UTILITY-BASED POWER CONTROL FOR PEER-TO-PEER COGNITIVE RADIO NETWORKS WITH HETEROGENEOUS QOS CONSTRAINTS*

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ABSTRACT

Transmit-power control is a critical task in cognitive radio (CR) networks. In the present contribution, adherence to hierarchies between primary and secondary users in a peer-to-peer CR network is enabled through distributed power control. Hierarchies are effected by imposing minimum and maximum bounds on a quality-of-service (QoS) metric, such as communication rate. These bounds translate to signal-to-interference-plus-noise ratio (SINR) constraints. Furthermore, a utility function captures each user's satisfaction with the received SINR. The novel power control strategy maximizes the total utility while respecting individual SINR constraints — a task recast as a convex optimization problem under a suitable relaxation. Sufficient conditions, realistic for practical CR networks, are provided to obtain the optimal power allocation from the solution of the relaxed problem. Finally, a low-overhead distributed algorithm for optimal power control is developed, and tested against competing alternatives via simulations.

Index Terms— Cognitive radios, distributed algorithms, optimization methods, power control, QoS constraints

1. INTRODUCTION

Cognitive radio is an emerging technology promising efficient spectrum utilization by dynamically adapting to the conditions of the operating environment [1]. In a CR network, primary users or licensees coexist with secondary and/or unlicensed users or lessees, who have limited access to network resources [1, §49]. Such a regulated access can be realized by bounding the maximum level of a commodity a user can receive, which may be communication rate (as in [2], [3]), bit error rate (BER), or any other QoS figure. Such bounds lead in turn to *heterogeneous* QoS requirements of the CR users.

Adjusting transmission power [1, §27] offers the potential to satisfy these requirements. The challenge however, is to mitigate co-channel interference, which is intimately coupled with individual power control decisions. This paper deals with a utility-based approach to power control in peer-to-peer CR networks, where the satisfaction of each user with the received QoS level is captured by a utility function, which depends on the received SINR; see also [4], [5], [3] and references therein.

So far, two sub-optimal algorithms have been reported for distributed power control in CR networks with diverse QoS constraints [3]. The contribution of the present work is two-fold: (i) optimal power control is obtained using convex optimization; and (ii) a practical distributed algorithm for optimal power control is developed to account for heterogeneous QoS requirements tailored to the CR paradigm (not considered in other works on utility-based power control [4], [5], [6]).

The remainder of the paper is organized as follows. The power control problem is stated in Sec. 2 and solved in Sec. 3. The resultant distributed algorithm is the subject of Sec. 4, while Sec. 5 presents simulations and Sec. 6 conclusions.

2. PROBLEM STATEMENT

Consider a wireless peer-to-peer network with a set of $\mathcal{M} := \{1,\ldots,M\}$ links, as in [4], [3], where each link $i \in \mathcal{M}$ entails a user with a dedicated transmitter (Tx_i) wishing to communicate with a corresponding receiver (Rx_i). All links are assumed sharing the same frequency band (referred to as single-channel in [4]), as e.g., in CDMA. Let h_{ij} denote the channel gain from Tx_i to Rx_j (assumed static); n_i the noise power at Rx_i; and p_i the transmission power of Tx_i. Suppose that Tx_i can transmit with at least $P_i^{\min}$ and at most $P_i^{\max}$ power budget, i.e., $p_i \in \mathcal{P}_i := [P_i^{\min}, P_i^{\max}]$. The received SINR γ_i at Rx_i is a function of the powers $\boldsymbol{p} := (p_1,\ldots,p_M)$ and is given by

$$\gamma_i := \frac{h_{ii}p_i}{n_i + \sum_{k\neq i} h_{ki}p_k}\,. \tag{1}$$

Each user link $i \in \mathcal{M}$ adopts a utility function $u_i(\gamma_i)$ that reflects the received QoS level. The utilities are selected concave, strictly increasing and twice continuously differentiable over $(0,\infty)$. We will focus on two important utilities (with $w_i > 0$ and $\alpha < 0$):

$$u_i(\gamma_i) = w_i \ln \gamma_i \quad \text{or} \quad u_i(\gamma_i) = w_i\alpha^{-1}\gamma_i^{\alpha}. \tag{2}$$

