{W}_f$.

We will consider that a conflict takes place if two aircraft traverse two waypoints in their respective routes that are too close to one another, within a small interval of time. To determine the bounds of such interval let us resort to a conservative strategy, and define $m_{A_{i,j}^{f,k}} = \max\{|t_{i+1}^f - t_i^f|, |t_{j+1}^k - t_j^k|\}$ as the smallest time interval that is allowed for aircraft f and k to reach their next waypoints $i+1$ and $j+1$ from the waypoints i and j, respectively, $\forall f, k \in \mathcal{F}, (i,j) \in \mathcal{W}_f \times \mathcal{W}_k$

Finally, we can define a partition of the aircraft set $\mathcal{F} = \bigcup_{i\in\mathcal{I}} \mathcal{F}_i$, $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset, \forall i, j \in \mathcal{I}$, where $f \in \mathcal{F}_i \Rightarrow \mathcal{F}^f \subset \mathcal{F}_i, \forall f \in \mathcal{F}, \forall i \in \mathcal{I}$ for splitting the problem into subproblems.

So, the CDR problem to tackle consists of detecting all conflicts in the *alert zone* (being this one an aerial sector or even the whole airspace) and avoiding them by using a solution provided by very tight 0-1 linear optimization models that are solved by using a state-of-the art optimization engine. The proposed models suggest some changes (as few as possible) in altitude and speed of the aircraft scheduled flight plans.

3 Collision Avoidance via flight level changes

3.1 Conflict Detection

The scheme proposed for aircraft conflict detection is very similar for the two types of CDR problems to tackle in this work, namely, CA via flight level changes and CA via flight level and speed changes. It, obviously, helps to decide if a conflict can be avoided, if any, but it also helps to finding at which pair of waypoints a conflict would occur. Moreover, the conflict detection scheme have some differences between both approaches. The basic idea for the flight level change scheme is as follows.

For a pair of aircraft $(f,k) \in \mathcal{F} \times \mathcal{F}$, there is a *potential conflict* at the pair of waypoints $(i,j) \in \mathcal{W}_f \times \mathcal{W}_k$ if the following conditions hold:

1. The waypoints i and j have a smaller distance than the minimum allowed (i.e. $5nm$),
2. The time instants are such that $t_i^f < t_{j+1}^k$ and $t_j^k < t_{i+1}^f$, since suppose, on the contrary, that e.g., the second inequality does not hold, then, when aircraft k reaches waypoint j, aircraft f is at waypoint $i+1$, at least, and, so, no conflict between the aircraft k and f is possible at the pair of waypoints (i,j)),
3. The flight levels are such that $\underline{z}_i^f \leqslant \overline{z}_j^k$ and $\underline{z}_j^k \leqslant \overline{z}_i^f$ since, otherwise, the aircraft f and k cannot be at the same flight level while traversing the waypoints i and j).

Let $\mathcal{P}^{f,k} \subset \mathcal{W}_f \times \mathcal{W}_k$ denote be the set of all *potencial waypoint conflicts* between the aricraft f and k, and $\mathcal{F}^f \subset \mathcal{F}$ be the set of *potential aircraft conflicts* where aircraft f is involved, for aircraft $f,k \in \mathcal{F}$. Notice that $k \in \mathcal{F}^f$ iff $\mathcal{P}^{f,k} \neq \emptyset$).

Similarly, for a pair of aircraft $(f,k) \in \mathcal{F} \times \mathcal{F}$, there is a *current conflict* at the pair of waypoints $(i,j) \in \mathcal{W}_f \times \mathcal{W}_k$ if $(i,j) \in \mathcal{P}^{f,k}$ and $z_i^f = z_j^k$.

Finally, let $C\mathcal{P}^{f,k} \subset \mathcal{W}_f \times \mathcal{W}_k$ denote the set of all *current waypoint conflicts* between aircraft f and k, and $C\mathcal{F}^f \subset \mathcal{F}$ be the set of *current aircraft conflicts* with aircraft f is involved, for aircraft $f,k \in \mathcal{F}$. Notice that $k \in C\mathcal{F}^f$ iff $C\mathcal{P}^{f,k} \neq \emptyset$.

As an illustration, let us consider the aerial zone depicted in Figure 1, where three aircraft cross their paths. Particularly, we can observe that waypoint i_2 is too close to the waypoints j_5 and j_6, which are within the safety disc drawn around waypoint i_2. Suppose that aircraft 1 is scheduled to fly through the waypoints i_2 and i_3 at time instants (e.g., seconds) 33 and 48, respectively, and aircraft 2 is scheduled to fly through the waypoints j_5 and j_6 at the time instances 54 and 71, respectively, (i.e. $t_{i_2}^1 = 33, t_{i_3}^1 = 48$, $t_{j_5}^2 = 54, t_{j_6}^2 = 71$). Then, there is not a potential conflict nor a current conflict at the pair of waypoints (i_2, j_5) (i.e., $(i_2, j_5) \notin C\mathcal{P}^{1,2} \subset \mathcal{P}^{1,2}$), since $t_{j_5}^2 > t_{i_3}^1$. However, there might be a conflict at the waypoints (i_2, j_6). So, the values $t_{j_6}^2$, $t_{j_7}^2$, $t_{i_2}^1$ and $t_{i_3}^1$ should be checked to evaluate it.

