

# Support Vector Robust Algorithms for Non-parametric Spectral Analysis

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**Abstract.** A new approach to the non-parametric spectral estimation on the basis of the Support Vector (SV) framework is presented. Two algorithms are derived for both uniform and non-uniform sampling. The relationship between the SV free parameters and the underlying process statistics is discussed. The application in two real data examples, the sunspot numbers and the Heart Rate Variability, shows the higher resolution and robustness in SV spectral analysis algorithms.

## 1 Introduction

Non-parametric spectral analysis of time series is a widely scrutinized framework. The most relevant of the classical spectral estimators, the Welch periodogram and the Blackman-Tukey correlogram, are based on Fourier-Transform representations, either for the observed time series or for its estimated autocorrelation function [1], so their main advantages are the low computational burden required and their simplicity. On the other hand, their spectral resolution is limited due to the effect of windowing. The spectral analysis of non-uniform sampled series has also been suggested by means of the Lomb periodogram [2,3]. In both cases, the Periodogram exhibits a high sensitivity to outliers.

An alternative approach to the classical non-parametric spectral analysis can be drawn from the Support Vector (SV) framework, which was first suggested to obtain maximum margin separating hyperplanes in classification problems [4,5,6]. In this work, we propose to modify the standard SV regression algorithm and the cost function to provide an adequate approach to non-parametric spectral analysis problems.

SV algorithms for the Welch periodogram and the Lomb periodogram are first derived. Two application examples, the sunspot numbers for the SV-Welch and the Heart Rate Variability (HRV) for the SV-Lomb algorithms are shown, to support the potential of this new approach.

\*\*\* This work has been partially supported by the project 07T/0046/2000 of the CAM

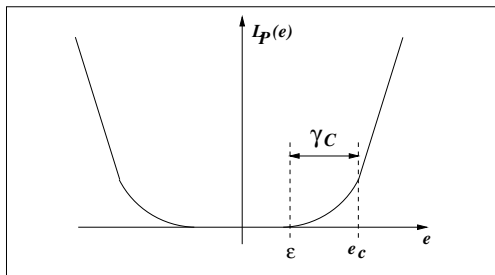


Fig. 1. Robust SV cost function.

## 2 SV-Welch Periodogram

The model for the harmonic decomposition of a sequence  $\{y_n\}$  can be expressed in terms of the discrete-time harmonic Fourier series. The spectral analysis of the observed signal can be denoted as follows,

$$y_n = \sum_{i=1}^{N_\omega} A_i \cos(\omega_i n + \phi_i) \tag{1}$$

where the unknown parameters are amplitudes  $A_i$ , phases  $\phi_i$  and frequencies  $\omega_i$  for a number  $N_\omega$  of sinusoidal components. This is a non-linear relationship, except in the case where frequencies are known, say  $\omega_i = 2\pi i/N$ . In this case, Eq. (1) can be linearly expressed by switching to Cartesian coordinates  $c_i = A_i \cos(\phi_i)$  and  $d_i = A_i \sin(\phi_i)$ , and the model can be written down as

$$y_n = \sum_{i=1}^{N_\omega} [c_i \cos(\omega_i n) + d_i \sin(\omega_i n)]. \tag{2}$$

Several robust cost functions have been used in SV regression, as the Vapnik’s loss function [4], Huber’s robust cost [7], or the ridge regression approach [8]. We propose here a more general cost function, which has the above mentioned ones as particular cases, we force the minimization of

$$\frac{1}{2} (\|\bar{c}\|^2 + \|\bar{d}\|^2) + \frac{1}{2\gamma} \sum_{k \in I_1} (\xi_k^2 + \xi_k^{*2}) + C \sum_{k \in I_2} (\xi_k + \xi_k^*) \tag{3}$$

constrained to

$$y_k - \sum_{i=1}^{N_\omega} c_i \cos(\omega_i k) - \sum_{i=1}^{N_\omega} d_i \sin(\omega_i k) \leq \varepsilon + \xi_k \tag{4}$$

$$-y_k + \sum_{i=1}^{N_\omega} c_i \cos(\omega_i k) + \sum_{i=1}^{N_\omega} d_i \sin(\omega_i k) \leq \varepsilon + \xi_k^* \tag{5}$$

$$\xi_k, \xi_k^* \geq 0 \tag{6}$$

for  $k = 1, \dots, N$ , where  $\xi_k^{(*)}$  are the losses, and  $I_1, I_2$  are the sets of samples for which losses are required to have quadratic or linear cost, respectively. Figure 1 depicts the relationship between the approximation error  $e_n$  and its corresponding loss. Note that for  $\gamma$  small enough this represents the regularized Vapnik's  $\varepsilon$ -insensitive cost, whereas for  $\varepsilon = 0$  it represents Huber's robust cost.