These utilities satisfy (with $\alpha = 0$ if $u_i(\gamma_i) = w_i \ln \gamma_i$)

$$\mathsf{C}_i(\gamma_i) := -\frac{\gamma_i u_i''(\gamma_i)}{u_i'(\gamma_i)} = 1-\alpha, \quad \forall\,\gamma_i > 0. \tag{3}$$

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The ratio $\mathsf{C}_i(\gamma_i)$ is instrumental in ensuring convexity of the power control problem; see [5, Ch.5], [4] and references therein. In future submissions, we will consider general utilities for which $\mathsf{C}_i(\gamma_i)$ is be allowed to vary with γ_i. Note that for the utilities in (2) it is also necessary to have $P_i^{\min} > 0$. This imposes no practical restriction, since setting $P_i^{\min}$ to a very small value effectively amounts to no transmission.

In the framework of CR networks, the maximum level of individual user commodities is bounded. For this reason, the focus here is on commodities that are one-to-one functions of γ_i. These include rate $\ln(1+\gamma_i)$ [2], [3], BER, or each user's utility function $u_i(\gamma_i)$. A bound on the maximum level of the commodity readily maps to an SINR constraint $\gamma_i \leq \gamma_i^{\max}$. Moreover, such a constraint is pertinent when further increase in γ_i cannot effectively increase the user's utility, as e.g., in fixed-rate services [3]. Also note that QoS guarantees for primary (or even secondary) users can be provided through a minimum SINR constraint $\gamma_i \geq \gamma_i^{\min}$.

Power control in networks with heterogeneous QoS constraints amounts to selecting powers $\boldsymbol{p}$ that maximize the total utility of the users, $\sum_{i=1}^{M} u_i(\gamma_i)$, while respecting the individual SINR requirements; i.e., the goal is

$$\max_{\{\boldsymbol{p}|p_i\in\mathcal{P}_i\,\forall i\in\mathcal{M}\}} \quad \sum_{i=1}^{M} u_i(\gamma_i) \tag{4a}$$

$$\text{subj. to} \quad \gamma_i^{\min} \leq \gamma_i \leq \gamma_i^{\max}, \quad \forall i \in \mathcal{M}. \tag{4b}$$

In general, problem (4) is non-convex in $\boldsymbol{p}$; certain instances of (4) though are known to be equivalent to convex problems. Specifically, in the *absense* of (4b) and for a general class of utility functions which includes (2), problem (4a) can be written as a convex optimization problem under the transformation $p_i = e^{y_i}$ [5]; see also [4]. (From [4] and [5] it can be inferred that minimum SINR constraints $\gamma_i \geq \gamma_i^{\min}$ can also be handled, although this is not explicitly treated.) Finally, under minimum SINR constraints only ($\gamma_i \geq \gamma_i^{\min}$), problem (4) can also be written as a geometric program (GP), if $u_i(\gamma_i) = w_i \ln \gamma_i$; and as a generalized GP, if $u_i(\gamma_i) = w_i\alpha^{-1}\gamma_i^{\alpha}$ [6]. (In the area of GP, the transformation $p_i = e^{y_i}$ is also standard [6].)

None of the aforementioned works can accommodate the maximum SINR constraint $\gamma_i \leq \gamma_i^{\max}$, pertinent to CR networks. It is the present paper's contribution to tackle the solution of (4), through a suitable relaxation, elaborated next.

3. OPTIMAL POWER CONTROL

Let q_i be an auxiliary variable, associated with link i, upper-bounding the true interference-plus-noise denominator in (1). Collecting all q_i's in a vector $\boldsymbol{q} := (q_1, \ldots, q_M)$, consider the following relaxed version of (4):

$$\max_{\substack{\{\boldsymbol{p}|p_i\in\mathcal{P}_i\,\forall i\in\mathcal{M}\}\\ \boldsymbol{q}\in\mathbb{R}_{++}^{M}}} \quad \sum_{i=1}^{M} u_i\big(h_{ii}p_iq_i^{-1}\big) \tag{5a}$$

$$\text{subj. to} \quad \gamma_i^{\min} \leq h_{ii}p_iq_i^{-1} \leq \gamma_i^{\max}, \forall i \in \mathcal{M} \tag{5b}$$

$$q_i \geq n_i + \sum_{k\neq i} h_{ki}p_k, \qquad \forall i \in \mathcal{M}, \tag{5c}$$

where $\mathbb{R}_{++}$ are the positive reals. Clearly, if (5c) were equality constraints, then problems (4) and (5) would be equivalent. Even though (5) is not jointly convex in $\boldsymbol{p}$, $\boldsymbol{q}$, it will be possible to transform it into an equivalent convex optimization problem.