On the other hand, we can observe in the figure that waypoint i_4 too close to the waypoints k_4, k_5 and k_6. So, suppose that e.g., $t_{i_4}^1 = 63, t_{i_5}^1 = 78, t_{k_4}^3 = 65$ and $t_{k_5}^3 = 85$. Then we find out that $t_{i_4}^1 < t_{k_4}^3 < t_{i_5}^1$ and, so, the first and second conditions

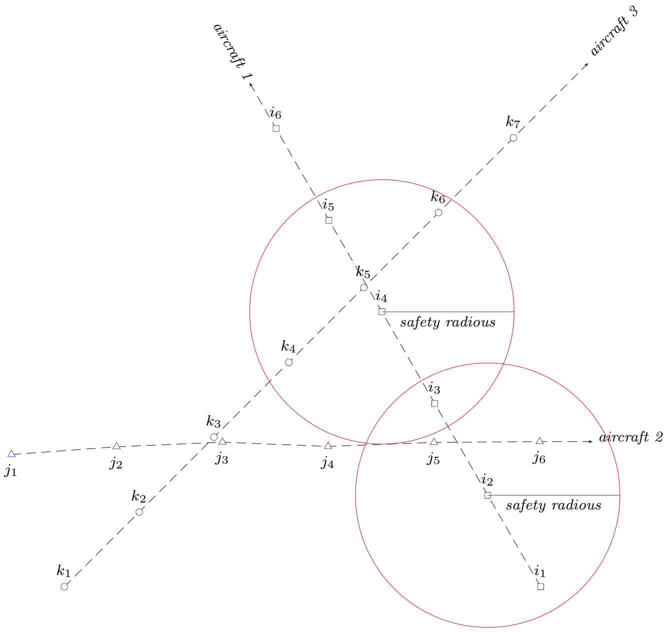

Fig. 1 Illustrative case for three flight routes

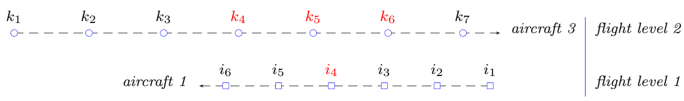

Fig. 2 Illustrative case for different flight levels of the routes of the aircraft 1 and 3

given above hold for the pair of waypoints (i_4, k_4) to belong to the sets $\mathcal{P}^{1,3}$ and $C\mathcal{P}^{1,3}$. To check if the third condition hold, the paths of the aircraft 1 and 3 depicted in Figure 2 should be analyzed on the axes x and z (i.e., abscissa and height). We can observe that both aircraft fly at different flight levels and, so, no current conflict takes place, thus $(i_4, k_4) \notin C\mathcal{P}^{1,3}$. However, suppose that $\underline{z}^3_{k_4} = 1, \overline{z}^3_{k_4} = 2$ and $\underline{z}^1_{i_4} = 1, \overline{z}^1_{i_4} = 1$, then $\underline{z}^3_{k_4} = \underline{z}^1_{i_4} = \overline{z}^1_{i_4} < \overline{z}^3_{k_4}$ and, thus, $(i_4, k_4) \in \mathcal{P}^{1,3}$, since aircraft 3 is allowed to fly at level 1 in waypoint k_4 and, so, a conflict may occur at the pair of waypoints (i_4, k_4) if such change is introduced by the model given below.

3.2 Model formulation for conflict resolution

The pure 0-1 model that we propose deals with the CDR problem by changing (i.e, climbing or descending) flight levels (i.e., altitude) for the aircraft in order to avoid *current conflicts*. It considers two objectives in a composite form, i.e., the maximization of rewards for the aircraft flying on the scheduled flight levels and the minimization of penalizations of flight level changes for the aircraft flying at other levels different from those scheduled ones. Both objectives are optimized at all the given waypoints. So, the model assigns flight level changes, if any, to the aircraft in order to guarantee that there will be no conflict among them.

Parameters

c_i^f and h_i^f, , reward and penalization for changing (i.e., climbing or descending) the scheduled flight level for aircraft f at its waypoint i, respectively, $\forall f \in \mathcal{F}, i \in \mathcal{W}_f$.

0-1 variables

$\phi_{i,h}^f = 1$, it will have the value 1 if aircraft f is at altitude level h at ith waypoint in its route path and 0, otherwise, $\forall f \in \mathcal{F}$, $i \in \mathcal{W}_f$, $h \in \mathcal{L}_i^f$.

$\nu_i^f = 1$, it will have the value 1 if aircraft f changes its altitude level from its waypoint i to the next one and 0, otherwise, $\forall f \in \mathcal{F}$, $i \in \mathcal{W}_f^-$.