The derivation of the dual problem shows the Cartesian components can be expressed as

$$c_l^W = \sum_{k=1}^N (\alpha_k - \alpha_k^*) \cos(\omega_l k); \quad d_l^W = \sum_{k=1}^N (\alpha_k - \alpha_k^*) \sin(\omega_l k) \quad (7)$$

where  $\alpha_k^{(*)}$  are the Lagrange multipliers for the constraints in (4) and (5). Therefore, the  $l^{th}$  coefficients are the cross-correlation of the Lagrange multipliers and the sinusoid with frequency  $\omega_l$ . These conditions are introduced into the Lagrange functional in order to remove the primal variables, and it is easy to show that if we write down

$$R_{cos}^W(m, k) = \sum_{i=1}^{N_\omega} \cos(\omega_i m) \cos(\omega_i k); \quad R_{sin}^W(m, k) = \sum_{i=1}^{N_\omega} \sin(\omega_i m) \sin(\omega_i k) \quad (8)$$

then the regularized  $L_D$  dual problem, to be maximized with respect to the dual variables only, can be solved by maximizing

$$L_D = -\frac{1}{2} (\bar{\alpha} - \bar{\alpha}^*)^T [\mathbf{R}_{cos}^W + \mathbf{R}_{sin}^W] (\bar{\alpha} - \bar{\alpha}^*) + (\bar{\alpha} - \bar{\alpha}^*)^T \bar{y} - \varepsilon \bar{\mathbf{1}}^T (\bar{\alpha} + \bar{\alpha}^*) - \frac{\gamma}{2} (\bar{\alpha}^T \mathbf{I} \bar{\alpha} + \bar{\alpha}^{*T} \mathbf{I} \bar{\alpha}^*) \quad (9)$$

constrained to  $0 \leq \alpha, \alpha^* \leq C$ .

It can be easily shown that the SV approach leads to the non-orthogonal, regularized representation of the observation data upon the signal subspace which is generated by the Cartesian representation of the sinusoidal components in the model. The implications of this interpretation are currently being studied.

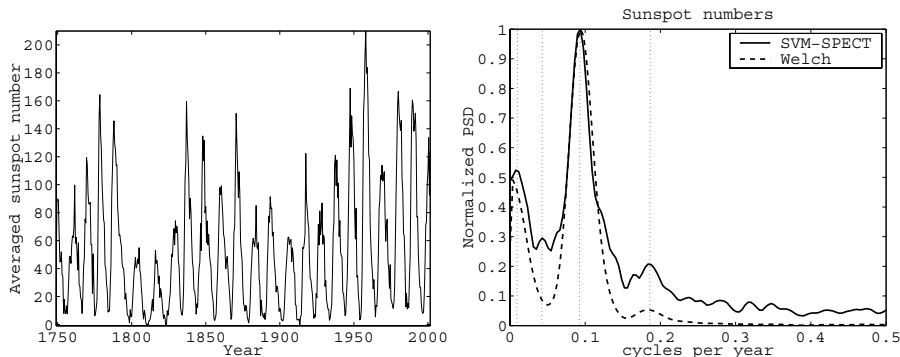
### 3 SV-Lomb Periodogram

For series of data samples  $\{y_{t_n}\}$  which have been non-uniformly sampled in the corresponding time instants  $\{t_1, \dots, t_N\}$ , the model is

$$y_{t_n} = \sum_{k=1}^{N_\omega} A_k \cos(\omega_k t_n + \phi_k) = \sum_{k=1}^{N_\omega} [c_k \cos(\omega_k t_n) + d_k \sin(\omega_k t_n)] \quad (10)$$

and in this case the solution coefficients are given by

$$c_l^L = \sum_{k=1}^N (\alpha_k - \alpha_k^*) \cos(\omega_l t_k); \quad d_l^L = \sum_{k=1}^N (\alpha_k - \alpha_k^*) \sin(\omega_l t_k). \quad (11)$$



**Fig. 2.** Sunspot numbers. **Left:** Averaged and decimated sunspot number series. **Right:** Normalized Welch and SV spectral estimators (natural units).

Taking into account

$$R_{\cos}^L(m, k) = \sum_{i=1}^{N_w} \cos(w_i t_m) \cos(w_i t_k); \quad R_{\sin}^L(m, k) = \sum_{i=1}^{N_w} \sin(w_i t_m) \sin(w_i t_k) \tag{12}$$

the problem is now equivalent to maximize the dual functional

$$L_D = -\frac{1}{2} (\bar{\alpha} - \bar{\alpha}^*)^T [\mathbf{R}_{\cos}^W + \mathbf{R}_{\sin}^W] (\bar{\alpha} - \bar{\alpha}^*) + (\bar{\alpha} - \bar{\alpha}^*)^T \bar{y} - \varepsilon \bar{\mathbf{1}}^T (\bar{\alpha} + \bar{\alpha}^*) - \frac{\gamma}{2} (\bar{\alpha}^T \mathbf{I} \bar{\alpha} + \bar{\alpha}^{*T} \mathbf{I} \bar{\alpha}^*) \tag{13}$$

constrained to  $0 \leq \alpha, \alpha^* \leq C$ .

