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Electrostatic forces between sharp tips and metallic and dielectric samples

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A detailed analysis of electrostatic interactions between a dc-biased tip and a metallic or insulating sample is presented. By using a simple method to calculate capacitances and forces, tip shape effects on the force versus tip-sample distance curves are dicussed in detail. For metallic samples the force law, except for a constant background, only depends on the tip radius of curvature. In contrast, for dielectric samples the forces depend on the overall geometry of the tip. Interestingly, we found that the contact (adhesion) force does not depend on the tip size and is bound by a simple expression which only depends on the applied bias and the sample dielectric constant. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1424478]

By applying a voltage between a force microscope tip and a sample, electrostatic force microscopy (EFM) has been used to analyze different surface properties at the nanoscale.[1–11] As in other scanning probe microscopy techniques, the interpretation of the EFM images is not always evident.[12] The detailed shape and dimensions of the tip must then be taken into account for a precise calculation of both force and capacitance.[13,14] Most of the theoretical studies on EFM have been focused on the force and capacitance between a microscope tip and a *metallic* sample.[13–15] However, tip shape effects on the electrostatic interactions with insulating samples have not been studied in detail. In this letter, we study the capacitances and electrostatic forces between a tip and a metallic or insulating sample as a function of different tip shape parameters.

We consider a metallic probe tip at a distance D from a flat homogeneous semi-infinite insulating surface characterized by a dielectric constant ϵ. A dc-bias V_0 is applied between the tip and sample. The tip is assumed to have a total length L and a conical shape with a half angle θ with rounded ends and an apex radius R (see Fig. 1). Our numerical calculations are based on a generalized image-charge method[16] originally developed to calculate the three-dimensional electron potential energy for arbitrary shaped (axial symmetric) tips in field emission diode geometries.

Let us first consider the interaction between the tip and a homogeneous metallic sample. As long as the tip-sample distance is smaller than the tip radius ($D/R < \approx 1$) the main contribution to the electrostatic force comes from the interaction of the tip apex with the sample.[13,15] It is then likely that, in this range of tip-sample distances, the force law would be close to that of the sphere-plane model.[13] The contribution of the macroscopic part of the conical tip is expected to have logarithmic dependence with L/D[13,17] and, for small distances, would give an almost constant contribution. Our results show that this is indeed the case. We have performed an extensive calculation of force F versus distance D curves for a wide range of tip-shape parameters (L,θ,R,D). For distances smaller than the tip radius, the results follow a simple force law given by

$$\frac{F}{\pi\epsilon_0 V^2} = A(\theta, L/R) + B\frac{R}{D}, \tag{1}$$

where $A(\theta,L/R)$ is a constant which depends on the "macroscopic" geometry and $B \approx -1$ for all geometries analyzed. Figure 2(a) shows a typical force versus distance curve for a tip with $\theta = 10°$, $L/R = 500$ (full dots) together with the best fit based on Eq. (1) (continuous line). The constant background A increases its absolute value with increasing angle θ for a fixed length L/R, while for fixed angle, A increases logarithmically with tip length L [see Figs. 2(b) and 2(c)]. For metallic samples, the tip radius can be determined from the slope of the force-distance characteristics (as long as $D < R$).

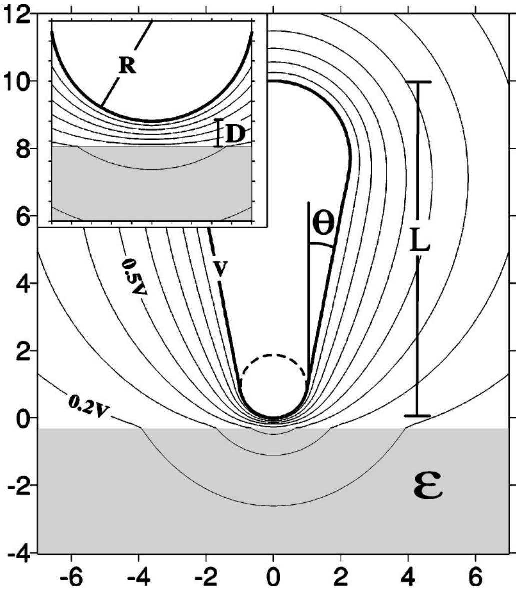

FIG. 1. Scheme of the tip-sample system. The tip shape is characterized by its angle θ, its length L, and the tip radius of curvature R. The equipotential lines correspond to a metallic tip at a bias V (with $L=10R$, $D=0.5R$, $\theta=10°$) in front of a dielectric sample with $\epsilon=5.4$.