To this end, apply the one-to-one change of variables $p_i = e^{y_i}$, $q_i = e^{z_i}$. Then the power constraints in (5a) map to $P_i^{\min} \cdot e^{-y_i} \leq 1$ and $(P_i^{\max})^{-1}e^{y_i} \leq 1$; the SINR constraints (5b) become $\gamma_i^{\min}h_{ii}^{-1}e^{z_i - yi} \leq 1$, $(\gamma_i^{\max})^{-1}h_{ii}e^{y_i - z_i} \leq 1$; and those in (5c) translate to $n_ie^{-z_i} + \sum_{k\neq i} h_{ki}e^{y_k - z_i} \leq 1$. The transformed constraints are convex in $\boldsymbol{y} := (y_1, \ldots, y_M)$ and $\boldsymbol{z} := (z_1, \ldots, z_M)$ since all left-hand sides are compositions of nonnegative sum of exponentials (which are convex functions) with affine mappings [7, Sec. 3.2].

What remains to show is that the objective in (5a) is concave in $\boldsymbol{y}, \boldsymbol{z}$. Since it is a nonnegative sum of $u_i(e^{y_i - z_i + \ln h_{ii}})$ terms, it suffices for $u_i(e^x)$ to be concave in the scalar $x \in \mathbb{R}$, i.e., that $\frac{d^2u_i(e^x)}{dx^2} \leq 0 \Leftrightarrow \mathsf{C}_i(\xi) = -\frac{\xi u_i''(\xi)}{u_i'(\xi)} \geq 1$ ($\xi = e^x$).

Now define matrix $\mathbf{A} := [a_{ij}]$ with $a_{ii} = 0 \,\forall i \in \mathcal{M}$ and $a_{ij} = h_{ji}/h_{ii} \,\forall j \neq i$. (It is common to collect channels h_{ij} in such a matrix; see e.g., [5].) The following result asserts that under mild conditions the solution of (5) also solves (4).[1]

Proposition 1 *Assume that: (a1) problem* (4) *is feasible; (a2) utilities* $u_i(\gamma_i)$ *are continuous and strictly increasing; (a3) matrix* $\mathbf{A}$ *is irreducible; (a4) there is no power vector* $\boldsymbol{p}$ *with* $p_i \in \mathcal{P}_i \,\forall i \in \mathcal{M}$ *s.t.* $\gamma_i = \gamma_i^{\max} \,\forall i \in \mathcal{M}$*; and (a5) the constraint* $P_i^{\min}$ *is sufficiently small s.t.* $h_{ii}P_i^{\min}/n_i < \gamma_i^{\max}$*. If* $\boldsymbol{p}^*, \boldsymbol{q}^*$ *solve problem* (5)*, then* (5c) *holds as equality at* $\boldsymbol{p}^*, \boldsymbol{q}^*$*; i.e.,*

$$q_i^* = n_i + \sum_{k\neq i} h_{ki}p_k^* \quad \forall i \in \mathcal{M}.$$

It is worth stressing that Prop. 1 holds for *any* strictly increasing utility, not only the ones in (2). Note further that the assumption $h_{ii}P_i^{\min}/n_i < \gamma_i^{\max}$ is innocuous, since $P_i^{\min}$ is selected so small that it amounts to no transmission. Moreover, the assumption on the irreducibility [5, Def. A.21] of $\mathbf{A}$ is common in power control problems; see e.g., [5, Sec. 5.5].

The non-achievability condition on the SINRs $\gamma_i^{\max}$ within the power constraints for *all* users is slightly more restrictive and should be checked before solving (5). It is important to remark that if the SINRs $\gamma_i^{\max}$ are achievable for all users, then the optimal total utility will be $\sum_{i=1}^{M} u_i(\gamma_i^{\max})$ and no further optimization is needed. If not though, the solution of (4) will yield the optimal power allocation.

