[Computer Methods and Programs in Biomedicine 207 (2021) 106125](https://doi.org/10.1016/j.cmpb.2021.106125)

 

Contents lists available at [ScienceDirect](http://www.ScienceDirect.com/)

Computer Methods and Programs in Biomedicine

journal homepage: [www.elsevier.com/locate/cmpb](http://www.elsevier.com/locate/cmpb)

A study of the sensitivity of biomechanical models of the spine for scoliosis brace design

Christos Koutras[∗,](#_bookmark0) Jesús Pérez, Kateryna Kardash, Miguel A. Otaduy

*Universidad Rey Juan Carlos, Madrid, 28933, Spain*

a r t i c l e i n f o a b s t r a c t

*Article history:*

Received 24 September 2020

Accepted 19 April 2021

*Keywords:*

Computational biomechanics Sensitivity analysis Computer model of torso Scoliosis

*Background and Objective:* The development of biomechanical models of the torso and the spine opens the door to computational solutions for the design of braces for adolescent idiopathic scoliosis. However, the design of such biomechanical models faces several unknowns, such as the correct identiﬁcation of relevant mechanical elements, or the required accuracy of model parameters. The objective of this study was to design a methodology for the identiﬁcation of the aforementioned elements, with the purpose of creating personalized models suited for patient-speciﬁc brace design and the deﬁnition of parameter estimation criteria.

*Methods:* We have developed a comprehensive model of the torso, including spine, ribcage and soft tissue, and we have developed computational tools for the analysis of the model parameters. With these tools, we perform an analysis of the model under typical loading conditions of scoliosis braces.

*Results:* We present a complete sensitivity analysis of the models mechanical parameters and a compar- ison between a reference healthy subject and a subject suffering from scoliosis. Furthermore, we make a direct connection between error bounds on the deformation and tolerances for parameter estimation, which can guide the personalization of the model.

*Conclusions:* Not surprisingly, the stiffness parameters that govern the lateral deformation of the spine in the frontal plane are some of the most relevant parameters, and require careful modeling. More sur- prisingly, their relevance is on par with the correct parameterization of the soft tissue of the torso. For scoliosis patients, but not for healthy subjects, we observe that the axial rotation of the spine also re- quires careful modeling.

© 2021 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

### Introduction

Scoliosis problems of moderate degree on adolescents are typ- ically treated using orthotic brace structures that push the spine. Scoliosis braces are designed in a variety of shapes and procedures. Most design methods rely to date on physical experimentation and prototyping, although computational strategies have been moder- ately studied; see [[16,52]](#_bookmark48) for some comparisons. Wider adoption of computational methods for the design of scoliosis braces suffers an important challenge: they require a personalized model of the patient’s torso biomechanics.

\* Corresponding author

*E-mail addresses:* christos.koutras@urjc.es (C. Koutras), jesus.perez@urjc.es (J. Pérez), kateryna.kardash@urjc.es (K. Kardash), miguel.otaduy@urjc.es (M.A. Otaduy).

* 1. *Background/Spine modeling and parameter estimation*

Designing a personalized biomechanical model entails two tasks: ﬁtting the morphology and connections of anatomical ele- ments to those of the patient, and parameterizing the mechanical models to match the response of the patient’s body. Biomechan- ical modeling of the spine has received a lot of attention, with popular approaches largely divided into two categories. One cate- gory follows the Finite Element Method (FEM); please see [[53]](#_bookmark67) for a survey of methods. Many FEM models have been developed for the lumbar [[20,22,55]](#_bookmark54), thoracic [[3,4]](#_bookmark35) or the cervical spine [[28]](#_bookmark65). Furthermore, Dicko et al. [[19]](#_bookmark53) developed a hybrid lumbar spine

<https://doi.org/10.1016/j.cmpb.2021.106125>

0169-2607/© 2021 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

model containing rigid bodies, FEM and contact mechanics, while Clin et al. [[14]](#_bookmark45) developed a novel method to include gravitational forces in an FE model. While these methods are potentially accu- rate, they require careful estimation of model parameters for per- sonalized design applications.

FE models of the torso have been coupled to brace models and patient geometry for personalized brace design in the context of adolescent idiopathic scoliosis [[23,31,37]](#_bookmark57). Some studies include the evaluation of the effectiveness of these techniques on large cohorts of patients [[17,25,51]](#_bookmark49).

Another category of approaches uses a simpler but more eﬃ- cient solution based on multi-body models. de Zee et al. [[18]](#_bookmark51) made a generic detailed rigid-body model of the lumbar spine. Bayo- glu et al. [[6]](#_bookmark36) developed a multi-body muscoloskeletal model of the human spine in order to study the spinal loads. Raabe and Chaudhari [[43]](#_bookmark56) investigated the jogging biomechanics using a full- body spine model developed in OpenSim, an open-source muscu- loskeletal simulation software. Le Navéaux et al. [[29]](#_bookmark66) developed biomechanical models based on multi-body dynamics to analyze the effects of implant density and distribution on curve correction and the resulting forces on the vertebrae. Ignasiak et al. [[27]](#_bookmark64) pre- dicted the dynamic spinal loading using a multi-body thoracolum- bar spine model with articulated rib cage.

In the category of multi-body models, a strong effort has been devoted to ﬁnding accurate simpliﬁcations of the models and de- signing parameter estimation techniques. In our design and param- eterization of a spine-and-torso model, we borrow insight and de- sign choices from this collection of work.

Panjabi et al. [[40]](#_bookmark53) studied the mechanical behavior of the hu- man lumbar and lumbosacral spine as shown by three dimen- sional load-displacement curves. Panjabi et al. [[39]](#_bookmark52) estimated the rotational stiffness coeﬃcients of the thoracic spine from experi- ments. Bisschop et al. [[8]](#_bookmark39) and Panjabi et al. [[39]](#_bookmark52) found the trans- lational stiffness coeﬃcients of the thoracic spine through exper- imental studies. Moroney et al. [[36]](#_bookmark48) estimated the load displace- ment properties of the cervical spine from experiments. Andriacchi et al. [[48]](#_bookmark62) estimated the stiffness coeﬃcients of the elastic proper- ties of the rib cage through simulations. Wilke et al. [[54]](#_bookmark69) exam- ined the ﬂexibility of every thoracic spinal segment in an in-vitro experiment. Liebsch et al. [[32,33]](#_bookmark70) investigated the kinematic and stiffness properties of the thoracic spine and the rib cage through experimental studies.

Moreover, many studies tried to estimate the mechanical pa- rameters of the soft tissue in the human’s body. Choi and Zheng

[[10]](#_bookmark40) estimated Young’s modulus and Poisson’s ratio of soft tissue from indentation using two different-sized indentors in a ﬁnite el- ement analysis. Song et al. [[47]](#_bookmark60) studied the elasticity of the living abdominal wall in laparoscopic surgery. Hostettler et al. [[26]](#_bookmark63) mea- sured the Bulk modulus and volume variation of the liver and the kidneys in vivo. McKee et al. [[35]](#_bookmark72) compared the reported values of Young’s modulus obtained from indentation and tensile deforma- tions of soft biological tissues.

* 1. *Background/Sensitivity analysis*

As discussed above, the biomechanics of the torso and the spine have been thoroughly studied, but hardly from the perspective of their accuracy for the computational design of scoliosis braces. Multiple questions arise in this regard, such as the identiﬁcation of the anatomical elements that play a relevant role, or the re- quired accuracy of the model parameters. In this work, we study these questions, with the goal of setting guidelines for the design of practical personalized models and the estimation of model pa- rameters. We do this by analyzing the sensitivity of model param- eters to guide parameter estimation.

Several previous works also analyzed the sensitivity of var- ious model components. Zander et al. [[56]](#_bookmark71) analyzed sensitivity to the position of the intervertebral centers of rotation in up- right standing using a musculoskeletal lumbar spine model. Bauer et al. [[5]](#_bookmark37) performed sensitivity analysis of intervertebral disc pa- rameters in a multibody-model of the lumbar spine. Senteler et al. [[45]](#_bookmark58) analyzed the sensitivity of interverteral joint forces to the center of rotation location. Putzer et al. [[41]](#_bookmark54) conducted a sen- sitivity analysis of lumbar spine loadings to anatomical parameters. Bayoglu et al. [[7]](#_bookmark38) analyzed the sensitivity of muscle and interverte- bral disc computations against potential errors in modelling muscle attachment sites. Xu et al. [[55]](#_bookmark70) conducted a mesh convergence and material sensitivity analysis on a ﬁnite element model of healthy lumbar spine. Clin et al. [[15]](#_bookmark47) analyzed the sensitivity of brace eﬃ- ciency to multiple brace designs.

