Using Multiobjective Optimization Algorithms and Decision Making Support to Solve Polymer Extrusion Problems

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Single screw extrusion is a major polymer processing operation. Its optimization is crucial for producing good quality products at suitable costs. This study addresses extrusion as a multiobjective optimization problem that can be solved using evolutionary algorithms incorporating decision making and robustness strategies for selecting solutions. This approach enables focusing the search for solutions in favored regions where the preference was defined either by the relative importance of the objectives or determined considering the robustness of solutions against perturbations in the design variables. The outcome of this strategy provides not only a better insight into the problem at hand, but also facilitates the choice of a single solution for practical implementation. The usefulness of the approach is illustrated by several case studies involving the definition of the most adequate operating conditions, of the best screw geometry and the two together. POLYM. ENG. SCI., 58:493–502, 2018. © 2017 Society of Plastics Engineers

INTRODUCTION

Plasticating screw extrusion involves the conversion of the inlet material (usually in pellet or powder form) into a homogeneous melt that is continuously pushed through a shaping die, to produce a molten extrudate with the required cross-section. Extrusion products are used in a variety of industries—including window and roofing profiles for construction, plastic tubes for engineering and medicine, vehicle trims and door frames for transportation, shelves and racks for retail, film cores and packaging tubes for food and cosmetics. Extrusion is also a unit operation for other industrially relevant manufacturing technologies such as plastics compounding, injection molding, and blow molding.

Single screw extruders use an Archimedes-type screw rotating at a constant controllable speed inside a hollow barrel that is kept under a set temperature profile. The solid polymer is typically delivered to the screw channel by gravity flow from a vertical hopper. It is then dragged forward due to friction forces and eventually melts due to conducted and dissipated heat. The melt is progressively mixed and pressurized and subsequently flows through the die [1–3]. Thus, extrusion is a complex process, its performance depending on a number of factors including polymer properties, operating conditions and screw and die geometries. Setting/adjusting the operating conditions for a given production or designing a screw with improved performance can constitute major challenges, as the process is characterized by multiple, often conflicting, objectives. For instance, a large mass output usually entails high power consumption that should be minimal for economic reasons, and may also jeopardize the physical properties of the extrudate due to poorer mixing. In practice, finding the trade-off between the different objectives involved in extrusion in order to ensure a competitive and adequate production is frequently performed on a trial-and-error basis, heavily dependent on personal knowledge and experience.

Process modelling can support decision making based upon quantitative predictions of process behavior. Modelling involves solving the relevant governing equations coupled to adequate boundary conditions and constitutive equations in order to pressure, stress, temperature and velocity [1–4]. These predictions are delivered for a given set of input values, thus it is up to the user to identify the input parameters (e.g., operating parameters and/or geometry) that will satisfactorily solve the extrusion problem. Unfortunately, solving the inverse problem, i.e., solving the governing equations in order to the operating parameters and/or geometrical variables is often mathematically ill-posed [5]. A few methodologies have been proposed to approach extrusion problems. Rauwendaal [1] derived analytical equations for distinct extrusion stages that addressed various individual process objectives. Other researchers coupled statistical methods to process modelling routines. For example, Potente [6] combined factorial experiments to modelling routines to optimize screw geometry. However, statistical methods usually generate a number of points that can be insufficient to describe a multimodal/complex response.