To check this, we rely on a classical power control algorithm for given SINR requirements [8]. Specifically, consider the iteration $\boldsymbol{p}(t+1) = \boldsymbol{I}\big(\boldsymbol{p}(t)\big)$, called standard power control algorithm (SPCA), where $\boldsymbol{I}(\boldsymbol{p}) := [I_1(\boldsymbol{p}), \ldots, I_M(\boldsymbol{p})]$ with

$$I_i(\boldsymbol{p}) := \min\Big\{P_i^{\max}, \gamma_i^{\max}\,\frac{1}{h_{ii}}\big(n_i + \textstyle\sum_{k\neq i} h_{ki}p_k\big)\Big\}.$$

From [8, Cor. 1] it follows that the algorithm converges, and upon convergence, all users will have $\gamma_i = \gamma_i^{\max}$ if and only if this is feasible under the constraint $p_i \leq P_i^{\max} \,\forall i \in \mathcal{M}$ (and then $p_i \geq P_i^{\min} \,\forall i \in \mathcal{M}$ due to $h_{ii}P_i^{\min}/n_i < \gamma_i^{\max}$);

[1] Proofs are omitted due to space limitations.

otherwise, at least one user will have $\gamma_i < \gamma_i^{\max}$. Furthermore, the SPCA can be implemented in a distributed fashion, without any exchange of information among users [8, Sec. VI].

Proposition 1 allows optimizing the power allocation (when not all $\gamma_i^{\max}$ are achievable) by solving the Karush-Kuhn-Tucker (KKT) conditions [7, Sec. 5.5.3] of the convex equivalent of (5). This is the theme of the ensuing subsection.

3.1. Solution of the KKT conditions

Let λ_i^ℓ, λ_i^{u}, μ_i denote Lagrange multipliers corresponding to min and max SINR constraints (5b) and (5c), respectively. The Lagrangian of the convex equivalent of (5) is

$$\begin{aligned} L(\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\lambda}^\ell, \boldsymbol{\lambda}^{\mathrm{u}}, \boldsymbol{\mu}) := \sum_i \mu_i \Big[e^{-z_i} \Big(n_i + \sum_{k \neq i} h_{ki} e^{y_k} \Big) - 1 \Big] \\ - \sum_i u_i (h_{ii} e^{y_i - z_i}) + \sum_i \lambda_i^\ell \Big(\gamma_i^{\min} h_{ii}^{-1} e^{z_i - y_i} - 1 \Big) \\ + \sum_i \lambda_i^{\mathrm{u}} \Big((\gamma_i^{\max})^{-1} h_{ii} e^{y_i - z_i} - 1 \Big). \quad (6) \end{aligned}$$

The Lagrangian is separable in z_i; hence, the z_i which minimizes the Lagrangian can be obtained given $\boldsymbol{y}$, λ_i^ℓ, λ_i^{u}, μ_i for each $i \in \mathcal{M}$ by taking $\partial L / \partial z_i = 0$. The latter yields:

$$u_i' \Big(h_{ii} \frac{e^{y_i}}{e^{z_i}} \Big) - \mu_i \frac{n_i + \sum_{k \neq i} h_{ki} e^{y_k}}{h_{ii} e^{y_i}} + e^{2 z_i} \frac{\lambda_i^\ell \gamma_i^{\min}}{(h_{ii} e^{y_i})^2} - \frac{\lambda_i^{\mathrm{u}}}{\gamma_i^{\max}} = 0 \quad (7)$$

Eq. (7) can be solved for e^{z_i} as a function of $\boldsymbol{y}$, λ_i^ℓ, λ_i^{u}, μ_i. In fact, *all quantities needed for solving* (7) *are known locally* at Tx_i or Rx_i. Specifically, these are the local Lagrange multipliers λ_i^ℓ, λ_i^{u}, μ_i, the received power $h_{ii} e^{y_i}$, and the measured SINR, $h_{ii} e^{y_i} / \big(n_i + \sum_{k \neq i} h_{ki} e^{y_k} \big)$.