To the best of our knowledge, no prior study analyzed the sen- sitivity of biomechanical models of the torso and the spine under the loading conditions of scoliosis braces. Our study, on the other hand, is aimed at guiding parameter estimation for the computa- tional design of braces. Furthermore, previous works on sensitivity analysis of spine models often focus on portions of the spine, or model elements of the spine to a degree of complexity beyond our needs. Instead, we apply sensitivity analysis to a comprehensive model of the spine and the torso, including soft tissue, with a de- gree of complexity suited for scoliosis brace design. Precisely one of our goals is the validation of the components of the model. From the point of view of methodology, we depart from sensitivity anal- ysis approaches based on sampling and statistical tools, and rely instead on local linear analysis and linear algebra tools, as done often in structural design [[21,50]](#_bookmark55). We extend these tools for the derivation of parameter tolerances based on prescribed bounds on the model’s accuracy.

* 1. *Contributions*

As the ﬁrst component of our methods (see [Section 2](#_bookmark1)), we de- velop a comprehensive model of the passive biomechanics of the torso, with a focus on the deformation of the thoracolumbar spine. We deform this model using reference loadings that mimic the action of typical scoliosis braces, and we perform a comparative study by applying the model to data from both healthy and patho- logical subjects.

As the second component of our methods (see [Section 3](#_bookmark15)), and to facilitate experimental analysis for a torso model with many degrees of freedom and many mechanical parameters, we design mathematical tools that operate in a linearized regime with re- spect to equilibrium conditions. In particular, we show how to de- rive a sensitivity matrix that relates model parameters to the de- formed conﬁguration of the torso, subject to implicit static equilib- rium constraints. Furthermore, we show how to leverage various linear algebra tools to carry out analysis operations on the sensi- tivity matrix.

The study of parameter sensitivity allows us to draw important conclusions for the design of personalized biomechanical models in the context of computational brace design. First and foremost, the impact of soft tissue cannot be neglected. Many biomechanical models focus on the musculoskeletal system, but we have found that the required accuracy on the parameters of the soft-tissue model is on par with the stiffness parameters that govern lat- eral motion of the spine in the frontal plane. In addition, we have found that the axial rotation stiffness of the spine is not relevant for healthy subjects, but it gains relevance for subjects suffering from scoliosis. Finally, we also derive tolerance values for the vari- ous model parameters as a function of admissible error bounds on spine deformation.

### Methods/Biomechanical model of the torso

In this section, we describe our biomechanical model of the torso, which represents the main biomechanical elements involved in brace-body interaction. The proposed biomechanical model will be used to predict the change in static spine conﬁguration due to surface forces. More speciﬁcally, we want to analyze the sensitiv- ity of the static spine conﬁguration with regard to the various me- chanical parameters of the torso. We start this section by providing an overview of the model, with an overall summary of its main in- gredients and design choices. The interested reader may then con- tinue with the rest of the section to understand the details of the model, or may jump to the next section to discover how the model is used for sensitivity analysis.

* 1. *Overview of the biomechanical model*

Let us start by identifying the main variables of our model. First, the degrees of freedom (DoFs) **x** gather the rigid transfor- mations of bones and the locations of discrete nodes that sam- ple the soft tissue of the torso. Second, the model parameters *ξ* gather stiffness values of bone joints and parameter values of the soft-tissue model. And third, the boundary conditions **f***ext* repre- sent external forces applied on the torso, which mimic the inter- action produced by scoliosis braces.

The purpose of the biomechanical model is to ﬁnd the static- equilibrium conﬁguration of the spine and torso, given a set of parameter values and known external forces. The biomechanical model can be interpreted as a function *f* that maps parameter

values *ξ* and external forces **f***ext* to a conﬁguration **x** that satisﬁes

static equilibrium. The model can then be summarized as

**x** = *f (***f***ext , ξ ),* (1)

To enable sensitivity analysis, we require a biomechanical model that is differentiable with respect to the mechanical param- eters. We achieve this by designing the biomechanical model as an unconstrained energy minimization problem, which combines po- tential energy due to gravity, soft-tissue elastic energy, and skeletal joint energies. Then, given a total energy function *E*, the model *f* in [(1)](#_bookmark4) can be rewritten as:

**x** = *arg*min *E (***x***, ξ )* − **f***T* **x***.* (2)

**x** *ext*

The minimization [(2)](#_bookmark5) is satisﬁed when the net forces on the torso are zero, which corresponds to static equilibrium conditions. With internal forces **f** = − **x***E*, the biomechanical model *f* is then deﬁned implicitly by the following static equilibrium condition:

∇

**f***(***x***, ξ )* + **f***ext* = 0*.* (3)

We make several design choices to limit the complexity of the model while retaining suﬃcient accuracy for the problem at hand. In particular, we consider the torso model passive, without muscle activation. We are looking at setting up an accurate model capa- ble of predicting how surface forces produced by a scoliosis brace act on the spine. While the exact instantaneous forces depend on the instantaneous muscle activity, scoliosis braces perform most of their action during periods of little muscle activity. Therefore, we

discard muscle activity in our study.



**Fig. 1.** A 6D joint between two vertebrae, showing the rigid conﬁgurations of the parent and child vertebrae, *(***x***p ,* **R***p )* and *(***x***c ,* **R***c )*, the attachment points, **r***p* and **r***c* , and the local orientation of the joint, **R**0 .

default parameters for the different joints, based on previous liter- ature in the ﬁeld. We continue with a discussion of our soft-tissue model and its connection to the articulated skeleton. We also dis- cuss in detail how we apply boundary conditions. Finally, we de- scribe the details of the static equilibrium formulation that com- bines all the previous elements.

* 1. *6D Joint model*

Let us consider two bones, a parent *p* and a child *c*, deﬁned by their respective translation and rotation, *(***x***p,* **R***p )* and *(***x***c,* **R***c )*, as shown in [Fig. 1](#_bookmark2). We consider that all bones have zero rotation in the rest conﬁguration. We place a 6D joint between the pair of bones, i.e., a joint that creates both a force and a torque that counteract the relative motion between the bones. Each joint is lo- cated and oriented according to its biomechanical functionality. In [Section 2.3](#_bookmark9) below, we discuss this process for the different types of joints in our model.

The location of a joint is deﬁned by a pair of attachment points, represented by two vectors, **r***p* and **r***c*, in the local reference frames of the parent and the child bones, respectively. We represent the local orientation of the joint by a rotation, **R**0, in the local refer- ence frame of the parent. In [Section 2.3](#_bookmark9), we specify how the lo- cal orientation of each joint is initialized for an input spine model. We use this local reference frame to decouple the different motion components of the joint, while retaining anisotropic behaviors. Ide- ally, one would perform personalized motion experiments to un- derstand the possible coupling between motion components [[11]](#_bookmark44), and then diagonalize the observed coupled stiffness to obtain a de- coupled local reference frame, which enables a simpler model and parameterization of the joint.

Given the attachment points, the local orientation of the joint,

and the conﬁgurations of the two bones, we can express the trans- lational displacement of the joint as the separation of the attach- ment points, transformed to the local frame of the joint:

**x** = **R***T* **R***T (***x***c* + **R***c* **r***c* − **x***p* − **R***p* **r***p ).* (4)

0

*p*

Similarly, we express the rotational displacement of the joint as the separation of the bone orientations, using an angle-axis repre- sentation, again transformed to the local frame of the joint:

 *θ* = ToAxisAngle(**R***T* **R***T* **R***c* **R**0 *.* (5)

0

*p*

We also decide to model the skeletal structure as a multi-body system of rigid bones connected by compliant joints. To satisfy differentiability of the model, we avoid using hard constraints at joints. Speciﬁcally, we use a model of anisotropic compliant 6D joint, and we continue this section with a detailed description of

this joint model. Next, we discuss the construction of the full ar-

Based on the translational and rotational displacements, we set translational and rotational springs. By setting a different stiff- ness for each DoF of the joint, we achieve an anisotropic behavior, which is crucial for capturing the elastic response of the torso. For

each joint, we deﬁne **kx** (= *kx, ky, kz* , a 3-vector of translational

(

stiffness values, and **k***θ* =

*kα, kβ , kγ* , a 3-vector of rotational stiff-

ticulated body that represents the spine, the rib cage, the sternum,

and their connections. We pay particular attention to the choice of

ness values. Joint force and torque are naturally formulated as the

negative gradients with respect to bone positions and rotations of

the following elastic energy:

*E* = 1 **x***T* diag*(***k** *)* **x** + 1 *θ T* diag*(***k** *) θ.* (6)

*i* 2 *x* 2 *θ*

**Table 1**

Default stiffness values (in kN/m for translation, N/rad for rotation) for the various 6D joints in our model, classiﬁed into four types.

Joint type *kx ky kz kα kβ kγ*

In [Section 2.4](#_bookmark11) below, we discuss the choice of default stiffness val- ues for all joints in the model.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Thoracic segment | 262 | 1720 | 262 | 154 | 137 | 154 |
| Lumbar segment | 245 | 1720 | 245 | 143 | 498 | 149 |
| Rib-vertebrae | 53.9 | 123 | 123 | 9.7 | 6.87 | 6.87 |
| Rib-sternum | 51.85 | 8 | 8 | 9.7 | 2.29 | 2.29 |

* 1. *Articulated-body model of the skeleton*

We design an articulated-body model for the skeleton of the torso, focusing on elements that are relevant for predicting the de- formation of the lumbar and thoracic spine. To this end, we model the spine including lumbar and thoracic vertebrae, the sternum, the rib cage, and the pelvis, as shown in [Fig. 3](#_bookmark27). We rigidly attach the costal cartilages to the sternum. In [Section 2.6](#_bookmark14) below we de- scribe the boundary conditions applied to the skeleton.

