Path Efficiency in Mobile Ad-Hoc Networks

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***Abstract*— The process of routing in large ad-hoc mobile networks is theoretically analyzed as the capacity of a packet to be directed form a source to a destination. The equivalence between directivity and an effective radius, which represents the actual knowledge of any node of its neighbourhood, is demonstrated. The mobility of the network is modelled as that resulting from the most probable distribution of mobile nodes. The results are conclusive: mobility reduces the throughput and delay performance of any routing algorithm with a finite effective radius.**

I. Introduction

The need for efficient routing in wireless ad-hoc networks, has given birth to a plethora of routing protocols [1]. Due to the particular characteristics of those networks, the criteria followed to design the routing algorithms are somewhat different from the ones used in traditional, wired or wireless, networks with infrastructure [2]. The optimisation criteria range from the minimisation of the number of hops to reach destination, number of retransmissions, energy efficiency [3] or topological considerations [4]. With the advent of "large-scale" wireless ad-hoc networks, those routing strategies have to be revisited with new constraints in mind, such as the scalability [5] and the capability for self-organization [6].

In this work we develop a theoretical framework that is oriented to evaluate the efficiency of routing protocols in a dense wireless ad-hoc networks. We characterise the routing strategies by means of the capability of directing the route from source to destination. To this end we define an *energy of the route* where this directivity is implied. This energy ranges from zero value in a random routing strategy to infinity for a routing strategy capable of resolving the shortest path from any source node to any destination node. We will focus in the study of the distribution of the distances, relative to the length of the routing path, for any source-destination pair of nodes, a distribution defined as the *End-to-End distribution*. We investigate the characteristics of any routing algorithm based on a universal measure which we define as the *effective radius* of the routing protocol.

A. Organization and Summary of Results

The network model and the description of the theoretical techniques used in this work are described in Section II. In Section III we obtain analytically the End-to-End distribution for a routing strategy which is entirely described by a directivity function. The results for the specific cases of random walk routing and optimal routing are also described in Section III. The case of Large-Scale Ad-Hoc Wireless Networks is treated in Section IV where the moments of the End-to-End distribution are analyzed for each of the three routing strategies defined in this work. A definition of the effective radius is also found in this section. We finally present the conclusions of this work, future directions and the limit of applicability of our results in Section V.

II. Problem Statement

A. Network Model

The network model that we will use is summarized graphically in Figure 1. The nodes of a wireless ad-hoc network are randomly distributed in a three-dimensional space. For the sake of clarity, graphical representations are shown in two dimensions even all the theoretical framework is formulated in three dimensions. The separation between the nodes fluctuates around a with a mean square $\left\langle (\Delta \mathbf{x}_n)^2 \right\rangle = a^2/3$. We formulate the routing process in terms of a chain of N hops $\Delta \mathbf{x}_n$ between node 0 (source) and node N (destination). The density of nodes is such that the radio coverage for each of the nodes allows the communications with neighboring nodes within the limitations of distances previously described. We will show later that, asymptotically, whether the distance between neighbouring nodes is fixed or fluctuating, the results are equal.

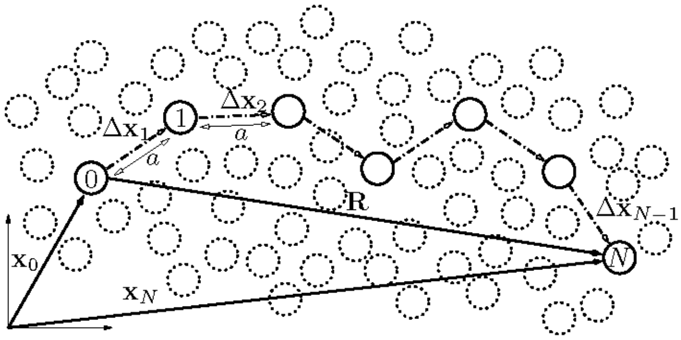

Fig. 1. Routing path in a wireless ad-hoc networks which is composed of N links $\Delta \mathbf{x}_n$ of length a connecting $\mathbf{x}_0$ and $\mathbf{x}_N$.

We assumed a perfect MAC in which every transmission is successful. This is done to establish a baseline comparison with respect to other works [7] and to obtain an absolute upper limit in the expected performance of the system.