The objective function includes two terms, namely, the reward for having the aircraft flying at the scheduled altitude levels and the penalization for flying at different altitude levels than the scheduled ones.

The model is as follows,

$$\max \sum_{f \in \mathcal{F}, i \in \mathcal{W}_f, h = z_i^f} c_i^f \cdot \phi_{i,h}^f - \sum_{f \in \mathcal{F}, i \in \mathcal{W}_f^-} h_i^f \cdot \nu_i^f \tag{1}$$

subject to:

$$\sum_{h \in \mathcal{L}_i^f} \phi_{i,h}^f = 1 \quad \forall f \in \mathcal{F}, i \in \mathcal{W}_f \tag{2}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_i^f}^{\overline{V}_i^f} \phi_{i+1,h+\ell}^f \quad \forall f \in \mathcal{F}, i \in \mathcal{W}_f^-, h \in \mathcal{L}_i^f \tag{3}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_{i-1}^f}^{\overline{V}_{i-1}^f} \phi_{i-1,h-\ell}^f \quad \forall f \in \mathcal{F}, i \in \mathcal{W}_f', h \in \mathcal{L}_i^f \tag{4}$$

$$\phi_{i,h}^f - \phi_{i+1,h}^f \leqslant \nu_i^f \quad \forall f \in \mathcal{F}, i \in \mathcal{W}_f^-, h \in \mathcal{L}_i^f \tag{5}$$

$$\phi_{i,h}^f + \phi_{j,h}^k \leqslant 1 \quad \forall f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k} h \in \mathcal{L}_i^f \cap \mathcal{L}_j^k \tag{6}$$

$$\phi_{i,h}^f, \nu_i^f \in \{0,1\} \quad \forall f \in \mathcal{F}, i \in \mathcal{W}_f, h \in \mathcal{L}_i^f. \tag{7}$$

Constraints (2) guarantee that all flights traverse every waypoint at only one flight level. Constraints (3)-(4) ensure "soft" flight level changes. Constraints (5) give the number of flight level variations from one waypoint with respect to the next one. Constraints (6) avoid the conflicts. Finally, expression (7) defines the integrality character of the 0-1 variables.

4 Collision Avoidance via flight level and speed changes

4.1 Definitions

Hereafter we expand the model presented in section 3.2 to take also into account speed changes. To that end, the following additional parameters and variables are defined.

Parameters

$\underline{t}_i^f$ and $\overline{t}_i^f$, lower and upper bounds for the feasible time instant at which aircraft f traverses the route segment $i \rightarrow (i+1)$, respectively, $f \in \mathcal{F}$, $i \in \mathcal{W}_f^-$.

$s_{i,j}^{f,k}$, reward for avoiding the conflicts between the aircraft f and k at the waypoints i and j due to time coincidence.

Variables

τ_i^f, variable that represents the time elapsed at the time instant aircraft f transverses waypoint i, for $f \in \mathcal{F}$, $i \in \mathcal{W}_f$.

$\gamma_{i,j}^{f,k}$, 0-1 variable such that it will have the value 1 if there is no conflict between the aircraft f and k at the waypoints i and j due to the timing (and, so, independently at which flight level they traverse their respective waypoints) and 0, otherwise, for $f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k}$.

$\beta_{i,j}^{f,k}$, 0-1 instrumental variable, see below.

4.2 Conflict Detection

As we mention in section 3.1, although the scheme for conflict detection is very similar for the both models that we propose in this work, there are some differences. Let the following slight modification. For a pair of aircraft $(f,k) \in \mathcal{F} \times \mathcal{F}$, there is a *potential conflict* at the pair of waypoints $(i,j) \in \mathcal{W}_f \times \mathcal{W}_k$ if both the conditions 1 and 3 stated in section 3.1 hold and, instead of condition 2, the following one holds too:

– The time instants are such that $t_1^f + \sum_{i'<i} \underline{t}_{i'}^f < t_1^k + \sum_{j' \leqslant j} \overline{t}_{j'}^k$ and $t_1^f + \sum_{i' \leqslant i} \overline{t}_{i'}^f > t_1^k + \sum_{j'<j} \underline{t}_{j'}^k$ since suppose, on the contrary, that e.g., the second inequality does not hold, then even if aircraft k reaches waypoint j the soonest possible time, aircraft f is at its waypoint $i+1$, at least, and no conflict between the aircraft k and f is possible at the pair of waypoints (i,j)),

Similarly to the problem with flight level changes only, for a pair of aircraft $(f,k) \in \mathcal{F} \times \mathcal{F}$, there is a *current conflict* at the pair of waypoints $(i,j) \in \mathcal{W}_f \times \mathcal{W}_k$ if $(i,j) \in \mathcal{P}^{f,k}$, $t_i^f < t_{j+1}^k$, $t_j^k < t_{i+1}^f$ and $z_i^f = z_j^k$.

4.3 Model formulation for conflict resolution

As in the pure 0-1 model, the first term in the objective function rewards the flights that do not change their scheduled flight level, the second term penalizes the number of "jumps" (climbing or descending) of the aircraft taken into consideration and the third term rewards the number of conflict resolutions by avoiding time coincidence. Notice that the model presented in section 3.2 does only consider the first two terms.