Since optimal powers $\boldsymbol{y}^*$ and optimal Lagrange multipliers $\boldsymbol{\lambda}^{\ell*}$, $\boldsymbol{\lambda}^{\mathrm{u}*}$, $\boldsymbol{\mu}^*$ cannot be obtained in closed form, namely by solving $\partial L / \partial y_i = 0$ directly, an iterative algorithm is needed. The exact form of a Lagrangian gradient-based algorithm and its distributed implementation are presented next.

4. DISTRIBUTED ALGORITHM

In this section, we present a distributed algorithm to solve the convex equivalent of (5). Let $\bar{z}_i$ denote the optimal value of z_i as a *function* of $\boldsymbol{y}$, λ_i^ℓ, λ_i^{u}, μ_i, obtained locally from (7). Then at any $\boldsymbol{y}$, $\boldsymbol{\lambda}^\ell$, $\boldsymbol{\lambda}^{\mathrm{u}}$, $\boldsymbol{\mu}$ (and corresponding $\bar{\boldsymbol{z}}$) we have

$$\begin{aligned} \frac{\partial L}{\partial y_i} \Big|_{(\boldsymbol{y}, \bar{\boldsymbol{z}}, \boldsymbol{\lambda}^\ell, \boldsymbol{\lambda}^{\mathrm{u}}, \boldsymbol{\mu})} = -u_i' \Big(h_{ii} \frac{e^{y_i}}{e^{\bar{z}_i}} \Big) h_{ii} \frac{e^{y_i}}{e^{\bar{z}_i}} + e^{y_i} \sum_{j \neq i} h_{ij} \mu_j e^{-\bar{z}_j} \\ - e^{\bar{z}_i} \lambda_i^\ell \gamma_i^{\min} (h_{ii})^{-1} e^{-y_i} + e^{-\bar{z}_i} \lambda_i^{\mathrm{u}} (\gamma_i^{\max})^{-1} h_{ii} e^{y_i}. \quad (8) \end{aligned}$$

Further, define a *beacon* variable $b_j := \mu_j e^{-\bar{z}_j}$ and observe that the variables b_j as well as the channels h_{ij} are the only non-local (to Tx_i or Rx_i) quantities that $\partial L / \partial y_i$ depends on.

Now let $Y_i^{\min} := \ln P_i^{\min}$, $Y_i^{\max} := \ln P_i^{\max}$, and $[.]_{Y_i^{\min}}^{Y_i^{\max}}$ define the projection onto $[Y_i^{\min}, Y_i^{\max}]$; and $[.]^+$ onto the non-negative reals. Then the optimal powers $\boldsymbol{y}^*$ and Lagrange multipliers $\boldsymbol{\lambda}^{\ell*}$, $\boldsymbol{\lambda}^{\mathrm{u}*}$, $\boldsymbol{\mu}^*$ can be obtained by gradient projection iterations (indexed by t) with constant stepsize β.

$$y_i(t+1) = \left[y_i(t) - \beta \frac{\partial L}{\partial y_i} \Big|_{(\boldsymbol{y}(t), \bar{\boldsymbol{z}}(t), \boldsymbol{\lambda}^\ell(t), \boldsymbol{\lambda}^{\mathrm{u}}(t), \boldsymbol{\mu}(t))} \right]_{Y_i^{\min}}^{Y_i^{\max}} \quad (9)$$

$$\lambda_i^\ell(t+1) = \left[\lambda_i^\ell(t) + \beta \left(\frac{e^{\bar{z}_i(t)} \gamma_i^{\min}}{h_{ii} e^{y_i(t)}} - 1 \right) \right]^+ \quad (10)$$

$$\lambda_i^{\mathrm{u}}(t+1) = \left[\lambda_i^{\mathrm{u}}(t) + \beta \left(\frac{h_{ii} e^{y_i(t)}}{\gamma_i^{\max} e^{\bar{z}_i(t)}} - 1 \right) \right]^+ \quad (11)$$

$$\mu_i(t+1) = \left[\mu_i(t) + \beta \left(\mu_i \frac{n_i + \sum_{k \neq i} h_{ki} e^{y_k(t)}}{e^{\bar{z}_i(t)}} - 1 \right) \right]^+. \quad (12)$$