B. Path Integral Method

In order to obtain the aforementioned End-to-End distribution, we will use the *path integral* method [8]. In a very simplified way, this method was developed in the realm of quantum

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mechanics in order to obtain the probability amplitude that a quantum particle went from position $\mathbf{x}_a$ at time τ_a to position $\mathbf{x}_b$ at time τ_b. The nature of quantum mechanics is such that, to calculate correctly such probability amplitude, it is necessary to *sum aver all the possible histories*. We take advantage of this method to calculate the End-to-End distribution of all the possible routes with N hops from node 0 to node N

C. Mobility Model

We consider an arbitrary distribution function of nodes which are enclosed in a physical volume V. We define a phase space (μ-space) defined by the position ($\mathbf{x}$) and the speed ($\mathbf{v}$) of the nodes. A node is confined to a finite region of this phase-space, because the values of speed and position are restricted (no infinite speed or volume). Cover this finite region of μ-space with volume elements of volume $\omega = d^3x \cdot d^3v$ and number them from 1 to K being K a very large number. We specify a number n_i of nodes found in the i-th cell to satisfy the following conditions

$$\sum_{i=1}^{K} n_i = N \qquad (1)$$

$$\sum_{i=1}^{K} \epsilon_i = E \qquad (2)$$

where ϵ_i is the kinetic energy of a node in the i-th cell

$$\epsilon_i = \frac{v_i^2}{2} \qquad (3)$$

where v_i os the speed of the node. An arbitrary set of integers $\{n_i\}$ satisfying Equations 1 and 2 defines an arbitrary distribution function. The value of the distribution function in the i-th cell, denoted by f_i, is

$$f_i = \frac{n_i}{\omega} \qquad (4)$$

This is the distribution function for one member of the ensemble. The equilibrium distribution function is the above averaged over the whole ensemble, which, for lack of further assumptions, assigns equal weight to all systems satisfying Equations 1 and 2

$$\bar{f}_i = \frac{\langle n_i \rangle}{\omega} \qquad (5)$$

So we will choose an arbitrary distribution function by choosing an arbitrary set of $\{n_i\}$, calculate the volume it occupies by counting the number of systems in the ensemble that have these occupation numbers and then, vary the distribution to maximize volume. Let us denote by $\Omega\{n_i\}$ the volume in Γ-space (which characterizes the whole ensemble of nodes) occupied by the distribution function corresponding to the numbers $\{n_i\}$. It is proportional to the number of ways of distributing N distinguishable nodes among K cells so that there are n_i of them in the i-th cell ($i = 1, 2, \ldots, K$).Therefore

$$\Omega\{n_i\} = \frac{N!}{n_1!n_2!n_3!\cdots n_K!} g_1^{n_1} g_2^{n_2} \cdots g_K^{n_K} \qquad (6)$$

where g_i is a number we will put equal to unity at the end of the calculation but which is introduced for mathematical convenience. Taking the logarithm of previous equation, we obtain

$$\log\Omega\{n_i\} = \log N! - \sum_{i=1}^{K} \log n_i! + \sum_{i=1}^{K} n_i \log g_i + \text{constant} \qquad (7)$$

If we assume each n_i is a very large integer, we can use Stirling's approximation

$$\log n_i! \approx n_i \log n_i - 1 \qquad (8)$$

We then have

$$\log\Omega\{n_i\} = N\log N - \sum_{i=1}^{K} n_i \log n_i + \sum_{i=1}^{K} n_i \log g_i + \text{constant} \qquad (9)$$

To find the equilibrium distribution we vary the set of integers $\{n_i\}$ subject to Equations 1 and 2 until $\log\Omega$ attains a maximum. Let $\{\bar{n}_i\}$ denote the set of integers that maximizes $\log\Omega$. If we use the method of the Lagrange multipliers, we have

$$\delta\left[\log\Omega\{n_i\}\right] - \delta\left(\alpha\sum_{i=1}^{K} n_i + \beta\sum_{i=1}^{K}\epsilon_i n_i\right) = 0 \qquad (10)$$

If we substitute 9 in 10, we obtain

$$\sum_{i=1}^{K}\left[-(\log n_i + 1) + \log g_i - \alpha - \beta\epsilon_i\right]\delta n_i = 0 \qquad (11)$$

Since δn_i are independent variations, we obtain the equilibrium condition by setting the summand equal to zero at $n_i = \bar{n}_i$

$$\begin{aligned} \log\bar{n}_i &= -1 + \log g_i - \alpha - \beta\epsilon_i \\ \bar{n}_i &= g_i e^{-\alpha-\beta\epsilon_i - 1} \end{aligned} \qquad (12)$$