We start with artist-created reference models of the various bones. Given image data of a particular subject in a relaxed stand- ing conﬁguration, we personalize the geometry of the bone models by morphing the reference models to register the image data. To this end, we apply a hierarchical manual deformation, guided by a visual overlay of the deformed model and the input scans. We start with global anisotropic scaling, followed by progressive ad- dition of radial-basis-function 3D deformers [[9]](#_bookmark41), and we conclude with anisotropic scaling of individual vertebrae. [Fig. 3](#_bookmark27) shows the skeletal model personalized for two subjects, one with a healthy spine and one suffering from scoliosis.

Once the personalized bones are deﬁned, we proceed by deﬁn- ing the location **r***p* and **r***c*, and local orientation **R**0 of joints, for each pair of parent and child vertebrae. There is no consensus in the literature about the placement of the intervertebral joints, especially during lateral bending. Thus, we decided to place the intervertebral joints in the centers of mass of the intervertebral discs, as this choice complies with the main approach in the lit- erature [[56]](#_bookmark71). If the discs are not visible in the subject’s images, we

place each joint in the middle point between the centers of mass of the two connected vertebrae. We have veriﬁed that our results are robust to small changes in joint location. To deﬁne the joint orientation **R**0 we proceed as follows. We deﬁne the Y axis (ax- ial rotation) along the line that connects the centers of mass of the two vertebrae. We then deﬁne the X axis (lateral translation in the frontal plane, ﬂexion/extension in the sagittal plane) by ﬁt- ting a global frontal plane, and ﬁnding the axis in the plane that is orthogonal to the Y axis of the joint. Finally, the Z axis (sagit- tal translation, lateral bending) is computed as the cross-product of the other two.

For the connections between vertebrae and ribs and the ones between ribs and sternum, we place the joints at the midpoint of the closest points between the surfaces of the two connected bones. When searching for the closest point to the sternum, we consider also the rigidly attached costal cartilages. We deﬁne the X axis of the joint (‘separation’ for translation, ‘twist’ for rotation) by aligning it to the center line of the rib near the joint. Then, we deﬁne the Y and Z axes (‘alignment’ for translation, ‘bending’ for rotation) arbitrarily in the orthogonal plane, as we use isotropic stiffness values for bending on those axes.

* 1. *Joint stiffness values*

We have deﬁned default stiffness values following the data re- ported in the study of Ignasiak et al. [[27]](#_bookmark64). While they classify joints based on the number of compliant DoFs (e.g., revolute or spheri- cal), in practice they also model all joints as 6D joints, and report stiffness parameters for stiff DoFs too. Other recent studies also provide experimental data for stiffness values, and could be used to modify the default parameters of the model [[32,33,54]](#_bookmark70).

The intervertebral joints represent the compound effect of in- tervertebral disks, ligaments, and facet joints. The 6D joints be- tween ribs and vertebrae gather the effect of both the costo- vertebral and costo-transverse joints, which in turn consist of the capsules and the surrounding ligaments. Similarly, the 6D joints between ribs and sternum gather the effect of the costal cartilages and the costo-sternal joints.

In some joints, Ignasiak et al. consider asymmetric stiffness be- havior, i.e., different translational stiffness for tension vs. compres- sion (in rib joints), or different rotational stiffness for ﬂexion vs. extension of vertebrae. For simplicity, we have modeled the joints as symmetric, using average stiffness values from the study of Ig- nasiak et al. The sensitivity analysis we discuss in [Section 4.3](#_bookmark26) sup- ports our choice, as the parameter tolerances cover the asymmetry in the model of Ignasiak et al.

Altogether, we classify four types of joints based on their de- fault stiffness values: thoracic vertebral joints, lumbar vertebral joints, rib-vertebrae joints, and rib-sternum joints. All the default stiffness values are listed in [Table 1](#_bookmark7).

* 1. *Soft tissue*

The soft tissue of the torso is composed of multiple and diverse elements, including muscles, organs, and fat. Biomechanical mod- els often focus on the muscles, as they tend to be concerned with the relation between muscle action and the motion of the muscu- loskeletal system. We are instead concerned with the passive re- sponse of the torso to surface forces, and in particular the mo- tion of the spine resulting from this response. Therefore, we ask ourselves what is the overall impact of the soft tissue on this re- sponse. Even though the soft tissue of the torso is clearly heteroge- neous, we model it as homogeneous on the large scale. As shown in [Section 4.3](#_bookmark26), this will allow us to analyze the overall sensitivity of the soft tissue in contrast to the rest of the parameters of the model. However, it is important to note that soft-tissue parameter tolerances indicate that homogeneity is not a valid assumption for accurate spine deformation.

We start from a 3D scan of the subject’s torso, and register it to the rest pose of the skeleton. We mesh the volume of the torso us- ing a Delaunay tetrahedralization of the surface [[46]](#_bookmark61). To attach the soft tissue and the skeletal bones, we sample the bones, and in- clude these samples as constraint nodes for the tetrahedralization. During the simulation, we constrain the motion of these soft-tissue nodes to the corresponding bones.

We have simulated the soft tissue using continuum elasticity equations with FEM discretization [[38]](#_bookmark50). We have tested different options for the choice of material model, and we found that the choice is not critical when analyzing the sensitivity for brace de- sign. Speciﬁcally, with a St. Venant-Kirchhoff material and a Neo- Hookean material, the displacements of vertebrae differ by less than 3% under the conditions of our experiments. The principal stretches (eigenvalues of the deformation gradient) range between

0.27 and 1.96, with the largest compressions close to the ribs, but far from the spine, where sensitivity is analyzed.

In the experiments reported in the paper, we have used the Neo-Hookean material. We parameterize the model using Lamé

constants, as they avoid artiﬁcial sensitivity due to nonlinearity of the material with respect to Poisson’s ratio. Using Lamé constants

We can formulate the static equilibrium [(3)](#_bookmark6) as the minimization of the total energy with respect to the DoFs, i.e.,

*μ* (shear modulus) and *λ*, the Neo-Hookean material is described

by the following energy density:

**x** arg min

**x**

=

*Ei (***x***)*

*i o*

+ r

*\ll(***x***) do* − **f***T*

**x***.* (8)

*\ll* = *μ (I* − 3*)* − *μ* log *J* + *λ (*log *J )*2*,* (7)

*ext*

Here, we sum the energies *E* of all 6D joints, deﬁned as in [(6)](#_bookmark8);

## 2 1 2

where *J* is the determinant of the deformation gradient **F**, and *I*1 is the ﬁrst invariant, i.e. the trace, of the right Cauchy-Green ten- sor **C** = **F***T* **F**. We choose default parameter values of 30 kpa for the Young modulus and 0.4 for Poisson’s ratio by averaging estimations from multiple references in the literature [[10,26,35,47]](#_bookmark40), and we set the corresponding default Lamé constants.

* 1. *Boundary conditions*

We distinguish two types of boundary conditions applied to the model: ﬁxed DoFs of the skeleton to provide balance, and external forces that mimic the action of a scoliosis brace. Let us start with how we ﬁx the skeleton.

We wish to simulate postures where the subject remains straight, but allowing some adaptation to the external forces, thus avoiding excessive stress. To this end, we make two choices in how we ﬁx the ﬁrst thoracic vertebra, T1, and the pelvis. For T1, we ﬁx its lateral and frontal translation, allowing for full rotation and vertical translation, which allow rotation of the spine and the to- tal height to slightly adapt to the external forces. For the pelvis, although most previous works consider it ﬁxed, some exceptions allow it to tilt [[13,37]](#_bookmark46); therefore, we also allow the pelvis to tilt in our model, and we ﬁx the rest of its DoFs. In practice, we have seen that more restrictive boundary conditions would suﬃce. We have tested the model with the rotation of T1 ﬁxed, and the displacements of vertebrae differ by less than 17% under our ex- perimental conditions, with little effect on parameter sensitivities. Similarly, we have tested the model with the pelvis fully ﬁxed, and the displacements of vertebrae differ by less than 8% under our experimental conditions.

We model the action of a brace on the torso and the spine through external forces **f***ext* applied on the surface of the torso. To determine these forces, we consider the geometry of a brace model and the default parameter values of the torso model, and we sim- ulate the deformation of the torso and the brace with sliding fric- tional contact between them. Then, we use these same external forces for the rest of our sensitivity study, hence they can be con- sidered ﬁxed. In [Section 4.1](#_bookmark25), we discuss details about the creation of the brace model for our experiments. In early versions of our model, we considered more drastic approximations to brace forces, such as applying forces directly on nearby ribs. We found that the main conclusions about the model and the relative sensitivity of parameters were essentially the same, although modeling the in- teraction of the brace as a surface force ﬁeld has revealed higher importance of the ﬂexion/extension stiffness.