The model is as follows,

$$\max \sum_{f\in\mathcal{F},i\in\mathcal{W}_f,h=z_i^f} c_i^f \phi_{i,h}^f - \sum_{f\in\mathcal{F},i\in\mathcal{W}_f^-} h_i^f \nu_i^f + \sum_{\forall f\in\mathcal{F},k\in\mathcal{F}^f,(i,j)\in\mathcal{P}^{f,k}} s_{i,j}^{f,k}\gamma_{i,j}^{f,k} \tag{8}$$

subject to constraints (2)-(5) and

$$\tau_1^f - t_1^f \leqslant \mu \quad \forall f\in\mathcal{F} \tag{9}$$

$$t_1^f - \tau_1^f \leqslant \mu \quad \forall f\in\mathcal{F} \tag{10}$$

$$\tau_{i+1}^f - \tau_i^f \leqslant \overline{t}_i^f \quad \forall f\in\mathcal{F}, i\in\mathcal{W}_f^- \tag{11}$$

$$\tau_{i+1}^f - \tau_i^f \geqslant \underline{t}_i^f \quad \forall f\in\mathcal{F}, i\in\mathcal{W}_f^- \tag{12}$$

$$\tau_{|\mathcal{W}_f|}^f - t_{|\mathcal{W}_f|}^f \leqslant \epsilon \quad \forall f\in\mathcal{F} \tag{13}$$

$$t_{|\mathcal{W}_f|}^f - \tau_{|\mathcal{W}_f|}^f \leqslant \epsilon \quad \forall f\in\mathcal{F} \tag{14}$$

$$\gamma_{i,j}^{f,k} \leqslant \frac{(\tau_i^f - \tau_j^k)}{m_{A_{i,j}^{f,k}}} + m_{i,j}^{f,k}\beta_{i,j}^{f,k} \quad \forall f\in\mathcal{F}, k\in\mathcal{F}^f, \quad (i,j)\in\mathcal{P}^{f,k} \tag{15}$$

$$\gamma_{i,j}^{f,k} \leqslant \frac{(\tau_j^k - \tau_i^f)}{m_{A_{i,j}^{f,k}}} + m_{i,j}^{f,k}(1-\beta_{i,j}^{f,k}) \quad \forall f\in\mathcal{F}, k\in\mathcal{F}^f, \quad (i,j)\in\mathcal{P}^{f,k} \tag{16}$$

$$\phi_{i,h}^f + \phi_{j,h}^k \leqslant 1 + \gamma_{i,j}^{f,k} \quad \forall f\in\mathcal{F}, k\in\mathcal{F}^f, \quad (i,j)\in\mathcal{P}^{f,k}, \quad h\in\mathcal{L}_{\mathcal{W}_f}^f \cap \mathcal{L}_{\mathcal{W}_k}^k \tag{17}$$

$$\tau_i^f \in \Re^+ \quad \forall f\in\mathcal{F}, i\in\mathcal{W}_f^- \tag{18}$$

$$\phi_{i,h}^f, \nu_i^f \in \{0,1\} \quad \forall f\in\mathcal{F}, i\in\mathcal{W}_f, \quad h\in\mathcal{L}_i^f \tag{19}$$

$$\gamma_{i,j}^{f,k}, \beta_{i,j}^{f,k} \in \{0,1\} \quad \forall f\in\mathcal{F}, k\in\mathcal{F}^f, \quad (i,j)\in\mathcal{P}^{f,k}, \tag{20}$$

where the parameter ϵ in constraints (13) and (14) is half the length of the time interval around the scheduled arrival time. Its purpose is to avoid to constrain the aircraft arrival time to an isolated value. The aim of this requirement is to avoid changing scheduled flight times in other air zones, which could lead to new conflicts where they had previously been avoided. The parameter μ in constraints (9) and (10) is half the length of the time interval around the scheduled "departure" time. It will allow a small margin to decide when the aircraft fly into the conflict zone. The parameter $m_{i,j}^{f,k}$ in constraints (15) and (16) is the smallest possible value, big enough to guarantee that the right-hand-side of both constraints is positive, since their left-hand-side is a 0-1 variable.

Constraints (9) and (10) set the initial time instant for the aircraft to arrive to the conflict zone. Constraints (11) and (12) ensure "soft"' speed changes. Constraints (13) and (14) force the aircraft to arrive at their destination waypoints at (almost) their previously assigned time instant. Constraints (17) avoid the conflicts together with the auxiliary constraints (15) and (16), whose purpose is to force the variables $\gamma_{i,j}^{f,k}$ to be

zero if aircraft f and k traverse the waypoints i and j, respectively, within a small time interval (i.e., the difference of their time instants be smaller than $m_{A_{i,j}^{f,k}}$). Finally, constraints (18)-(20) define the character of the variables.
Note: As in the pure 0-1 model, the integrality condition of variable ν_i^f can be relaxed (i.e., let $\nu_i^f \in \Re^+$), as it can be done with variable $\gamma_{i,j}^{f,k}$ for similar reasons.