The most probable distribution is, by Equations 12 and 4

$$\bar{f}_i = Ce^{-\beta\epsilon_i} \qquad (13)$$

where C is a constant. If the nodes are uniformly distributed in space, so that f is independent of $\mathbf{x}$

$$\int f(\mathbf{x}, \mathbf{v}, t) = \frac{N}{V} \qquad (14)$$

it can be shown that

$$C = \left(\frac{\beta}{\pi}\right)^{3/2}\frac{N}{V} \qquad (15)$$

If we define the density of the nodes as $\mu = N/V$, we finally have

$$\bar{f}_i = \mu\left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta\epsilon_i} \qquad (16)$$

This is the Maxwell-Boltzmann distribution. Using this distribution, we calculate the most probable speed of any node and results in

$$\bar{v} = \left[\frac{\int d^3v \cdot v f(v)}{\int d^3v f(v)}\right]^{1/2} = \sqrt{\frac{2}{\beta}} \qquad (17)$$

and the root mean squared speed v_{rms} is equal to

$$v_{\text{rms}} = \left[\frac{\int d^3v \cdot v^2 f(v)}{\int d^3v f(v)}\right]^{1/2} = \sqrt{\frac{3}{\beta}} \tag{18}$$

III. End-to-end Distribution of Routing Strategies

A. Directed Routing

We formulate now the directionality of the hop from one node to the following in a simple expression that we will define as the *energy of the direction*. This energy will address the difference between the RRS (where the directionality was random) and the Directed Routing Strategy (DRS). To construct this functional, let us define a given route described by the one-parameter continuous function $\mathbf{r}(s)$. The tangent to that trajectory is given by by $\mathbf{u}(s) = \partial\mathbf{r}(s)/\partial s$. The variation of this tangent vector through the partial differential $\partial\mathbf{u}(s)/\partial s$. If we construct a quadratic functional from this variation of the "direction" of the route, we will be able to calculate the energy stored by a particular route of length L as

$$E^L = \int_0^L \frac{\kappa}{2}\left(\frac{\partial\mathbf{u}(s)}{\partial s}\right)^2 ds \tag{19}$$

where κ is defined as the *elastic constant*.

The actual route is not continuous, $\partial\mathbf{u}(s)/\partial s$ being substituted by the difference of the vectors that link two consecutive pair of nodes n and $n+1$. The actual form of the energy should be

$$E^N_{\text{DRS}} = \frac{\kappa}{2a}\sum_{n=1}^{N}(\mathbf{u}_n - \mathbf{u}_{n-1})^2 \tag{20}$$

With that functional of the directivity defined, the end-to-end distribution of the DRS over a distance

$$\mathbf{R} \equiv \mathbf{x}_b - \mathbf{x}_a = a\sum_{n=1}^{N}\mathbf{u}_n \tag{21}$$

is obtained from the path integral with specific directions of the initial and final pieces

$$\begin{aligned} P_N(\mathbf{u}_b,\mathbf{u}_a;\mathbf{R}) &= \frac{1}{A}\prod_{n=1}^{N-1}\left[\int\frac{d^2\mathbf{u}_n}{A}\right]\delta^3(\mathbf{R} - a\sum_{n=1}^{N}\mathbf{u}_n) \\ &\times \exp\left[-\frac{2\pi a}{A^2}\sum_{n=1}^{N}(\mathbf{u}_n - \mathbf{u}_{n-1})^2\right] \end{aligned} \tag{22}$$

where A is a measure factor given by

$$A = \sqrt{\frac{2\pi a}{\kappa\beta}} \tag{23}$$

To obtain the desired end-to-end distribution, we integrate 22 over all initial directions and average over the initial ones to obtain is given by

$$P_N^{\text{DRS}}(\mathbf{R}) = \int d^2\mathbf{u}_b\int\frac{d^2\mathbf{u}_a}{4\pi}P_N(\mathbf{u}_b,\mathbf{u}_a;\mathbf{R}) \tag{24}$$

The former equation is quite difficult to evaluate and, in general, the expressions of the End-to-End distributions are quite difficult to obtain analytically. Therefore, we shall work with the moments of the distribution instead of the distribution itself, which are found more easily. Thus, the moments of the distributions can be written as

$$\langle R^{2l}\rangle = \int d^2\mathbf{u}_b\int\frac{d^2\mathbf{u}_a}{4\pi}R^{2l}P_N(\mathbf{u}_b,\mathbf{u}_a;\mathbf{R}) \tag{25}$$

We will take care only of the even moments of the distribution, as its dependence on the actual distance is rotationally invariant. Therefore, odd moments vanish.