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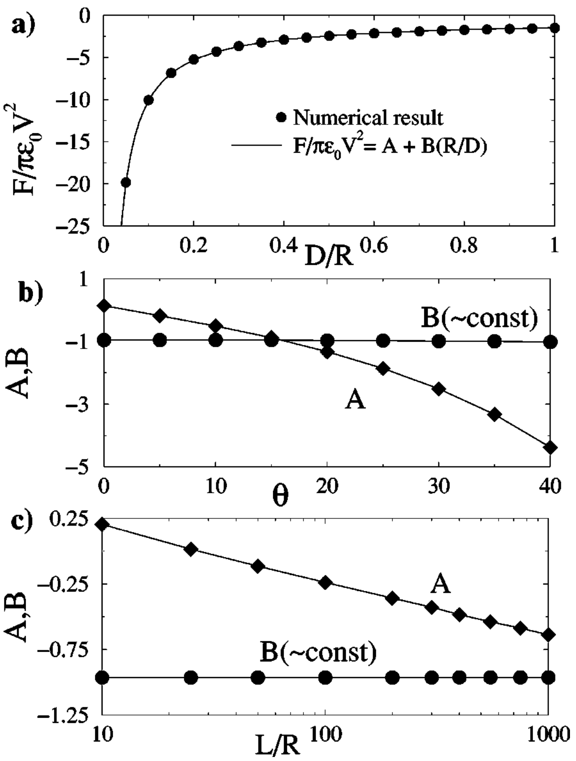

FIG. 2. (a) Electrostatic force between a tip (tip radius R, $L=500R$, $\theta=10°$) and a plane metallic sample vs the tip sample distance D/R. Continuous line is the best fit of the numerical results (dots) to Eq. (2). (b) Best fitting parameters A and B [see Eq. (2)] as a function of the tip angle θ for a fixed tip length $L=500R$. (c) Best fitting parameters A and B as a function of the tip length L/R for a fixed angle $\theta=10°$.

In contrast with metallic surfaces, the electrostatic tip interaction with a dielectric sample shows a very interesting, and sometimes apparently paradoxical, behavior as a function of tip shape parameters. Some of the differences between metallic and dielectric samples can be illustrated with the simplest spherical tip. The exact solution for the sphere can be written as an infinite sum over multiple image charges (see for example Ref. 18). In the limit $D\rightarrow 0$, i.e., when the sphere is in contact with the dielectric sample (and assuming there is no charge transfer between them), we found a simple closed expression for the electrostatic contact force

$$\frac{F_0^{\mathrm{sp}}}{\pi\epsilon_0 V^2} = -\frac{2}{3\beta}\left[\frac{\beta}{(1-\beta)^2}+\ln(1-\beta)\right], \tag{2}$$

where $\beta=(\epsilon-1)/(\epsilon+1)$. In other words, the contact force *does not depend on the radius of curvature*. This is a general property that follows from a simple scaling argument. Since the capacitance is a first order homogeneous function of the length variables $[C(kL,kR,kD,\theta)=kC(L,R,D,\theta)]$, the force is invariant under spatial scaling, $F(kL,kR,kD,\theta)=F(L,R,D,\theta)$.[17] In the limit $D\rightarrow 0$, and setting $k=1/R$, the force F_0 is a function of the ratio L/R, $F_0(L/R,\theta)$. Interestingly, in the limit where $L\rightarrow\infty$ (or D, $R\rightarrow 0$) the force only depends on the angle θ. In other words, for a dielectric sample the *electrostatic contact force does not depend on the tip size but only depends on the tip shape*.

In Fig. 3(a) we plot our results for the contact force versus L/R for different angles θ [$L/R=2$ corresponds to a spherical tip, i.e., $F_0(L/R=2,\theta)=F_0^{\mathrm{sp}}$]. For a given tip length, the contact force increases with the tip radius. Since $F_0=F_0(L/R,\theta)$, this implies that, for a given radius, the force decreases with increasing tip length, i.e., the longer the tip-cone is the smaller the contact force is. In the limit $L/R\rightarrow\infty$, the contact force saturates to the contact force of a sharp cone of angle θ. The strongest contact force corresponds to a spherical tip $L/R=2$, i.e., contact forces are always smaller than $|F_0|$ given by Eq. (2) (see Fig. 4). For a given cone angle θ, there is a lower bound for the contact force given by the force of a sharp cone, $F_0(\infty,\theta)$. This implies that, for a given angle θ the electrostatic contact force is bounded between the results for a spherical tip and those of a sharp cone, $F_0(\infty,\theta)<F_0<F_0^{\mathrm{sp}}$. In Fig. 4 we have plotted the contact force versus β for these two limits.[19]