4.4 Tightening the model

Reducing the parameter $m_{i,j}^{f,k}$.

The easiest candidate for the parameter would be the total time considered in the problem, but a tighter candidate can be calculated as follows,

$$m_{i,j}^{f,k} = \frac{\max\left\{|\sum_{s<i}\bar{t}_s^f - \sum_{t<j}\underline{t}_t^k|, |\sum_{s<i}\underline{t}_s^f - \sum_{t<j}\bar{t}_t^k|\right\}}{m_{A_{i,j}^{f,k}}} + 1. \tag{21}$$

Again, we can even reduce $m_{i,j}^{f,k}$ by taking into account that so far, the aircraft are forced to arrive at their destination waypoints at their assigned arrival time instants. Then, let us use in expression (21) the following formulae: $\min\left\{\sum_{s<i}\bar{t}_s^f, t_{|\mathcal{W}_f|}^f - \sum_{s\geqslant i}\underline{t}_s^f\right\}$ and $\max\left\{\sum_{s<i}\underline{t}_s^f, t_{|\mathcal{W}_f|}^f - \sum_{s\geqslant i}\bar{t}_s^f\right\}$ instead of $\sum_{s<i}\bar{t}_s^f$ and$\sum_{s<i}\underline{t}_s^f$, respectively. Similarly, we can replace $\sum_{t<j}\bar{t}_t^k$ and $\sum_{t<j}\underline{t}_t^k$ with analogous expressions.

Special set of constraints

The above model for collision avoidance via flight level and speed changes just presented above can be tightened by appending the constraints

$$\gamma_{i,j}^{f,k} = \gamma_{i+1,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k} \tag{22}$$
$$\gamma_{i,j}^{f,k} = \gamma_{i,j+1}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k} \tag{23}$$
$$\gamma_{i,j}^{f,k} = \gamma_{i-1,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k} \tag{24}$$
$$\gamma_{i,j}^{f,k} = \gamma_{i,j-1}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathcal{F}^f, (i,j) \in \mathcal{P}^{f,k}. \tag{25}$$

Constraints (22)-(25) actually reduce the LP feasible space, while exclude some non optimal 0-1 solutions, thus resulting in a much tighter model and, then, allowing to obtain a a smaller elapsed time for solving the problem. To understand their meaning and why the excluded 0-1 solutions are not optimal, let us recall first how the variables $\gamma_{i,j}^{f,k}$ work. If the waypoints i and j are too close, the conflict between the aircraft f and k is avoided, since the time instant at which each aircraft traverses the respective waypoint are sufficiently distant, then $\gamma_{i,j}^{f,k} = 1$ and, otherwise, it is zero. So, the above constraints force to avoid a particular set of possible conflicts between the two aircraft f and k (i.e., conflicts in consecutive waypoints), by one and only one of the possible manoeuvres, e.g., solving them all by changing the flight level. For an illustration, consider the situation depicted in Figure 1, and suppose that $(i_4, k_4), (i_4, k_5), (i_4, k_6) \in \mathcal{P}^{1,3}$, then if e.g., the potential conflict in (i_4, k_4) is avoided by delaying aircraft 3 so that both aircraft 1 and 3 do not coincide on time at that waypoint, then the potential conflict in (i_4, k_5) should be avoided taken advantage of such delay without needing to force a new maneuver, e.g., forcing aircraft 1 descending one flight level.

5 Computational experience

We report the results of the computational experience obtained while optimizing the pure 0-1 model and the mixed 0-1 model presented in sections 3.1 and 4.2, respectively. The models have been implemented in a c++ experimental code and have been optimized by using the sate-of-the-art engine CPLEX v12.1 [11]. The computations were carried out in a PC Intel Core 2 Duo 4, 2 GHz and 2 Gbytes of RAM.

Two sets of testbeds of randomly generated instances have been used in our experimentation, 24 instances for the first testbed and 25 instances for the the second one. For each instance 10 simulations have been performed, such that the averages of the computational results are reported. The simulations differ one from the other for each instance in the the conflict zone, and the arrival time instances of the aircraft (chosen at random throughout a uniform distribution) to the conflict zone along the time horizon through any of the four sides of the conflict zone (all of them with equal probability) and any waypoint of the sides (we have used a normal distribution with a standard deviation equal to 1). A random number of potential flight levels ranges between 1 and 8 per aircraft.

The second term in the objective function (2) have been used for the pure 0-1 model (i.e., minimizing the number of flight level changes). The constraints (22)-(25) have been also appended in the mixed 0-1 model, where the number of conflict resolutions by speed changing is maximized and the number of flight level changes is minimized. So, the following objective function has been used for this model, $\min \sum_{f\in\mathcal{F},i\in\mathcal{W}_f,h\in\mathcal{L}_i^f} \nu_i^f + \sum_{\forall f\in\mathcal{F},k\in\mathcal{F}^f,(i,j)\in\mathcal{P}^{f,k}} (-10)\cdot\gamma_{i,j}^{f,k}$).