The updates for y_i, λ_i^ℓ, λ_i^{u}, μ_i take place at the transmitter of link $i \in \mathcal{M}$. As in [4], this is possible provided that Tx_i knows: (i) the SINR $h_{ii} e^{y_i} / \big(n_i + \sum_{k \neq i} h_{ki} e^{y_k} \big)$ at every timeslot t and the channel h_{ii} (through feedback from Rx_i); (ii) the channels h_{ij} to Rx_j (by reciprocity if Rx_j transmits a training signal); and (iii) the beacon variables b_j. Note that each b_j is known at Tx_j, so *every transmitter must broadcast its beacon variable to all other transmitters*. Nevertheless, it is only a *scalar* quantity that must be broadcasted. This type of message passing in utility-based power control is also used in [4], [6, Ch. 3], while a simpler scheme is advocated in [5, Sec. 6.5.4].

We contend that the updates (9)–(12) can be implemented in a distributed fashion. Indeed, observe that the updates (10)–(12) need only quantities locally available at each Tx_i. Specifically, $\bar{z}_i$ can be evaluated at Tx_i, if the current SINR and channel h_{ii} are fed back from Rx_i. Similarly, the interference-plus-noise $n_i + \sum_{k \neq i} h_{ki} e^{y_k}$ depends only on the current SINR, h_{ii} and power e^{y_i}. For the evaluation of $\partial L / \partial y_i$ in (9), the variables b_j need to be acquired at Tx_i as described earlier in (iii), and the channels h_{ij} are available by assumption (ii). (All other quantities involved in $\partial L / \partial y_i$ are known at Tx_i by (i).)

5. NUMERICAL RESULTS

We tested our algorithm in a power control problem for a peer-to-peer CR-CDMA network consisting of $M = 8$ users with heterogeneous QoS constraints. With d_{ij} denoting the distance between Tx_i and Rx_j, the channels h_{ij} follow a (deterministic) path loss model with $h_{ii} = d_{ii}^{-4}$ and $h_{ij} = B^{-1} d_{ij}^{-4}$ for $i \neq j$, where $B = 128$ is the spreading gain. All Tx_i-Rx_i pairs are placed randomly with uniform distribution. Specifically, each Tx_i is placed on a square with side 10 m and each Rx_i is placed on a square with side 3 m centered at its corresponding Tx_i and at distance at least 1 m from it (if not, the position of Rx_i is redrawn). Table 1 lists the coordinates (on the plane) of 8 Tx-Rx pairs selected randomly as described. Since $h_{ij} > 0 \, \forall \, i, j$, matrix $\mathbf{A}$ is irreducible [5, Lem. A.22], and (a3) in Prop. 1 holds.

Logarithmic utilities (i.e., $u_i(\gamma_i) = \ln \gamma_i$) are adopted, as well as the heterogeneous QoS requirements used in [3] (given in terms of rates), mapped to SINR for our simulation. Specifically, we set $\gamma_i^{\min} = 8$, $\gamma_i^{\max} = 20$ for users 2, 3, and 4, $\gamma_i^{\min} = 20$, $\gamma_i^{\max} = 140$ for users 5, 7, and 8, and $\gamma_i^{\min} = 140$, $\gamma_i^{\max} = 20000$ for users 1 and 6. (The assignment of $\gamma_i^{\min}$, $\gamma_i^{\max}$ to

Pair	Coordinates Tx_i; Rx_i	Pair	Coordinates Tx_i; Rx_i
1	(4.80,5.15);(4.92,3.67)	5	(6.17,3.18);(6.95,4.40)
2	(5.61,6.06);(6.11,7.51)	6	(6.85,5.88);(8.07,6.70)
3	(6.16,9.67);(4.70,10.93)	7	(5.10,1.30);(4.45,0.12)
4	(6.62,8.22);(5.17,9.39)	8	(7.14,2.54);(5.83,1.05)

Table 1. Coordinates of 8 Tx-Rx pairs (shown in 2 columns).

