Algebra Biomedical Engineering

Study Guide

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CONTENTS

Chapter I. Systems of linear equations

- Linear equations and their solutions
- Systems of linear equations and elementary operations
- Matrices and matrix operations
- Systems of linear equations and matrices
- Solution sets of linear systems
- Invertible matrices
- Determinants

Objectives: Review basic concepts of linear systems of equations. Introduce the matrix formalism used in the rest of the course and tools to compute and describe the solution sets of linear systems. Establish the connection between the solution sets of a linear system and the inverse and the determinant of the associated matrix.

Chapter II. Vector spaces

- Vector spaces and subspaces
- Linear combinations and spans
- Null space and column space of a matrix
- Linear independence
- Bases and dimensions
- Coordinates

Objectives: Introduce two fundamental concepts in linear algebra: vector spaces and linear independence. Define and study two vector spaces associated with any matrix, null and column space, and establish its connection with the rank of a matrix. Characterize the bases of a vector space and study the representation of vectors in different bases.

Chapter III. Linear mappings and diagonalization

- Linear mappings and matrices
    - Basic properties
    - Kernel and image of a linear mapping
    - Associated matrices
    - Associated matrices and change of bases
- Eigenvalues, eigenvectors, and diagonalization
    - Eigenvalues and eigenvectors
    - Multiplicity of eigenvalues
    - Diagonalization of linear operators

Objectives: Define and study basic properties of linear mappings. Study the kernel and image of a linear mapping as vector spaces. Study the representation of linear mappings as matrices with respect to different bases. Define eigenvectors and eigenvalues. Develop the theory of diagonalizable linear operators. Characterize diagonalizable linear operators in terms of the multiplicity of their eigenvalues.

Chapter IV. Inner products and orthogonality

- Inner product spaces
- Gram matrices of an inner product
- Cauchy-Schwarz inequality
- Orthogonality
- Orthogonal sets and bases
- Gram-Schmidt orthogonalization
- Orthogonal projection and minimization
- Least-squares problems
- Orthogonal diagonalization

Objectives: Introduce inner products and inner product spaces. Study the properties of orthogonal sets and bases as well as orthogonal projections with respect to orthogonal bases. Present the Gram-Schmidt orthogonalization algorithm. Study some applications of orthogonalization such as solving least-squares problems and orthogonal diagonalization of linear operators.

TIMELINE

Week	Contents	Reference
1	Chapter I. Linear equations and their solutions, systems of linear equations and elementary operations.	Lecture notes pages 23-31.
2	Chapter I. Matrices and matrix operations, systems of linear equations and matrices, solution sets of linear systems.	Lecture notes pages 31-48.
3	Chapter I. Invertible matrices, determinants.	Lecture notes pages 48-56.
4	Chapter II. Vector spaces and subspaces.	Lecture notes pages 57-63.
5	Chapter II. Linear combinations and spans, null space and column space of a matrix.	Lecture notes pages 63-69.
6	Chapter II. Linear independence.	Lecture notes pages 69-74.
7	Chapter II. Bases and dimension.	Lecture notes pages 74-79.
8	Chapter II. Coordinates.	Lecture notes pages 79-84.
9	First midterm exam	

10	Chapter III. Introduction and basic properties, kernel and image of a linear mapping.	Lecture notes pages 85-90.
11	Chapter III. Associated matrices, associated matrices and the change of basis.	Lecture notes pages 90-101.
12	Chapter III. Eigenvalues and eigenvectors. Multiplicity of eigenvalues, diagonalization of linear operators.	Lecture notes pages 101-115.
13	Chapter IV. Inner product spaces, Gram matrices of an inner product, Cauchy-Schwarz inequality.	Lecture notes 116-123.
14	Chapter IV. Orthogonality, orthogonal sets and bases, Gram-Schmidt orthogonalization.	Lecture notes 126-132.
15	Chapter IV. Orthogonal projections and minimization, least-squares problems, orthogonal diagonalization.	Lecture notes 132-138.
January	Second midterm exam	