Tables 1 and 3 show the problem dimensions in the 24 instances in the testbed for the pure 0-1 model and the 25 instances in the testbed for the mixed 0-1 model. The headings are as follows: $|\mathcal{F}|$, number of aircraft; *CZ*, conflict zone side length (in nautical miles); $|\mathcal{T}|$, time horizon (in secs.); $|\bigcup_{f\in\mathcal{F}} C\mathcal{F}^f|$, number of *current aircraft conflicts*; $|\bigcup_{f\in\mathcal{F}} \mathcal{F}^f|$, number of *potencial aircraft conflicts*; $|\bigcup_{f\in\mathcal{F},k\in C\mathcal{F}^f} C\mathcal{P}^{f,k}|$, number of *current waypoint conflicts*; and $|\bigcup_{f\in\mathcal{F},k\in\mathcal{F}^f} \mathcal{P}^{f,k}|$, number of *potencial waypoint conflicts*. We can observe that the number of aircraft, conflict zone side length and time horizon have realistic dimensions.

The number of conflicts that took place in the simulations for each instance has been measured in 4 different ways, namely, the number of *current aircraft conflicts*, the number of *potential aircraft conflicts*, the number of *current waypoint conflicts*, and the number of *potential waypoint conflicts*.

Tables 2 and 4 show the dimensions of the pure 0-1 and mixed 0-1 models, respectively. The headings are as follows: m and m^*, number of constraints before and after CPLEX preprocessing, respectively; rm: Ratio (in %) between m and m^* (i.e., $\frac{m^*\cdot 100}{m}$); *n01* and nc, number of 0-1 and continuous variables, respectively; n and n^*, number of variables before and after CPLEX preprocessing, respectively; rn, ratio (in %) between n and n^* (i.e., $\frac{n^*\cdot 100}{n}$). We can observe in these tables how high are the dimensions of the models.

Tables 5 and 6 report the computational results. The headings are as follows: z_{lp}, solution value of the LP relaxation; z_s, solution value of the stronger LP relaxation (i.e., the value of the LP model after appending the cuts identified by CPLEX); z_{ip}, solution value of the original CDR problem; GAP_{lp} and GAP_s, related optimality gaps

Table 1 Dimensions of the flight level change problem

Case	$\|\mathcal{F}\|$	CZ	$\|\mathcal{T}\|$	$\|\bigcup_{f\in\mathcal{F}} C\mathcal{F}^f\|$	$\|\bigcup_{f\in\mathcal{F}} \mathcal{F}^f\|$	$\|\bigcup_{f\in\mathcal{F},k\in C\mathcal{F}^f} C\mathcal{P}^{f,k}\|$	$\|\bigcup_{f\in\mathcal{F},k\in\mathcal{F}^f} \mathcal{P}^{f,k}\|$
p01	25	50	300	15	43	36	270
p02	25	50	600	27	70	79	691
p03	25	100	300	8	20	29	177
p04	25	100	600	12	34	40	345
p05	25	200	600	5	12	18	145
p06	50	200	900	22	45	100	908
p07	50	200	1800	20	67	68	1295
p08	50	200	3600	18	77	65	1650
p09	50	400	1800	10	25	50	681
p10	50	400	3600	12	49	52	1301
p11	65	200	900	36	80	138	1338
p12	65	200	1800	37	125	132	2361
p13	65	200	3600	31	124	107	2588
p14	65	400	1800	20	49	89	1208
p15	65	400	3600	18	69	79	1861
p16	75	200	900	49	100	200	1826
p17	75	200	1800	46	168	187	3026
p18	75	200	3600	39	171	122	3398
p19	75	400	1800	26	58	125	1458
p20	75	400	3600	25	98	98	2471
p21	100	400	3600	43	177	173	4433
p22	100	600	3600	30	93	146	2682
p23	200	400	1800	195	463	868	11610
p24	200	400	3600	163	673	693	17665

computed as $\frac{z_{ip}-z_{lp}}{z_{ip}}\%$ and $\frac{z_{ip}-z_s}{z_{ip}}\%$, respectively; nn, number of CPLEX branch-and-cut nodes; t_{lp}, t_s and t_{ip}, elapsed times (secs.) to obtain the solution values z_{lp}, z_s and z_{ip}, respectively; t_t, total elapsed time from the starting of the optimization; nc, total number of cuts identified and appended by CPLEX.

Note: Some results for the pure 0-1 model, namely z_{lp}, z_s, z_{ip}, GAP_{lp}, GAP_s and nn, have not been included in Table 5, since they are zero in all instances of the testbed. Additionally, the model is so tight that the LP solution gives integer values for the (0-1) variables and then, the CPLEX branch-and-cut phase is not been required in any of the instances, being the total elapsed time close to zero in 21 out of 24 instances, and very small for the other three remaining instances.

Finally, it is worthy to point out the impressive total time t_t (in secs.) that has been required for providing the optimal solution of the mixed 0-1 models, see Table 6.