If we introduce the angular distribution of a random chain of length $L = Na$ as

$$\begin{aligned} P_N(\mathbf{u}_b,\mathbf{u}_a|L) &= \frac{1}{A}\prod_{n=1}^{N-1}\left[\int\frac{d^2\mathbf{u}_n}{A}\right] \\ &\times \exp\left[-\frac{2\pi a}{A^2}\sum_{n=1}^{N}(\mathbf{u}_n - \mathbf{u}_{n-1})^2\right] \end{aligned} \tag{26}$$

We will be able to calculate the trivial moment $l = 0$ (which will provide us with the proper normalization of the distribution) as

$$\langle 1\rangle = \int d^2\mathbf{u}_b\int\frac{d^2\mathbf{u}_a}{4\pi}P_N(\mathbf{u}_b,\mathbf{u}_a|L) = 1 \tag{27}$$

Equation 26 can be solved analytically

$$P_N(\mathbf{u}_b,\mathbf{u}_a|L) = \sum_{l=0}^{\infty}\exp\left[-L\frac{1}{2\kappa\beta}L_2\right]\sum_{\mathrm{m}}Y_{l\mathrm{m}}(\mathbf{u}_b)Y^*_{l\mathrm{m}}(\mathbf{u}_a) \tag{28}$$

where $L_2 = l(l+1)$ and $Y_{l\mathrm{m}}(\mathbf{u}_n)$ represent the harmonic polynomials. Using the orthogonality properties of the harmonic polynomials and rewriting the former integral in terms of Gegenbauer polynomials [9] and using their recursion relations, we are able to obtain the first nontrivial moment of the End-to-End distribution (details of the exact calculation will be published elsewhere). Before writing the expression of $\langle R^{2l}\rangle$ let us define the following quantity

$$\xi \equiv \kappa\beta \tag{29}$$

which we call the *persistence radius*. The exact expression of the first nontrivial moment of the End-to-End distribution can be written as

$$\langle R^2_{\text{DRS}}\rangle = 2\left[\xi L - \xi^2\left(1 - e^{-L/\xi}\right)\right] \tag{30}$$

Additional moments are increasingly difficult to calculate.

B. Random Routing

For Random Routing Strategy, we have previously calculated the End-to-End distribution with the path integral, but to validate such approach and to give further insight on the results, we will calculate it with a different approach. Let us have the route that we defined in Section II, but instead of fluctuating, we will have a fixed distance between a between consecutive nodes. If we have no preferred angle of direction to hop from one node to the following, we will have a

Random Routing Strategy. In three dimensions, the probability distribution of the end-to-end distance vector $\mathbf{x}_b - \mathbf{x}_a$ of such an route is given by

$$P_N(\mathbf{R}) = \prod_{n=1}^{N} \left[\int d^3\Delta x_n \frac{1}{4\pi a^2} \delta\left(|\Delta \mathbf{x}_n| - a\right) \right] \times \delta^3(\mathbf{R} - \sum_{n=1}^{N} \Delta \mathbf{x}_n) \quad (31)$$

If we look at equation 31 in terms of the Fourier transform of the one-link probabilities $\tilde{P}_1(\mathbf{k})$, we will obtain the desired integral as

$$P_N(\mathbf{R}) = \int \frac{d^3k}{(2\pi^3)} \left[\tilde{P}_1(\mathbf{k})\right]^N e^{i\mathbf{k}\mathbf{R}} = \frac{1}{2\pi^2 R} \int_0^\infty dk k \sin kR \left[\frac{\sin ka}{ka}\right]^N \quad (32)$$

If we solve the previous integral, we should find that

$$P_L^{\mathsf{RRS}}(\mathbf{R}) = \sqrt{\frac{3}{2\pi a}} e^{-3R^2/2La} \quad (33)$$

as found in the previous Section (we changed N subscript by $L = Na$).