An alternative route to approaching extrusion problems consists in linking global optimization methods and process modelling. While the former searches for the best solution(s) within the feasible search space, the latter is used to evaluate the performance of each solution considered during the optimization. Covas et al. [7] determined the optimal operating settings for single screw extrusion considering the weighted sum of four objectives and a genetic algorithm. Later, various objectives were simultaneously optimized for single [8] and twin screw extrusion [9], the results being validated experimentally. The

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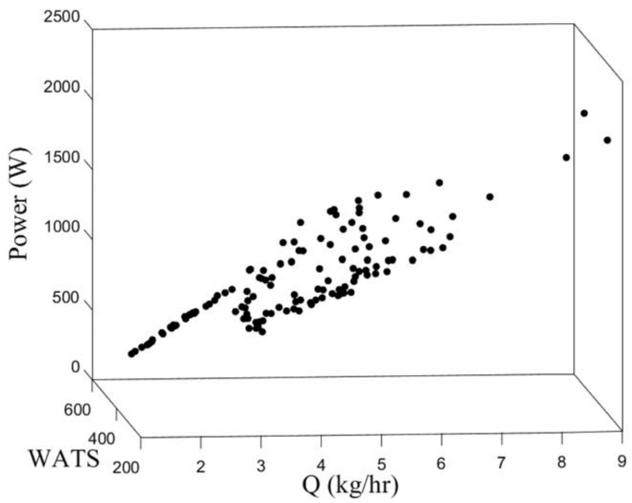

FIG. 1. Example of a Pareto front for the optimization of the operating conditions of a single screw extrusion process aiming at maximizing the output (Q) and degree of distributive mixing (WATS), and minimization of mechanical power consumption (Power).

methodology was also applied to extrusion scale-up [10]. This approach is known as multiobjective optimization (MO) and generates a set of Pareto optimal solutions [11, 12]. These are the global solutions to the problem that cannot be improved in terms of any of the individual objectives without compromising some of the other objective values. The user—usually known as decision maker (DM) in the field of optimization—can then select the most adequate solution from a list, taking into account his/her own preferences.

Application of MO to plasticating extrusion is not straightforward due to the nonlinear interactions between the search variables, the multimodality of the search space, the high number, and conflicting character of the objectives and the need that the solutions obtained meet convergence and diversity requirements [9]. The set of Pareto optimal solutions can be quite large, making the task of the DM difficult and tedious. The problem is illustrated in Fig. 1, where a Pareto plot is shown for setting the operating conditions [screw speed (N) and barrel temperatures (Ti)], in order to maximize the output (Q) and the degree of distributive mixing (calculated as WATS, as suggested in [13]) and to minimize the mechanical power consumption (Power) for a specific extruder, die and polymer (for a detailed analysis, see [9]). Given the complexity of the three-dimensional Pareto surface, selection of the best solution is not evident. Table 1 presents the best solutions to maximize and/or minimize each of the three objectives. These solutions correspond to the extreme points of the plot, i.e., to the solutions that take into account a single objective and, therefore, are not the best answer to the problem. Thus, it is clear that the best overall solution, hopefully within the Pareto front, results from a compromise defined by the preferences of the DM.

The aim of this study is to develop the methodology able to support decision making when solving polymer extrusion problems. This methodology is based on MO that integrates the decision maker's preferences and allows to obtaining not only optimal solutions but also those most relevant to the DM. More specifically, the DM preferences will be defined before the search and taken in from two different perspectives. One concerns the relative importance of the objectives, which is relatively easy to define for most process engineers. The other considers the robustness of the solutions against inescapable small perturbations in the process. The method can run automatically, requiring little intervention from the DM.

Process Modelling

The typical configuration of a single screw extruder is depicted in Fig. 2. The screw (with diameter D) rotates inside a hollow barrel (with length L). The latter contains a lateral opening for material inlet through a hopper, while the die is fixed at the opposite end. Both barrel and die are enveloped by heater bands. The screw has three geometrically distinct sections (with lengths L_1, L_2 and L_3, respectively), with constant channel depth (H_1), channel depth varying linearly (between H_1 and H_3) and constant depth (H_3), respectively. The screw helix is defined by its pitch (P) and flight thickness (e). The operator sets the screw rotation speed (N) and barrel/die temperature profile (T_b). As the material is poured into the hopper, it is successively subjected to: (1) gravity induced flow of discrete solid particles in the hopper towards the screw channel; (2) friction drag of the solids along the screw together with dissipated heat and conducted heat from the barrel; (3) melting of a thin layer of material adjacent to the inner barrel wall; (4) progressive melting of the remaining material, according to a mechanism involving segregation of melt and surviving solids; (5) viscous drag of the molten material with pressure generation; and (6) pressure flow through the die [1, 2]. This process is influenced by the geometry of the extruder components such as the barrel, screw, and die [14]. Important effects are also produced by the operating conditions and the characteristics of the material. The latter encompass physical properties (friction coefficients, solids and