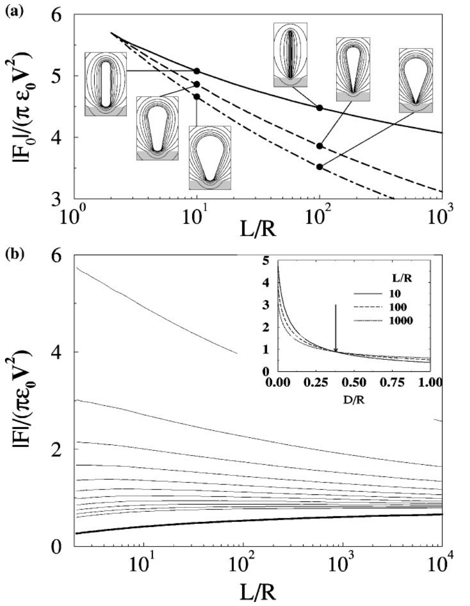

FIG. 3. (a) Absolute value of the contact force $|F_0|$ between a metallic tip and a plane dielectric sample ($\epsilon=5.4$) vs the ratio between tip length and radius L/R for different tip angles $\theta=0°,10°,20°$ ($\theta=0$ corresponds to a nanotube-like tip). (b) Absolute value of the electrostatic force $|F|$ vs L/R at different tip-sample distances D/R ($0,0.05,0.1,\ldots,0.5$ from top to bottom). Thick line corresponds to $D/R=1$. The inset shows the force vs distance D/R for different tip lengths L/R. ($\theta=10°$, $\epsilon=5.4$).

The situation is reversed when the tip-sample distance is larger than the tip radius ($D\gtrsim R$). In this case, the force must be a function $F(L/D,\theta)$ which decreases with the tip-sample distance. Then, for a fixed distance D ($L\gg D\gtrsim R$), the force increases with the tip length L. In the limit $L\rightarrow\infty$, the force

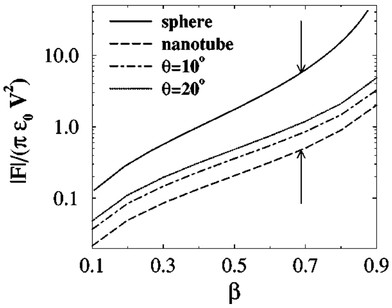

FIG. 4. Electrostatic contact (adhesion) force, $\approx F_0(\infty,\theta)$, vs sample dielectric contrast $\beta=(\epsilon-1)/(\epsilon+1)$ for sharp conical tips of different angles θ. Continuous line corresponds to the contact force of a spherical tip F_0^{sp}. The contact force for any tip size and shape is bounded between F_0^{sp} and the force corresponding to its opening angle θ. The arrows indicate the force window for a nanotube-like tip and a sample with $\epsilon=5.4$.

approach to that of a sharp cone independently of the tip sample distance. This is illustrated in Fig. 3(b) where we have plotted the force versus tip length at different tip-sample distances. As it can be seen, there is a particular distance D/R ($D/R \approx 0.37$ for $\theta = 10°$ and $\epsilon = 5.4$) at which the force is almost independent of the tip length L/R. This means that force versus distance curves for different tip lengths cross at approximately the same distance as shown in the inset of Fig. 3(b). The force at the crossing point corresponds to $\approx F_0(\infty, \theta)$ (see Fig. 4).

In summary, we have analyzed different tip-shape effects in the electrostatic force between a dc-biased tip and metallic and dielectric samples. For metallic samples the force law, except for a constant background, only depends on the tip radius of curvature. For dielectric samples the electrostatic contact forces do not depend on the tip size but on the overall geometry of the tip. We have shown that the maximum force in contact is obtained for a spherical tip and it is independent of the tip radius.

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[19] Notice that this holds only for the forces on the tip. In EFM, for large tip-sample distances the forces on the cantilever may become dominant [see Ref. 14 and J. Colchero *et al.* (unpublished)].