6 Conclusions and Future Work

Two integer linear optimization models for Conflict Detection and Resolution in a set of aircraft in the airspace have been proposed. The first one is a pure 0-1 linear model which avoid conflicts by means of altitude changes, and the second one a mixed 0-1 linear model whose strategy is based on altitude and speed changes. The very small elapsed time for both models shows that they can be used in real time, particularly in the medium term.

Table 2 Dimensions of the pure 0-1 model

Case	m	m*	rm(%)	n	n*	rn(%)
p01	3052	1493	48.9	1081	537	49.7
p02	5222	3758	72.0	1735	1264	72.9
p03	2265	1906	84.2	844	714	84.6
p04	3275	2385	72.8	1231	909	73.8
p05	2007	1428	71.2	787	562	71.4
p06	8634	7431	86.1	3115	2675	85.9
p07	6876	6596	95.9	2631	2509	95.4
p08	6668	5771	86.5	2564	2211	86.2
p09	5558	5425	97.6	2095	2035	97.1
p10	5071	4941	97.4	1941	1883	97.0
p11	11648	11376	97.7	4164	4048	97.2
p12	13082	12788	97.8	4777	4648	97.3
p13	10396	10024	96.4	3963	3799	95.9
p14	9240	8242	89.2	3427	3037	88.6
p15	6867	6698	97.5	2595	2520	97.1
p16	15032	14668	97.6	5296	5140	97.1
p17	16359	15942	97.5	6007	5828	97.0
p18	13847	13592	98.2	5178	5064	97.8
p19	11848	11529	97.3	4400	4262	96.9
p20	10838	9440	87.1	4054	3524	86.9
p21	17117	16583	96.9	6406	6179	96.5
p22	14893	14519	97.5	5519	5353	97.0
p23	67260	65831	97.9	22654	22058	97.4
p24	33092	32789	99.1	11681	11547	98.9

Several extensions for improving the performance of both models can be proposed, particularly the possibility of selecting alternative routes and allowing aircraft climbing or descending to the next flight level in more than one step as well as allowing to relate flight level changes to speed. It is a subject of future research work.

Another piece of work consists of tackling the problem where the aircraft can perform the three types of manoeuvres: altitude, speed and angle changes. We had addressed in this work the first two ones. However, the most difficult one is the angle change, since some nonlinearities can appear in the constraints of the model. Its designing and testing its validity with state-of the art exact mixed integer nonlinear optimization solvers as well metaheuristics as the Variable Neighborhood Search approach will be the subject of our future research work.

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Table 3 Dimensions of the flight level and speed changes problem

Case	$\|\mathcal{F}\|$	CZ	$\|\mathcal{T}\|$	$\|\bigcup_{f\in\mathcal{F}} C\mathcal{F}^f\|$	$\|\bigcup_{f\in\mathcal{F}} \mathcal{F}^f\|$	$\|\bigcup_{f\in\mathcal{F},k\in C\mathcal{F}^f} C\mathcal{P}^{f,k}\|$	$\|\bigcup_{f\in\mathcal{F},k\in\mathcal{F}^f} \mathcal{P}^{f,k}\|$
m01	10	50	300	2	7	6	48
m02	10	50	600	3	9	8	75
m03	10	100	300	1	3	3	21
m04	10	100	600	1	4	5	51
m05	10	200	600	1	2	4	40
m06	20	50	300	9	27	20	162
m07	20	50	600	17	46	48	406
m08	20	100	300	6	13	19	109
m09	20	100	600	6	15	24	203
m10	20	200	600	4	7	16	104
m11	25	50	300	15	43	36	270
m12	25	50	600	27	70	79	691
m13	25	100	300	8	20	29	177
m14	25	100	600	12	34	40	345
m15	25	200	600	5	12	18	145
m16	50	200	900	22	45	100	908
m17	50	200	1800	20	67	68	1295
m18	50	200	3600	18	77	65	1650
m19	50	400	1800	10	25	50	681
m20	50	400	3600	12	49	52	1301
m21	75	200	900	49	100	200	1826
m22	75	200	1800	46	168	187	3026
m23	75	200	3600	39	171	122	3398
m24	75	400	1800	26	58	125	1458
m25	75	400	3600	25	98	98	2471

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Table 4 Dimensions of the mixed 0-1 model