* 1. *Static equilibrium*

We simulate the deformation of the full torso in reaction to ex- ternal forces **f***ext* by solving for static equilibrium conﬁgurations, as outlined in [Section 2](#_bookmark3). Let us group in a single vector **x** all the DoFs of the torso, consisting of the free FEM nodes of the soft tissue, and the translational and rotational DoFs of the bones. For transla- tion, we use the position of the center of mass of each bone. For orientation, we use the relative rotation with respect to the initial rest conﬁguration, represented using axis angle.

*i*

the energy density *\ll* of the soft tissue, deﬁned as in [(7)](#_bookmark13) and in-

tegrated over the whole volume *o* of the torso; and the work of external forces **f***ext* . As noted earlier in [Section 2.5](#_bookmark10), we discretize and integrate the soft-tissue energy using tetrahedral FEM.

Note that in our model we do not include gravitational forces. The reason for this is that the input model, obtained from medical images in a standing position, is already pre-loaded with gravita- tional forces, hence adding gravitational forces to this model would produce wrong deformations. Moreover, the direct effect of brace forces on the deformation of the spine is much larger than the in- direct effect due to redistribution of mass, i.e., changes in the grav- itational forces. A more accurate approach, which becomes neces- sary for the correct estimation of stresses in the spine, would be to estimate the rest shape of the spine without gravitational forces, following an optimization approach as done by Clin et al. [[14]](#_bookmark45).

### Methods/Sensitivity analysis

In this section, we describe how we study the parameterization of the torso model. We are interested in two effects of the param- eterization. One is the sensitivity of the different parameters, i.e., which of them have a larger effect on the deformation of the spine under external loads. The other one is the deﬁnition of parameter tolerances based on prescribed error bounds on the deformation of the spine. Deﬁning in which range parameters should be in or- der to ensure accurate spine deformations can serve the design of parameter estimation procedures.

We start this section by providing an overview of our method- ology for sensitivity analysis of the parameterization. We leverage tools from differential calculus and linear algebra to derive a sen- sitivity matrix that relates changes in the parameters to changes in a feature vector. The derivation of the sensitivity matrix requires an implicit solution to static deformations of the spine. By com- puting a singular value decomposition (SVD) of the sensitivity ma- trix, we show how to set parameter tolerances based on deforma- tion bounds. We conclude the section by deﬁning a speciﬁc feature vector that describes the deformation of the spine, and a reparam- eterization of the model based on parameter ratios, which is better suited for sensitivity analysis.

* 1. *Overview of the methodology*

To analyze the sensitivity of the torso model, we start by deﬁn- ing a feature vector **z** that captures relevant information of the ge- ometric state of the model. This feature vector can be evaluated as a function **z***(***x***)* of the DoFs of the model. We choose a feature vec- tor that captures positions and rotations of vertebrae, as detailed later in [Section 3.3](#_bookmark18).

In addition to the feature vector, we must deﬁne the speciﬁc parameter set to be analyzed. The joint stiffness and soft-tissue pa- rameters introduced in [Section 2](#_bookmark1) do not share a common scale, and would complicate a combined sensitivity analysis and the compari- son of parameter sensitivities. For sensitivity analysis, we introduce instead a reparameterization of the model based on *parameter ra- tios*. For each scalar parameter (i.e., a joint stiffness or a Lamé con- stant in our case), we deﬁne its actual value *k* based on a default

value *k*¯ and a ratio *ξk* , such that *k* = *ξk k*¯ . The parameter vector *ξ*

is then composed of the different parameter ratios. We also group parameter ratios for components of the model with similar biome-

chanical function, and hence design a compact parameter vector, as detailed later in [Section 3.4](#_bookmark20).

We seek to compute the sensitivity of the feature vector **z** with respect to the default model parameters. Under the assumption of small deformations, this sensitivity can be characterized by the Ja- cobian of the feature vector with respect to the vector of parame-

ter ratios *ξ* , evaluated at *ξ* = 1 (i.e., with default parameter values).

We refer to this Jacobian as the *sensitivity matrix* **S**.

To compute the sensitivity matrix, we can simply apply the chain rule:

*∂***z** *∂***z** *∂***x**

**S** = *∂ξ* = *∂***x** *∂ξ .* (9)

The Jacobian of the feature vector with respect to the DoFs of the torso model, *∂***z** , is trivial to compute, as it entails only deriva- tives of rigid transformations with respect to the translations and rotations of vertebrae. On the other hand, the Jacobian of the DoFs with respect to parameter ratios, *∂***x** , requires special attention. As

*∂***x**

*∂ξ*

the parameters of the torso change, its deformation is deﬁned by the static equilibrium [(8)](#_bookmark12). The solution to this problem is given by the static equilibrium condition [(3)](#_bookmark6), i.e., zero net forces. By apply- ing the Implicit Function Theorem to this condition, we can obtain the Jacobian of the DoFs with respect to parameter ratios:



**Fig. 2.** To facilitate the combined analysis of translational and rotational motion of the vertebrae, we place an oriented bounding box around each vertebra, and compute a feature vector consisting of the eight corners of the box.

where **v***k, j* is the *k*th component of **v** *j* , a right singular vector of **S**, i.e., a column of **V**. In [Appendix B](#_bookmark32), we include the derivation of this sensitivity-based tolerance.

Unlike the uniform tolerance [(12)](#_bookmark22), the sensitivity-based toler- ance [(13)](#_bookmark23) allows the parameter estimation to focus on the most relevant parameters, with loose bounds for insensitive parameters. In [Section 4.3](#_bookmark26), we use these results to set tolerances on the pa- rameters of the torso model according to prescribed bounds on the deformation error.

**f***(***x***, ξ )* + **f**

= 0 → *∂***f** *∂***x** + *∂***f** = 0 → *∂***x** = − *∂***f** −1 *∂***f** *.* (10)

*3.3. Spine feature vector*

This expression requires the Jacobians of forces with respect to the DoFs, *∂***f** , and the parameter ratios, *∂***f** , which we evaluate analyt-

*ext*

*∂***x** *∂ξ*

*∂ξ*

*∂ξ*

*∂***x**

*∂ξ*

*∂***x** *∂ξ*

ically.

Under linear approximation, i.e., valid for small changes of pa- rameters, the sensitivity matrix allows exploring the effects of parameters without recomputing torso deformations, which is a costly task. For any parameter change, we can obtain a linear ap- proximation of the resulting deformation. As a corollary, by ana- lyzing the columns of the sensitivity matrix **S** and computing their norm, we can sort the parameters according to their sensitivity. In [Section 4.3](#_bookmark26) we perform this operation on several experiments and we discuss the results. However, prior to this, in [Section 4.2](#_bookmark24) we validate the correctness of the linear approximation for sensitivity analysis.

*3.2. Model tolerance based on SVD*

Based on the sensitivity matrix **S**, a change of parameters *ξ* produces a change in the feature vector **z** = **S** *ξ* . Given an error bound *E* on spine deformation (i.e., on the feature vector), we wish to ﬁnd the largest parameter change *ξ* , such that the error bound is satisﬁed, i.e., **z** = **S** *ξ < E*.

1 1 1 1

We propose two approaches to set tolerances on parameters: one with uniform tolerance, the other with sensitivity-based toler- ance. Both approaches leverage the SVD of the sensitivity matrix:

**S** = **U** *E* **V***T ,* (11)

with *E* a diagonal matrix of singular values *σ j* , and **U** and **V** or- thogonal matrices.

{ }

Given *Nξ* parameters, one can bound the change of the feature vector **z** by bounding uniformly all parameter changes { *ξk* }:

Several metrics exist for the classiﬁcation and quantiﬁcation of scoliosis. One popular metric is the Cobb angle [[44]](#_bookmark57). It measures the largest angle between the superior endplate of a vertebra and the inferior endplate of some other vertebra, thus characterizing the maximum bending of the spine. Alternatively, Lenke’s classi- ﬁcation [[30]](#_bookmark68) uses three different spine curves measured between ﬁxed vertebrae. The Cobb angle is not differentiable, and both Cobb angle and Lenke’s curves characterize all the deformation of the spine with just one or a few scalar values; therefore, they are not well suited for sensitivity analysis of model parameters. Instead, we design a feature vector that accounts for the deformation of the complete spine, and which is differentiable.

We place an oriented bounding box (OBB) [[24]](#_bookmark59) around each ver- tebra of the spine, as shown in [Fig. 2](#_bookmark17). We build a feature point

**z***i* using the position of each corner of the OBB, as this feature point accounts for both translation and rotation of the vertebra. By concatenating the feature points of all OBB corners, we obtain the

feature vector of the spine, **z**. Note that the feature vector ignores other elements of the model, such as the rib cage, as they do not enter into the computation of the Cobb angle either.