## 4 Application Examples

**Periodogram and sunspot numbers.** A classical application of the classical spectral estimation is the search of periodicity in sunspot numbers. This experiment has been taken from [1, pp. 161-4], where preprocessing is detailed. The updated series of 3036 monthly sunspot numbers for the years from 1750 to 2001 were analyzed, aiming to study the details in the band between 0 and 2 cycles per year. The averaged Welch and SV-Welch periodograms were calculated. The free parameters in SV were  $\varepsilon = 0$ ,  $C = 100$  and  $\gamma = 0.01$  as this set was previously shown to give an enhanced harmonic spectral structure.

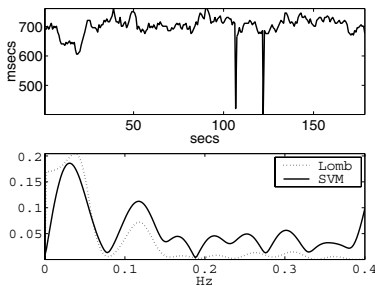
Figure 2 shows the normalized periodograms. The Welch estimator points the 10.5 year dominant component, one sub-harmonic and one harmonic. For the same frequency resolution, the SV estimator marks 4 components, which is coherent with the analysis performed in [1], where different non-parametric methods highlighted different components (0.95, 0.188, 0.25 and 0.45 cycles per

year). The peak frequencies were very similar in both the SV estimator and the several classical analysis, but the first one was able to mark all the peaks in a single spectral representation.

**Robust HRV analysis.** The natural oscillations of the time between consecutive heart beats are known as HRV, and it is related with the modulation of the sympathetic and the vagal nervous system on the heart rhythm [9]. In healthy conditions, the power of the oscillations observed in the LF band (from 0.04 to 0.15 Hz) is balanced with the power in the HF band (from 0.15 to 0.4 Hz). The association of the time between two consecutive beats to the time where the first couple beat happens leads to a non-uniform sampled series. Besides, ectopic beats frequently appear, but they are not related with the modulation of the autonomic system and they should be excluded from the analysis.

The Lomb periodogram has allowed to overcome the non-uniform sampling problem. Nevertheless, the detection of ectopic beats has to be manually validated by a medical expert, and this is a much time-consuming task. This fact has been a main limitation for the application of the HRV into the habitual clinical practice, and it suggests the introduction of a robust, ectopic-insensitive spectral analysis.

The SV-Lomb can be adapted by tuning its free parameters according to the nature of the HRV signal. The impact of the ectopic beats can be limited by constraining the amplitude of the Lagrange multipliers ( $\gamma C$ ) to a low enough level. Additionally, the SV-Lomb method has been observed to model the high-frequency noise from the high-frequency sinusoidal components which are non-uniform sampled, hence overestimating the HF power. This effect can be limited by choosing an insensitivity level  $\epsilon$  and a moderate frequency resolution.



**Fig. 3.** Heart Rate Variability. **Up:** time signal (dotted) and SV-Lomb approximation (continuous). **Down:** Lomb periodogram (dotted) and SV-Lomb (continuous) normalized spectral estimators.

Figure 3 shows an application example to a HRV series. The free parameters were chosen as  $C = 4$ ,  $\gamma = 1$ ,  $\epsilon = 3$  and the frequency resolution  $\Delta f = 0.001 Hz$ . Note the SV-Lomb estimators are less affected by the inclusion of ectopic beats in the window than Lomb periodogram is. In the first case, the spectral estimator holds the power in the LF and HF bands, whereas the Lomb periodogram spectrum is flatter

due to the influence of the wide-band spectral contamination of the ectopics. The overestimation of the frequency components near to 0.4 Hz in the SV-Lomb estimator can be observed. Further and deeper study is to be done in order to explore the possibilities of this method in HRV series analysis.

## 5 Conclusion

The application of the SV framework to classical, non-parametric spectral analysis is a promising framework. For uniform sampling applications, the spectral resolution of SV is higher than the corresponding classical methods. For non-uniform sampling, a more enhanced harmonic spectral structure can be captured by the SV method when compared to the Lomb periodogram. The SV spectral analysis seems to be a good approach for the robust measurement of the HRV in a clinical environment. Further work is to be done in several directions. First, the theoretical and statistical analysis of the SV spectral algorithms have to be completed. Second, the robust approach to the HRV problem has to be delimited and performed. Third, other application fields should be proposed and explored, where the advantages of the SV framework can be of interest, such as voice processing or astronomy data analysis. Finally, other spectral SV algorithms, such as signal-and-noise subspace decomposition, ARMA modeling and deconvolution, will be developed.

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