Case	m	m*	rm(%)	n01	nc	n	n*	rn(%)
m01	960	746	77.7	353	51	404	313	77.5
m02	1440	1064	73.9	503	71	574	424	73.9
m03	562	453	80.6	219	36	255	208	81.6
m04	1056	796	75.4	367	53	420	319	76.0
m05	957	795	83.1	333	51	384	318	82.8
m06	2783	2085	74.9	1002	136	1138	839	73.7
m07	6039	4215	69.8	2009	239	2248	1508	67.1
m08	2347	1907	81.3	875	131	1006	812	80.7
m09	3971	3064	77.2	1364	192	1556	1189	76.4
m10	2192	1665	76.0	793	117	910	699	76.8
m11	4570	3475	76.0	1612	214	1826	1361	74.5
m12	9918	7012	70.7	3204	365	3569	2412	67.6
m13	3295	2572	78.1	1207	171	1378	1054	76.5
m14	5976	4381	73.3	2054	273	2327	1697	72.9
m15	3265	2600	79.6	1181	178	1359	1084	79.8
m16	15897	12094	76.1	5339	705	6044	4539	75.1
m17	21321	14073	66.0	6920	866	7786	5289	67.9
m18	27032	17451	64.6	8584	1057	9641	6472	67.1
m19	12690	9302	73.3	4262	580	4842	3614	74.6
m20	21820	13947	63.9	7015	883	7898	5450	69.0
m21	29244	21485	73.5	9725	1215	10940	7891	72.1
m22	47939	32576	68.0	15320	1854	17174	11780	68.6
m23	55906	36855	65.9	17455	2132	19587	13376	68.3
m24	26502	19513	73.6	8847	1186	10033	7465	74.4
m25	41860	27504	65.7	13194	1651	14845	10268	69.2

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Table 5 Computational results for the pure 0–1 model

Case	t_{lp}	t_s	t_{ip}	t_t	nc
p01	$<.01$	0	$<.01$	$<.01$	$<.01$
p02	$<.01$	$<.01$	$<.01$	$<.01$	15
p03	$<.01$	$<.01$	$<.01$	$<.01$	1
p04	$<.01$	$<.01$	$<.01$	$<.01$	19
p05	$<.01$	$<.01$	$<.01$	$<.01$	$<.01$
p06	$<.01$	$<.01$	$<.01$	$<.01$	44
p07	$<.01$	$<.01$	$<.01$	$<.01$	72
p08	$<.01$	$<.01$	$<.01$	$<.01$	166
p09	$<.01$	$<.01$	$<.01$	$<.01$	1
p10	$<.01$	$<.01$	$<.01$	$<.01$	$<.01$
p11	$<.01$	$<.01$	$<.01$	$<.01$	38
p12	$<.01$	$<.01$	$<.01$	$<.01$	10
p13	$<.01$	$<.01$	$<.01$	$<.01$	5
p14	$<.01$	$<.01$	$<.01$	$<.01$	54
p15	$<.01$	$<.01$	$<.01$	$<.01$	4
p16	$<.01$	1	1	1	80
p17	$<.01$	1	$<.01$	$<.01$	14
p18	$<.01$	$<.01$	$<.01$	$<.01$	37
p19	$<.01$	$<.01$	$<.01$	$<.01$	16
p20	$<.01$	$<.01$	$<.01$	$<.01$	19
p21	$<.01$	$<.01$	$<.01$	$<.01$	6
p22	$<.01$	$<.01$	1	1	80
p23	2	18	15	18	311
p24	$<.01$	4	3	4	58

Table 6 Computational results for the mixed 0-1 model

Case	z_{lp}	z_s	z_{ip}	$GAP_{lp}(\%)$	$GAP_s(\%)$	nn	t_{lp}	t_s	t_{ip}	t_t	nc
m01	442.02	25.00	25.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m02	702.86	44.00	44.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m03	197.56	20.00	20.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m04	486.63	50.00	50.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m05	387.03	31.00	31.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m06	1490.52	58.00	58.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	6
m07	3892.76	258.00	258.00	2080.65	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	10
m08	1005.62	33.00	33.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m09	1929.84	210.00	210.00	1405.56	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	14
m10	962.81	56.00	-56.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m11	2502.83	94.00	94.00	3090.24	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	2
m12	6635.38	544.00	544.00	1132.49	0.00	0	$<.01$	1	$<.01$	$<.01$	56
m13	1601.03	89.00	89.00	3297.22	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	0
m14	3254.44	437.00	437.00	772.58	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	22
m15	1375.16	169.00	169.00	-	-	0	$<.01$	$<.01$	$<.01$	$<.01$	53
m16	8628.41	1244.65	1241.00	779.14	0.20	4	$<.01$	$<.01$	1	1	133
m17	12670.84	4914.69	4821.00	204.00	1.65	3	$<.01$	1	$<.01$	$<.01$	85
m18	16327.43	8069.55	7578.00	126.69	4.81	141	$<.01$	1	5	5	169
m19	6555.91	1993.00	1993.00	447.78	0.00	0	$<.01$	$<.01$	$<.01$	$<.01$	67
m20	12816.86	7624.00	7259.00	76.33	3.96	117	$<.01$	1	2	3	83
m21	17517.54	3214.14	3213.00	514.68	0.04	0	$<.01$	2	2	3	307
m22	29704.67	12036.53	11348.00	172.10	5.80	435	$<.01$	5	24	24	677
m23	33618.61	18632.84	17065.90	104.55	9.41	482	$<.01$	4	30	31	627
m24	14087.53	4059.69	3985.00	308.12	3.63	3	$<.01$	1	1	1	108
m25	24435.39	13424.85	12601.00	104.32	6.22	173	$<.01$	2	9	9	338