* 1. *Parameter set*

With 61 joints, 6 scalar stiffness values per joint, and 2 soft- tissue parameters, brute-force sensitivity analysis would employ a parameter vector with 368 elements. Instead, we assign common parameter ratios to groups of joint stiffness values, and perform sensitivity analysis on a vector *ξ* of 20 distinct parameter ratios:

* + - Translation and rotation stiffness of intervertebral joints

{*kv,x , kv,y , kv,z , kv,α, kv,β, kv,γ* }.

* + - Translation and rotation stiffness of rib-vertebrae joints

1 *ξk*1 *<* /

*E*

*Nξ σ*max

*,* (12)

{*kr,x , kr,y , kr,z , kr,α, kr,β , kr,γ* }.

* Translation and rotation stiffness of rib-sternum joints

where *σ*max is the largest singular value. In [Appendix A](#_bookmark42), we include the derivation of this uniform tolerance.

Alternatively, one can bound the change of the feature vector

**z** by bounding each parameter change *ξk* according to sensitiv- ities:

{*ks,x , ks,y , ks,z , ks,α, ks,β , ks,γ* }.

* + Lamé constants of the soft tissue, {*μ, λ*}.

For simplicity, in the paper we often refer to parameter ratios with the names of their corresponding model parameters. It is im- portant to note that all similar joints share a common parameter

1 *ξk*1 *<*

*Nξ* /L*i*

*E*

*σi* max *j*

(√*σ j* 1**v**

*k, j*

1 *,* (13)

ratio for sensitivity analysis, but in the case of lumbar and thoracic vertebrae the default stiffness values are different, as indicated in

[Table 1](#_bookmark7). Therefore, a change of, e.g., the axial rotation parameter ratio of intervertebral joints will produce a different actual axial rotation stiffness, *kv,β* , for lumbar and thoracic vertebrae.

### Results

In this section, we discuss the data and conditions of our ex- periments, the validation of the sensitivity matrix, and conclusions about model parameters obtained by applying sensitivity analysis. In a supplementary document, we provide a comprehensive sum- mary of the experimental results. Here, due to space constraints, we report only the most relevant data. All experiments reported in the study were run using the SOFA simulation framework [[2]](#_bookmark33). The simulation code is written in C++, and SOFA offers a Python interface to easily access simulation and model data.

* 1. *Experimental conditions*

[Fig. 3](#_bookmark27) summarizes the data and conditions of our experiments. We have extracted torso models from images of two subjects, fol- lowing the procedure described in [Section 2.3](#_bookmark9). We refer to these two subjects as *healthy* and *scoliosis* subjects, respectively, in the rest of the document. The healthy subject, shown on the left in [Fig. 3](#_bookmark27) exhibits a Cobb angle of 3.2 degrees at rest, while the scol- iosis subject, shown on the right, exhibits a Cobb angle of 31.8 de- grees.

In our experiments, we have used boundary conditions that represent the forces exerted by a scoliosis brace, as described in [Section 2.6](#_bookmark14). In particular, we try to mimic the conditions of a 3- point brace applied to the torso of the scoliosis subject. Given the model of the torso of the scoliosis subject, we have designed a model of a brace in the form of a Boston brace [[42]](#_bookmark55) (shown in [Fig. 3](#_bookmark27)). A 3-point brace applies a lateral force at the height of the main spine curvature, with two opposing forces above and below. Based on this strategy, we have manually modiﬁed the geometry of the brace trying to improve the spine deformation, until we reach a total (distributed) load of roughly 250 N on the skin of the torso. This results in a Cobb angle of 29.8 degrees. [Fig. 3](#_bookmark27) also shows the resulting pressure distribution on the skin. Many studies tried to estimate the forces exerted by a scoliosis brace on the patient, us- ing pressure sensors [[1,12,34,49]](#_bookmark34). These studies report force values in the range from 100 to 600 N. We chose to apply a force of 250 N, as an approximate average of the reported values.

As discussed in [Section 2.6](#_bookmark14), we compute the brace force using a full simulation of contact between the brace and the torso, but we then ﬁx these external forces for sensitivity analysis. For the healthy subject, we deﬁne a correspondence mapping to the skin of the scoliosis subject, and we use this mapping to transfer the

same brace forces. Of course, on the healthy subject these forces produce a negative effect and increase the Cobb angle to 7.2 de- grees.

* 1. *Validation of the sensitivity matrix*

As discussed in [Section 3](#_bookmark16), the sensitivity matrix **S** provides a linear approximation of the deformation due to changes in param- eters. Before carrying out sensitivity analysis of the model param- eters using the sensitivity matrix, we validate its accuracy. We do this by comparing the deformation predicted through the sensitiv- ity matrix **S**, with the actual change of deformation, on both the healthy and scoliosis subjects.

We have tested changes of 1% and 10% to all parameters, i.e., values of parameter ratios of 1.01 and 1.1 respectively. [Fig. 4](#_bookmark28) sum- marizes the error induced by the sensitivity matrix for the six most relevant parameters, for parameter changes of 10%. Errors for the rest of the parameters are negligible. In the ﬁgure, we depict the relative RMS error of the feature vector of each vertebra, normal- ized by the largest RMS feature vector among all vertebrae.

We can conclude that the sensitivity matrix is accurate, as the maximum error for a parameter change of 10% is less than 10% of the maximum displacement. We also identify six parameters

that produce larger error, roughly in this order for the scolio- sis subject: shear modulus *μ*, intervertebral axial rotation stiff- ness *kv,β* , intervertebral lateral bending stiffness *kv,γ* , interverte-

bral ﬂexion/extension stiffness *kv,α* , intervertebral lateral transla- tion stiffness *kv,x* , and Lamé’s parameter *λ*. For the healthy subject, the error is distributed differently, as shown in [Fig. 4](#_bookmark28).

From an analysis of the error, we can also observe that larger error is present at the lower thoracic and upper lumbar part. This is no surprise, as this is the region where the lateral force is acting.

* 1. *Sensitivity analysis*

The sensitivity matrix allows us to compare sensitivity of the deformation to the various parameters, and hence understand which parameters, and in what combinations, require tighter tol- erance for accurate prediction of deformations. We start by eval- uating sensitivity with respect to individual parameters. This boils down to analyzing the columns in **S** separately. [Fig. 5](#_bookmark29) illustrates the change in deformation of the spine produced by a 100% change in each parameter, for both subjects shown in [Fig. 3](#_bookmark27). The images are color-coded per vertebra, based on the RMS of the feature vector of each vertebra. They display deformations for the six dominant parameters, roughly in this order: shear modulus *μ*, intervertebral lateral bending stiffness *kv,γ* , intervertebral lateral translation stiff- ness *kv,x* , intervertebral axial rotation stiffness *kv,β* , intervertebral



**Fig. 3.** We use two test cases for our study: a reference healthy subject (left) and a subject suffering from scoliosis (right). The grayscale images depict the torso models at rest, with the skeletal elements of our model and the soft-tissue (drawn semitransparent). The color images show the deformation of each vertebra under external forces (blue means no deformation and red means maximum deformation), as well as the resulting Cobb angle. Note that these images are drawn from the back. The center of the ﬁgure also shows the surface force ﬁeld used in the experiments, which was obtained by simulating contact of the scoliosis subject with the pictured brace. The same force ﬁeld was transferred to the surface of the healthy subject. (For interpretation of the references to color in this ﬁgure legend, the reader is referred to the web version of this article.)



**Fig. 4.** Validation of the sensitivity matrix. The images illustrate, for both subjects and the test conditions indicated in [Fig. 3](#_bookmark27), the error (in logarithmic scale) between deformations estimated using the sensitivity matrix vs. deformations computed by solving full static equilibrium problems. The images show errors resulting from a 10% change of individual parameters, computed for each vertebra independently, and normalized with respect to the largest vertebra displacement. We depict only the parameters with highest sensitivity.



**Fig. 5.** Sensitivity of the parameters of the torso model. The images illustrate, for both subjects and the test conditions depicted in [Fig. 3](#_bookmark27), the displacement of vertebrae (in logarithmic scale) corresponding to a 100% change of individual parameters. We depict only the most sensitive parameters.



**Fig. 6.** Eigenanalysis of sensitivity. The bar plots illustrate, for both subjects depicted in [Fig. 3](#_bookmark27), the four main right singular vectors of the sensitivity matrix, scaled by their corresponding singular values and normalized to indicate RMS values per feature point (i.e., vertebra OBB corner). We depict only the vector components corresponding to the most sensitive parameters.

ﬂexion/extension stiffness *kv,α* , and Lamé’s parameter *λ*. A com- parison between the scoliosis and healthy subjects reveals slightly higher sensitivity for the scoliosis subject. The difference is most notable for the intervertebral axial rotation stiffness *kv,β* , which is not sensitive at all in the case of the healthy subject.