If we express the Fourier transform of $P_N(\mathbf{R})$ in terms of the moments of the end-to-end distribution of the RRP, we obtain

$$\tilde{P}_N(\mathbf{k}) = \sum_{l=0}^{\infty} \frac{(-1)^l (k)^{2l}}{(2l)!} \frac{1}{2l+1} \left\langle R^{2l} \right\rangle \quad (34)$$

where the moments are

$$\left\langle R_{\mathsf{RRS}}^{2l} \right\rangle = a^{2l} (-1)^l (2l+1)! \sum_{m_i} \prod_{i=1}^{l} \frac{1}{m_i!} \left[\frac{N 2^{2i} (-1)^i B_{2i}}{(2i)! 2i}\right]^{m_i} \quad (35)$$

where the sum over m_i obeys the constraint $l = \sum_{i=1}^{l} i \cdot m_i$ and B_i are the Bernouilli numbers.

C. Optimal Routing

In a Optimal Routing Strategy (ORS), the packet is able to find the shortest path from source to destination, thus establishing a straight line between source and destination. Its End-to-End distribution is trivial to find as

$$P_L^{\mathsf{ORS}}(\mathbf{R}) = \frac{1}{4\pi R^2} \delta(R - L) \quad (36)$$

from which is straightforward to find that the moments of the ORS are

$$\langle R_{\mathsf{ORS}}^n \rangle = \int d^3R R^n P_L^{\mathsf{ORS}}(\mathbf{R}) = \int_0^\infty d^3R R^n \delta(R - L) = L^n \quad (37)$$

IV. Routing in Large-Scale Ad-Hoc Wireless Networks

A. Effective Radius of RS and the Large-Scale Limit

Let us now revisit the moments of the End-to-End distribution for the three different routing strategies examined here (DRS, RRS and ORS) in the limit of a large number of nodes N at finite $a^2 N$.

We first take this limit in Equation 35, where we find

$$\left\langle R_{\mathsf{RRS}}^{2l} \right\rangle = \frac{(2l+1)!!}{3^l} (aL)^l \quad (38)$$

If we continue with Equation 30, we can see that, in the limit for large L/ξ we have

$$\left\langle R_{\mathsf{DRS}}^2 \right\rangle = 2\xi L \left(1 - \frac{\xi}{L} + \cdots\right) \quad (39)$$

We can pursue a tedious calculation of the fourth moment of the DRS End-to-End distribution and, after the limits are taken, we have

$$\left\langle R_{\mathsf{DRS}}^4 \right\rangle = 4\frac{5}{3}\xi^2 L^2 \left(1 - 2\frac{26}{15}\frac{\xi}{L} + \ldots\right) \quad (40)$$

We can observe that the first term in both the second and the fourth moment are in accord with Equation 38 but, instead of a distance between nodes of a, we have an *effective radius* of the DRS

$$a_{\mathsf{eff}} \equiv 2\xi \quad (41)$$

Please acknowledge that, as the persistence radius ξ in Equation 29 in only dependent in the directivity (κ) and the mobility (β) parameters, the effective radius will shrink or expand only due incremental/decremental speed and/or knowledge of its surroundings. So, in the large-scale limit, we can see that the Directed Routing Strategy can be seen as a Random Routing Strategy with a greater extent of influence. As a consequence, we can rewrite the general expression for the moments of the DRP as, approximately

$$\left\langle R_{\mathsf{DRS}}^{2l} \right\rangle \approx \frac{(2l+1)!!}{3^l} (a_{\mathsf{eff}} L)^l \quad (42)$$

Finally we rewrite the Optimal Routing Strategy moments of the End-to-End Distribution as

$$\left\langle R_{\mathsf{ORS}}^{2l} \right\rangle = L^{2l} \quad (43)$$

So we can see a transition from a Random Routing Strategy whose influence extends only to its nearest neighbours, to a Random Routing Strategy but with a greater topological influence to, finally, an Optimal Routing Strategy whose influence extends to the whole of the network. To illustrate the distribution of the End-to-End distances with respect to the actual length of the routes, we have calculated numerically the End-to-End distribution for Routing Strategies with different effective radii.

We can summarize the behaviour of the routing strategies as a whole by building the following moments function

$$\left\langle R^{2l} \right\rangle \propto (aL)^{2l\nu} \quad (44)$$

We will define ν as the *critical exponent*. This critical exponent would range from $\nu = 1/2$ (Random Walk) to $\nu = 1$ (Shortest Path).