	Lagrangian	QoS-ps-DSA	QoSe-DSA	ADP	SPCA
$\sum u_i$	32.43	12.1	12	33.7	
γ_1	140	**1.4e-07**	**7.1e-08**	**81**	**70.4**
γ_2	20	20	20	**43.3**	20
γ_3	20	20	20	**191.1**	20
γ_4	20	20	20	**6.2**	20
γ_5	32.9	52.5	91.3	55.2	**81.4**
γ_6	786.2	655.3	904.7	443	**734.2**
γ_7	140	140	140	**544.9**	140
γ_8	30	32.2	25.6	**7.5**	**24.1**

Table 2. Total utility (top) and SINR per user (bottom) achieved by different algorithms.

users is random.) We also set for all i, $P_i^{\max}/n_i = 40$ dB, $P_i^{\max}/P_i^{\min} = 90$ dB and a stepsize $\beta = 0.25$ for this example. Note that for these values, $h_{ii}P_i^{\min}/n_i < P_i^{\min}/n_i = -50\,\text{dB} < \gamma_i^{\max}$, as required by (a5) in Proposition 1.

Achievability of the SINRs $\gamma_i^{\max}$ can be checked with the SPCA (typically, no more than 20 iterations are required). The resulting SINRs are listed in the last column of Table 2. Clearly, users 1, 5, 6, and 8 have achieved SINR $\gamma_i < \gamma_i^{\max}$ (shown in boldface), confirming that (a4) in Prop. 1 holds and the utility maximization algorithm (9)–(12) needs to be used.

Total utility and SINR per user of this paper's algorithm (9)–(12) along with those of QoS-ps-DSA, QoSe-DSA [3] (for the single-channel case) and ADP [4] are listed in Table 2, where the SINRs violating the constraints are shown in boldface. Clearly, QoS-ps-DSA and QoSe-DSA, which incorporate QoS provisioning, failed to satisfy all users' rate requirements, although these were feasible. Also observe that they have not maximized the total utility. Moreover, we remark that the ADP has achieved the highest utility, which is expected, since it solves (4a) without the constraints (4b). Finally, note that despite the high value of $P_i^{\max}/n_i = 40$ dB, several users actually operate at much lower SINR, e.g., $\gamma_4 = 13$ dB, indicating that the network is not operating at high SINR.

Fig. 1 depicts the convergence of powers and Lagrange multipliers for our Lagrangian algorithm. Although the convergence is relatively fast, the figure indicates that it may take an order of magnitude more iterations to converge than its game-theoretic counterparts in [3] (but the algorithms in [3] do not guarantee satisfaction of the constraints). Note further that none of the Lagrange multipliers μ_i is finally zero, showing that the constraints (5c) are active for all i, in agreement with Prop. 1.

6. CONCLUSIONS

The present work tackled several aspects of the power control problem in peer-to-peer CR networks. The hierarchy between

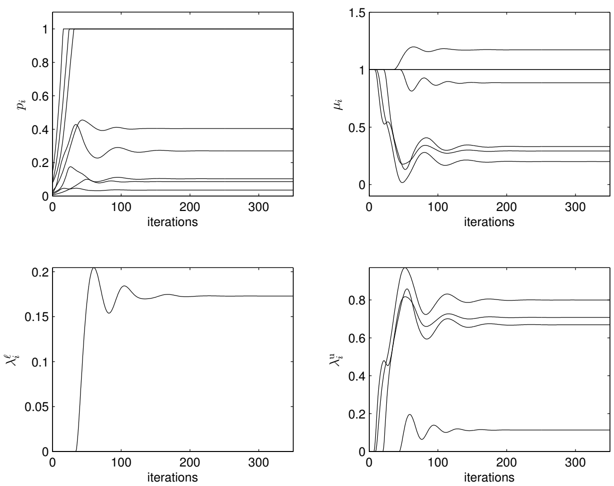

Fig. 1. Convergence of powers and Lagrange multipliers.

primary and secondary users was manifested through appropriate minimum and maximum SINR constraints, while a utility function was employed as a QoS indicator for each user. The optimal power control was obtained by considering a relaxed version of the original problem, which was shown equivalent to a convex problem; interestingly, a solution of the original problem could be recovered from the relaxed one under mild assumptions. Finally, a distributed algorithm for optimal power control requiring exchange of a scalar quantity was developed.[2]

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