[Figure 6](#_bookmark30) presents the four ﬁrst right singular vectors for both subjects, following the mathematical derivation described in [Section 3.2](#_bookmark19). The singular vectors are scaled by their correspond- ing singular value, and divided by 144, where 144 is the number of vertebra corners, to indicate RMS magnitudes per corner. From the combination of results in [Figs. 5](#_bookmark29) and [6](#_bookmark30) we can draw some conclusions. Soft-tissue parameters, intervertebral lateral stiffness (both translation and rotation), intervertebral axial rotation stiff- ness, and intervertebral ﬂexion/extension stiffness are clearly the parameters that require careful estimation for accurate model pre- diction. Moreover, the intervertebral parameters gain relevance for scoliosis subjects, but the correct estimation of soft-tissue parame- ters cannot be neglected. Intervertebral axial rotation stiffness is hardly relevant for healthy subjects, but it gains importance for scoliosis subjects. This conclusion matches expectations, as scolio- sis subjects tend to suffer some axial rotation of the spine in their rest conﬁguration.

√

Lastly, we have derived tolerances for model parameters, fol- lowing the mathematical derivation in [Section 3.2](#_bookmark19). In [Table 2](#_bookmark31), we report both uniform tolerance [(12)](#_bookmark22) and sensitivity-based tol- erance [(13)](#_bookmark23) for the model parameters, for an error bound of 5 mm. We report tolerances only for the scoliosis subject. We apply the 5 mm error bound in two different ways. First, we consider an ab-

solute bound on all vertebra corners. To this end, we study sepa- rately the sensitivity matrix **S***i* of the feature point **z***i* of each cor- ner, and we retain the tightest tolerance for each parameter. Sec- ond, we consider an RMS error of 5 mm on the complete spine, and we study the complete sensitivity matrix **S**. With 144 verte- bra corners, this amounts to an error bound of *E* = 144 × 5 mm

= 6 cm for the expressions in [Section 3.2](#_bookmark19).

√

For an absolute bound of 5 mm per vertebra corner, the uni- form parameter tolerance is 26%. With sensitivity-based tolerance, only ﬁve parameters (highlighted in the table) require a tighter tol-

erance (in this order: intervertebral lateral bending stiffness *kv,γ* , shear modulus *μ*, intervertebral axial rotation stiffness *kv,β* , in- tervertebral ﬂexion/extension stiffness *kv,α* , intervertebral lateral translation stiffness *kv,x* ). For all other parameters, sensitivity-

based tolerances are relaxed. Some tolerances are so wide that they exit the linear regime. Those values cannot be taken liter- ally; they represent that the tolerances allowed are very large. But it is possible to draw some important conclusions. The wide tol- erances validate several of our model design choices, such as the approximations in rib joints and cartilages. We also pay partic- ular attention to the parameterization of the intervertebral ﬂex- ion/extension stiffness *kv,α* . Previous work used asymmetric stiff- ness for this rotation (with a deviation of 17% with respect to the average value) [[27]](#_bookmark64). Our analysis validates that, under the loading conditions we studied, a choice of symmetric stiffness is suﬃcient for an error bound of 5 mm, as the parameter tolerance is precisely 17%. However, asymmetric stiffness would be required to enforce a tighter error bound and/or different loading conditions.

### Discussion

This work describes the development of a comprehensive model of torso and spine biomechanics, together with computa- tional tools that enable sensitivity analysis of the model and its parameters. This analysis sets guidelines for the parameter estima- tion of personalized torso and spine models in the context of com- putational design of scoliosis braces.

**Table 2**

Tolerances (in percentage) for all parameters in the model, to guarantee an error bound of 5 mm in spine deformation. We compute tolerances in two ways: uniform tolerance (12) (column ‘All’), and sensitivity-based per-parameter tolerance (13) (rest of the columns). We also enforce bounds in two ways: a maximum error of 5 mm among vertebra OBB corners, and an RMSE of 5 mm across corners.

 *μ*

7

10

*λ*

36

39

From the analysis, we draw the following main conclusions. Not surprisingly, the intervertebral lateral stiffness (both transla- tion and rotation) is one of the factors that requires careful model- ing. But perhaps more surprisingly, the shear modulus of the soft tissue in the torso requires equal care. For subjects suffering from scoliosis, but not for healthy subjects, intervertebral axial rotation stiffness must be carefully modeled too. For moderate accuracy, all other mechanical parameters of the torso model can be largely approximated. In particular, complex anatomical elements such as rib joints can use simple approximate models. Even the interverte- bral ﬂexion/extension stiffness, which in other works is considered asymmetric to distinguish ﬂexion and extension, can be approxi- mated unless high accuracy is needed.

*ks,y*

200

134

*ks,z*

136

192

*ks,α*

274

284

*ks,β*

2464

486

*ks,γ*

256

236

The study described in the paper could be further extended to understand additional factors. In the experiments, brace forces were kept constant during sensitivity analysis, and minor adapta- tions of these forces were ignored. The soft tissue of the torso was modeled as homogeneous, and the possible effects of material het- erogeneity remain unclear. One of such effects could be errors on the strain and stress distributions, even if the overall positions are similar. Similarly, large segments of the spine were modeled as ho- mogeneous, but local changes of intervertebral stiffness are known to occur in reality, both due to deformities such as scoliosis itself, or simply because more deformed areas tend to be stiffer. Nev- ertheless, note that our model is not personalized; therefore, the stiffness values can be considered default parameters. For a per- sonalized model, the parameters for each joint should be different and they should meet the tolerances indicated by the sensitivity analysis. All experiments were carried out ignoring muscle forces, under one of two possible assumptions, that the average force over time is negligible, or that scoliosis braces produce most of their ef- fect through periods of little muscle activity. It would be interest- ing to extend the study with a model of muscle activity, a larger body model to include full handling of the pelvis and the cervi- cal region, and motion of the body in various postures. The study was executed on spine data sets of a subject suffering from scolio- sis and a healthy subject, but the range of scoliosis types is large. The study could be extended by analyzing sensitivity on data sets corresponding, for example, to different types within Lenke’s clas- siﬁcation [[30]](#_bookmark68).

*kv,z*

77

74

 *kv,α kv,β kv,γ*

*kr,x*

380

131

*kr,y*

709

446

*kr,z*

892

363

*kr,α*

131

128

*kr,β*

290

135

*kr,γ*

26

53

*ks,x*

638

340

17

26

14

24

7

11

The proposed methodology for sensitivity analysis is not re-

*kv,y*

42

61

stricted to the torso model presented in this paper. The method- ology is general, and could be applied to other models, with dif- ferent components and/or different design choices. In this regard, it is important to design parameterizations which balance appro- priately the relevance of different model components and their pa- rameters. To this end, we parameterize our model using parameter ratios over default parameter values, and we group parameter ra- tios of multiple model components. Our approach could be gener- alized by normalizing parameters based on some objective metric, such as their contribution to the system energy.

Parameters

All

 *kv,x*

20

35

Tol. (%), 5 mm max. Tol. (%), 5 mm RMSE

26

46

We started this work as part of a broader project, concerned with the development of personalized brace design tools for ado- lescent idiopathic scoliosis. In the design of such tools, we encoun- tered unknowns about the model choices, but most importantly we encountered many unknowns about the required accuracy in the parameterization. To address these unknowns, we developed and applied the methodology for sensitivity analysis presented in

this paper. Thanks to the conclusions of the paper, we have de- veloped a methodology for personalized parameter estimation, and we are currently applying this methodology to a cohort of scolio- sis patients. This continuation study will allow us to evaluate the accuracy of the personalized models, and will also provide data to

### Appendix B. Sensitivity-based tolerance

Alternatively, one can square the bound [(A.1)](#_bookmark43), and substitute the SVD [(11)](#_bookmark21) to relate the parameter change to the error bound as:

*E*2 *> ξ T* **S***T* **S** *ξ* = *σ* 2 *(***v***T ξ )*2*,* (B.1)

estimate better default parameter values. *j j*

*j*

{ }

Finally, even though the tools developed in this work are gen-

eral, the conditions of the study and the conclusions are particular to the analysis of the effect of scoliosis braces. The tools and the analysis could be generalized to other applications of biomechani- cal models of the torso, such as posture or gait analysis, or even to

where the summation runs over all singular values *σ j* and corre-

sponding right singular vectors **v** *j* of **S**.

{ }

In this summation, one can bound each addend by a fraction of the bound proportional to the corresponding singular value, i.e.,

*σ j*

**v** *j ξ*

*<* L *σ ,* ∀ *j* ⇒

*j*

*j*

L *σi*

(B.2)

*i*

biomechanical models of other parts of the body.

2 ( *T* 2

*σ j E*2

*i*

*i*

√*σ* 1**v***T ξ* 1 *<* √ *E ,* ∀ *j.*

### Funding

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie

Given *Nξ* parameters, one can bound the dot product **v***T ξ* by

bounding each addend by a fraction 1 of the total bound:

*j*

*Nξ*

/*σ* 1**v** 1 1 *ξ* 1 *<*  *E ,* ∀ *j.* (B.3)

*j*

*k, j*

*k*

*Nξ*

/L*i σ*

*i*

By choosing the singular value that produces the largest weight

Sklodowska-Curie grant agreement No. 764644, Rainbow. This pa-

per only contains the author’s views and the Research Executive

Agency and the Commission are not responsible for any use that may be made of the information it contains.