One of the weaknesses of the method describe until now is that, except for Proactive (Table-Driven) Protocols, no a-priori determination of the effective radius can be done. For Reactive or Mixed Protocols one can only make educated guesses. We propose the use of the distributions above to fit measured (through simulation or measurement) distributions of path lengths in networks using a given routing protocol. Through this fit, we would be able to determine its effective radius and determine the optimality of its deployment in a given network.

B. Efficiency vs. Flooding: Experimental Results with DSR

If we use a routing strategy with constant finite effective radius a_{eff} (e.g. a table-driven routing protocol), as we go to the large-scale limit is easy to see that

$$\lim_{N\to\infty} \left\langle R^{2l}_{\text{DRS}} \right\rangle = \left\langle R^{2l}_{\text{RRS}} \right\rangle \tag{45}$$

It is obvious, therefore, from the quantitative analysis above, that routing protocols with finite effective radius will be completely inefficient in large networks, so the solution is clearly in the side of hierarchical algorithms.

We proceed now to show the results of simulations with the *ns-2* network simulator. Two face-centered hexagonally-distributed two-dimensional networks with sizes of 91 and 217 nodes are simulated. We use the IEEE 802.11 MAC and free-space loss propagation model. The selected routing algorithm is the Dynamic Source Routing (DSR) algorithm. A message is generated from any node of the network with any other node as destination. This procedure is repeated for any node of the network. The path is extracted from the trace files generated by *ns-2* and the length of the path L for any source-detination pair is computed. The euclidean distance between source and destination (R) is also computed. From the ratio of these two quantities of all the generated paths (R/L) a normalized histogram is extracted, thus resulting in the experimental End-to-End distributions plotted in Figure 2.

We can see that, due to the geometrical effect of the face-centered hexagonal 2D lattice, numerous peaks appear as some route configurations are favoured over others. We opted for a regular lattice rather than a random one, due to the added cost that would be supported as averaging through different spatial configurations would be needed.

It is evident the reduction in effectiveness of 55% (as measured by the effective radius) with the increase in size of the network as predicted by our theory. Apart from the almost perfect fitting of the theoretical distributions to the experimental ones in the long-route regime, the most striking feature of the results for the 217-node network, is that it is to be fitted to a 3D distribution, not to bidimensional one. Thus, although a 2D network is simulated, 3D features arise in larger networks.

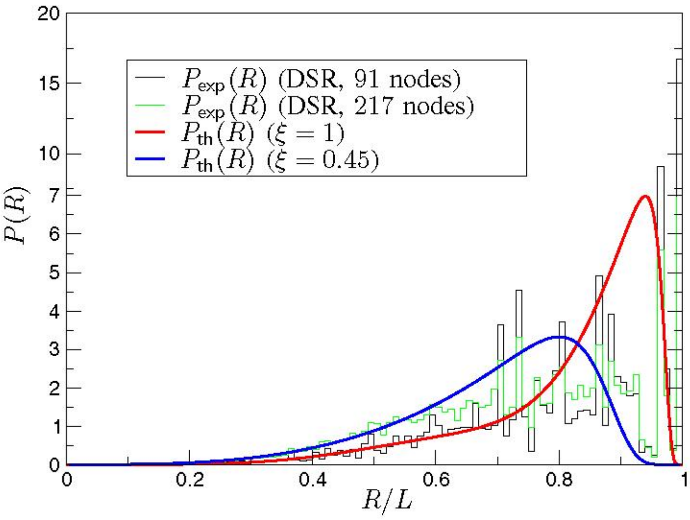

Fig. 2. Experimental End-to-End Distributions (histograms) obtained from simulations in *ns-2* with the DSR algorithm. Two different network sizes are evaluated, 91 and 217 nodes. Alongside, theoretical End-to-End Distributions for 3D networks respectively,with $\xi = 1$ and $\xi = 0.45$ are plotted.

V. CONCLUSIONS

In this work we introduced a novel method to analyze theoretically the routing strategies that are to be used in Large-Scale Wireless Ad-Hoc Networks.We have shown that in the large-scale limit, any routing strategy with finite effective radius will behave as a Random Walk. We have shown theoretically that mobility impacts negatively on the performance of the network for large-scale networks and large number of hops. We have shown results of simulations for networks of different sizes and have shown the decrease of the effective radius as the size of the network increases. We have also shown that 3D features arise in the experimental distributions of 2D networks.

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