/*σ j* 1**v**

*k, j*

1, we obtain the sensitivity-based tolerance:

# *E*

### Ethical approval

1 *ξk*1 *< N*

/L *σ*

max (√*σ* 1**v**

1 *.* (B.4)

All data used in this paper was provided, already anonymized, by AnatoScope SA. Data acquisition was carried out after ethical clearance. No other aspect of this work triggered ethical issues.

*ξ*

*i*

*i*

*j*

*j*

*k, j*

### Declaration of Competing Interest

The authors declare that they have no known competing ﬁnan- cial interests or personal relationships that could have appeared to inﬂuence the work reported in this paper.

### Acknowledgements

The authors would like to thank the team members of Anato- Scope SA Francois Faure, Matthieu Nesme, Thomas Lemaire, Uly- see Vimont, Nicolas Comte, Elie Cattan, Vincent Luboz and Maxime Tournier for their valuable contribution in the simulation soft- ware, the provision of the subjects’ geometry and the fruitful dis- cussions about the mechanical concept. Furthermore, they would like to thank Hamed Shayestehpour and Faezeh Moshfeghifar for their valuable contribution in the conception of the biomechanical model and Cristian Romero for his priceless help in the visualiza- tion of the results.

### Appendix A. Uniform tolerance

We wish to ﬁnd the largest parameter change *ξ* such that the resulting change of feature vector **z** satisﬁes an error bound *E*, i.e.,

1 1

1 1

*E >* 1 **z**1 = 1**S** *ξ* 1*.* (A.1)

For any parameter change, the resulting change of the feature vec- tor can be bounded based on the largest singular value *σ*max by 1**S** *ξ* 1 *< σ*max 1 *ξ* 1. Then, the bound [(A.1)](#_bookmark43) is satisﬁed if

### Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.cmpb.2021.106125](https://doi.org/10.1016/j.cmpb.2021.106125).

### References

1. [A. Ahmad, N.A. Abu Osman, H. Mokhtar, W. Mehmood, N.A. Kadri, Analysis of the interface pressure exerted by the Chêneau brace in patients with dou- ble-curve adolescent idiopathic scoliosis, Proc. Inst. Mech.Eng. Part H 233 (9) (2019) 901–908.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0001)
2. [J. Allard](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [S. Cotin](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [F. Faure](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [P.-J. Bensoussan](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [F. Poyer](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [C. Duriez](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002), [H. Delingette](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002),

[L. Grisoni, SOFA - an open source framework for medical simulation, in: MMVR 15 - Medicine Meets Virtual Reality, in: Studies in Health Technology and In- formatics, vol. 125, IOP Press, Palm Beach, United States, 2007, pp. 13–18.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0002)

1. [R.M.C. Aroeira, A.E.M. Pertence, D.T. Kemmoku, M. Greco, The effect of hy- pokyphosis on the biomechanical behavior of the adolescent thoracic spine, J. Braz. Soc. Mech. Sci. Eng. 40 (3) (2018) 128.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0003)
2. [R.M.C. Aroeira, A.E.d.M. Pertence, D.T. Kemmoku, M. Greco, Three-dimensional geometric model of the middle segment of the thoracic spine based on graph- ical images for ﬁnite element analysis, Res. Biomed. Eng. 33 (2) (2017) 97–104.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0004)
3. [S. Bauer, E. Keller, D. Paulus, Sensitivity analysis of intervertebral disc param- eters: MBS model of the lumbar spine, Int. J. Eng. Appl.Sci. 3 (5) (2016).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0005)
4. [R. Bayoglu, P.E. Galibarov, N. Verdonschot, B. Koopman, J. Homminga, Twente spine model: a thorough investigation of the spinal loads in a complete and coherent musculoskeletal model of the human spine, Med. Eng. Phys. 68 (2019) 35–45.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0006)
5. [R. Bayoglu, O. Guldeniz, N. Verdonschot, B. Koopman, J. Homminga, Sensitiv- ity of muscle and intervertebral disc force computations to variations in mus- cle attachment sites, Comput. Methods Biomech. Biomed.Eng. 22 (14) (2019) 1135–1143.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0007)
6. [A. Bisschop](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008), [M.G. Mullender](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008), [I. Kingma](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008), [T.U. Jiya](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008), [A.J. van der Veen](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008), [J.C. Roos](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008),

[J.H. van Dieën, B.J. van Royen, The impact of bone mineral density and disc degeneration on shear strength and stiffness of the lumbar spine following laminectomy, Eur. Spine J. (2012).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0008)

1. [M. Botsch, L. Kobbelt, Real-time shape editing using radial basis functions, Comput. Graphics Forum 24 (3) (2005).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0009)
2. [A. Choi, Y. Zheng, Estimation of Young’s modulus and Poisson’s ratio of soft tis- sue from indentation using two different-sized indentors: ﬁnite element anal- ysis of the ﬁnite deformation effect, Med. Biol. Eng. Comput. 43 (2) (2005) 258–264.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0010)
3. [J. Cholewicki, J. Crisco, T. Oxland, I. Yamamoto, M. Panjabi, Effects of posture](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0011)

[and structure on three-dimensional coupled rotations in the lumbar spine: a](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0011)

*E > σ*

max

 *E*

1 *ξ* 1 ⇒ 1 *ξ* 1 *< σ .* (A.2)

max

[biomechanical analysis, Spine 21 (1996) 2421–2428.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0011)

1. [C.L. Chung, D.M. Kelly, J.R. Sawyer, J.R. Steele, T.S. Tate, C.K. Bateman, D.J. Di-](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0012) [Angelo, Mechanical testing of a novel fastening device to improve scoliosis](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0012)

Given *Nξ* parameters, one can bound the norm *ξ* by bound- ing uniformly all parameter changes *ξk* , which yields the uni- form tolerance:

{ }

1 1

# *E*

[bracing biomechanics for treating adolescent idiopathic scoliosis, Appl. Bion- ics Biomech. 2018 (2018).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0012)

1. [J. Clin, C.-E. Aubin, H. Labelle, Virtual prototyping of a brace design for the cor- rection of scoliotic deformities, Med. Biol. Eng. Comput. 45 (5) (2007) 467–473.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0013)
2. [J. Clin, C.-É. Aubin, N. Lalonde, S. Parent, H. Labelle, A new method to include](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0014)

1 *ξk*1 *<* /

*Nξ σ*max

*.* (A.3)

[the gravitational forces in a ﬁnite element model of the scoliotic spine, Med. Biol. Eng. Comput. 49 (8) (2011) 967–977.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0014)

1. [J. Clin, C.-E. Aubin, S. Parent, A. Sangole, H. Labelle, Comparison of the biome- chanical 3D eﬃciency of different brace designs for the treatment of scoliosis using a ﬁnite element model, Eur. Spine J. 19 (7) (2010) 1169–1178.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0015)
2. [N. Cobetto, C.E. Aubin, S. Parent, J. Clin, S. Barchi, I. Turgeon, H. Labelle, Effec- tiveness of braces designed using computer-aided design and manufacturing (CAD/CAM) and ﬁnite element simulation compared to CAD/CAM only for the conservative treatment of adolescent idiopathic scoliosis: a prospective ran- domized controlled trial, Eur. Spine J. 25 (2016) 3056–3064.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0016)
3. [N. Cobetto, C.-É. Aubin, S. Parent, S. Barchi, I. Turgeon, H. Labelle, 3D correction of AIS in braces designed using CAD/CAM and FEM: a randomized controlled trial, Scoliosis Spinal Disord. 12 (1) (2017) 1–8.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0017)
4. [M. de Zee, L. Hansen, C. Wong, J. Rasmussen, E.B. Simonsen, A generic detailed rigid-body lumbar spine model, J. Biomech. 40 (2007) 1219–1227.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0018)
5. [A.H. Dicko, N. Tong-Yette, B. Gilles, F. Faure, O. Palombi, Construction and val- idation of a hybrid lumbar spine model for the fast evaluation of intradiscal pressure and mobility, World Acad. Sci. Eng.Technol. (2015).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0019)
6. [E. Dong, L. Shi, J. Kang, D. Li, B. Liu, Z. Guo, L. Wang, X. Li, Biomechanical char- acterization of vertebral body replacement in situ: effects of different ﬁxation strategies, Comput. Methods Programs Biomed. (2020) 105741.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0020)
7. [H. Ersoy, A. Mug˘an, Design sensitivity analysis of structures based upon the singular value decomposition, Comput. Methods Appl. Mech.Eng. 191 (32) (2002) 3459–3476.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0021)
8. [S.M. Finley, D.S. Brodke, N.T. Spina, C.A. DeDen, B.J. Ellis, FEBio ﬁnite element models of the human lumbar spine, Comput. Methods Biomech. Biomed.Eng. 21 (6) (2018) 444–452.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0022)
9. [D. Gignac, C.-É. Aubin, J. Dansereau, H. Labelle, Optimization method for 3D bracing correction of scoliosis using a ﬁnite element model, Eur. Spine J. 9 (3) (2000) 185–190.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0023)
10. S. Gottschalk, M.C. Lin, D. Manocha, OBBTree: a hierarchical structure for rapid interference detection, in: Proceedings of the 23rd Annual Conference on Computer Graphics and Interactive Techniques, in: SIGGRAPH ’96, Asso- ciation for Computing Machinery, New York, NY, USA, 1996, pp. 171–180, doi:[10.1145/237170.237244](https://doi.org/10.1145/237170.237244).
11. [A. Guy, H. Labelle, S. Barchi, E. Audet-Duchesne, N. Cobetto, S. Parent, M. Rai-](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0025) [son, C.-É. Aubin, Braces designed using CAD/CAM combined or not with ﬁnite element modeling lead to effective treatment and quality of life after 2 years: a randomized controlled trial, Spine 46 (1) (2021) 9–16.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0025)
12. [A. Hostettler, D. George, Y. Rémond, S.A. Nicolau, L. Soler, J. Marescaux, Bulk modulus and volume variation measurement of the liver and the kidneys in vivo using abdominal kinetics during free breathing, Comput. Methods Pro- grams Biomed. 100 (2) (2010) 149–157.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0026)
13. [D. Ignasiak, S. Dendorfer, S.J. Ferguson, Thoracolumbar spine model with artic- ulated rib cage for the prediction of dynamic spinal loading, J. Biomech. (2015).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0027)
14. [T.L. Lasswell, D.S. Cronin, J.B. Medley, P. Rasoulinejad, Incorporating ligament laxity in a ﬁnite element model for the upper cervical spine, Spine J. 17 (11) (2017) 1755–1764.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0028)
15. [F. Le Navéaux, A.N. Larson, H. Labelle, X. Wang, C.-É. Aubin, How does implant distribution affect 3d correction and bone-screw forces in thoracic adolescent idiopathic scoliosis spinal instrumentation? Clin. Biomech. 39 (2016) 25–31.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0029)
16. [L.G. Lenke](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030), [R.R. Betz](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030), [J. Harms](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030), [K.H. Bridwell](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030), [D.H. Clements](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030), [T.G. Lowe](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030),

[K. Blanke, Adolescent idiopathic scoliosis: a new classiﬁcation to determine extent of spinal arthrodesis, J. Bone Joint Surg. 83 (8) (2001) 1169–1181.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0030)

1. [Y.-C. Liao, C.-K. Feng, M.-W. Tsai, C.-S. Chen, C.-K. Cheng, Y.-C. Ou, Shape mod- iﬁcation of the boston brace using a ﬁnite-element method with topology op- timization, Spine 32 (26) (2007) 3014–3019.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0031)
2. [C. Liebsch, N. Graf, K. Appelt, H.-J. Wilke, The rib cage stabilizes the human thoracic spine: an in vitro study using stepwise reduction of rib cage struc- tures, PLoS One 12 (6) (2017) e0178733.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0032)
3. [C. Liebsch, N. Graf, H.-J. Wilke, In vitro analysis of kinematics and elastostatics of the human rib cage during thoracic spinal movement for the validation of numerical models, J. Biomech. 94 (2019) 147–157.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0033)
4. [I. Loukos, C. Zachariou, C. Nicolopoulos, D. Korres, N. Efstathopoulos, Analysis of the corrective forces exerted by a dynamic derotation brace (DDB), Prosthet. Orthot. Int. 35 (4) (2011) 365–372.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0034)
5. [C.T. McKee, J.A. Last, P. Russell, C.J. Murphy, Indentation versus tensile mea- surements of Young’s modulus for soft biological tissues, Tissue Eng. Part B 17 (3) (2011) 155–164.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0035)
6. [S.P. Moroney, A.B. Schultz, J.A. Miller, G.B. Andersson, Load-displacement prop- erties of lower cervical spine motion segments, J. Biomech. 21 (9) (1988) 769–779.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0036)
7. [W.-Z. Nie, M. Ye, Z.-D. Liu, C.-T. Wang, The patient-speciﬁc brace design and biomechanical analysis of adolescent idiopathic scoliosis, J. Biomech. Eng. 131 (4) (2009).](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0037)
8. [R. Ogden, Non-linear Elastic Deformations, Dover Publications, 1997](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0038).
9. [M.M. Panjabi, R.A. Brand Jr., A.A. White III, Three-dimensional ﬂexibility and stiffness properties of the human thoracic spine, J. Biomech. 9 (4) (1976) 185–192.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0039)
10. [M.M. Panjabi, T. Oxland, I. Yamamoto, J.J. Crisco, Mechanical behavior of the human lumbar and lumbosacral spine as shown by three-dimensional load-](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0040)

[-displacement curves, J. Bone Joint Surg. Am. 76 (3) (1994) 413–424.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0040)

1. [M. Putzer, I. Ehrlich, J. Rasmussen, N. Gebbeken, S. Dendorfer, Sensitivity of lumbar spine loading to anatomical parameters, J. Biomech. 49 (6) (2016) 953–958.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0041)
2. [D. Périé, C. Aubin, Y. Petit, M. Beauséjour, J. Dansereau, H. Labelle, Boston brace correction in idiopathic scoliosis: a biomechanical study, Spine 28 (15) (2003) 1672–1677.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0042)
3. [M.E. Raabe, A.M. Chaudhari, An investigation of jogging biomechanics us- ing the full-body lumbar spine model: model development and validation, J. Biomech. 49 (7) (2016) 1238–1243.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0043)
4. [A. Safari, H. Parsaei, A. Zamani, B. Pourabbas, A semi-automatic algorithm for estimating cobb angle, J. Biomed. Phys. Eng. 9 (3) (2019) 317.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0044)
5. [M. Senteler, A. Aiyangar, B. Weisse, M. Farshad, J.G. Snedeker, Sensitivity of intervertebral joint forces to center of rotation location and trends along its migration path, J. Biomech. 70 (2018) 140–148.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0045)
6. [J.R. Shewchuk, Tetrahedral mesh generation by Delaunay reﬁnement, in: Pro- ceedings of the Fourteenth Annual Symposium on Computational Geometry, 1998, pp. 86–95.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0046)
7. [C. Song, A. Alijani, T. Frank, G. Hanna, A. Cuschieri, Elasticity of the living ab- dominal wall in laparoscopic surgery, J. Biomech. 39 (3) (2006) 587–591.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0047)
8. [T. Andriacchi, A. Schultz, T. Belytschko, J. Galante, A model for studies of me- chanical interactions between the human spine and rib cage, J. Biomech. 7 (1974) 497–507.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0048)
9. [J. Van den Hout, L. Van Rhijn, R. Van den Munckhof, A. Van Ooy, Interface corrective force measurements in boston brace treatment, Eur. Spine J. 11 (4) (2002) 332–335.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0049)
10. [F. van Keulen, R. Haftka, N. Kim, Review of options for structural design sen- sitivity analysis. Part 1: linear systems, Comput. Methods Appl. Mech.Eng. 194 (30) (2005) 3213–3243.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0050)
11. [C. Vergari](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [Z. Chen](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [L. Robichon](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [I. Courtois](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [E. Ebermeyer](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [R. Vialle](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051), [T. Langlais](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051),

[R. Pietton, W. Skalli, Towards a predictive simulation of brace action in adoles- cent idiopathic scoliosis, Comput. Methods Biomech. Biomed.Eng. (2020) 1–8.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0051)

1. [C. Vergari, I. Courtois, E. Ebermeyer, H. Bouloussa, R. Vialle, W. Skalli, Experi- mental validation of a patient-speciﬁc model of orthotic action in adolescent idiopathic scoliosis, Eur. Spine J. 25 (10) (2016) 3049–3055.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0052)
2. [W. Wang, G.R. Baran, R.R. Betz, A.F. Samdani, J.M. Pahys, P.J. Cahill, The use of ﬁnite element models to assist understanding and treatment for scoliosis: a review paper, Spine Deformity 2 (1) (2014) 10–27.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0053)
3. [H.-J. Wilke, A. Herkommer, K. Werner, C. Liebsch, In vitro analysis of the seg- mental ﬂexibility of the thoracic spine, PLoS One 12 (5) (2017) e0177823.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0054)
4. [M. Xu, J. Yang, I.H. Lieberman, R. Haddas, Lumbar spine ﬁnite element model for healthy subjects: development and validation, Comput. Methods Biomech. Biomed.Eng. 20 (1) (2017) 1–15.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0055)
5. [T. Zander, M. Dreischarf, H. Schmidt, Sensitivity analysis of the position of the intervertebral centres of reaction in upright standing–a musculoskeletal model investigation of the lumbar spine, Med. Eng. Phys. 38 (3) (2016) 297–301.](http://refhub.elsevier.com/S0169-2607%2821%2900200-5/sbref0056)