TABLE 1. Best results for the individual objectives considered in the optimization of the operating conditions of a single screw extrusion process.

		Operating conditions				Objectives		
Aim	Objective	N (rpm)	T_1 (°C)	T_2 (°C)	T_3 (°C)	Q (kg/h)	Power (W)	WATS
Maximize	Min (Q)	10.8	209.9	175.6	150.8	1.43	296.9	488.0
	Max (Q)	59.9	155.2	189.3	199.5	8.83	2038.0	238.4
Minimize	Min (Power)	59.4	156.2	206.1	154.2	8.45	2231.0	239.9
	Max (Power)	10.8	209.9	175.6	150.8	1.43	296.9	488.0
Maximize	Min (WATS)	59.9	155.2	189.3	199.5	8.83	2038.0	238.4
	Max (WATS)	10.8	209.9	175.6	150.8	1.43	296.9	488.0

Data taken from the Pareto front plotted in Fig. 1.
The minimum and maximum objective values are shown in bold.

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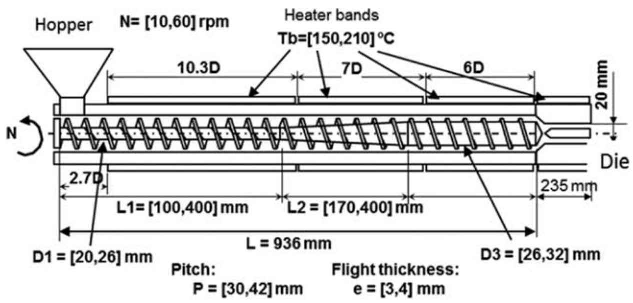

FIG. 2. Decision variables (geometrical and operating parameters) for a single screw extruder. The range of variation is defined between square brackets.

melt density, etc.), thermal properties (heat conduction coefficients, melting temperature, heat capacity, etc.) and rheological properties (shear-dependent viscosity).

For process modeling purposes, each of the above steps can be mathematically described by constitutive equations relating to mass, momentum, and energy conservation, together with a rheological law and the relevant boundary conditions. Coherent linkage between contiguous steps is assured by proper boundary conditions. Details of the modelling routine developed by the authors and of its experimental validation can be found elsewhere [15]. For a given set of inputs, the model predicts the main process responses, such as mass output, Q, average melt temperature of the polymer at die exit, T_{melt}, mechanical power consumption, Power, length of screw required for melting the polymer, L_{melt}, degree of distributive mixing (in terms of the average deformation induced, WATS), as well as the evolution of pressure, temperature, shear rate, etc., along the screw. Changes in the values of the input variables will cause alterations in the process responses.

MULTIOBJECTIVE OPTIMIZATION

Background

When approaching extrusion as a multiobjective optimization problem (MOP), a set of Pareto optimal solutions is obtained, representing different trade-offs between the individual (some of them conflicting) objectives. In the absence of additional information, the Pareto optimal solutions are equally important. In order to select a single solution, the DM must express his/her preferences 11.

Multiobjective evolutionary algorithms (MOEAs) [11, 12] are appropriate to solve MOPs due to their population-based nature, which enables an approximation to the Pareto set in a single run. By incorporating the DM preferences into MOEAs, the search can focus on the interesting regions of the space. Then, high-resolution Pareto optimal regions can be obtained instead of the whole Pareto set containing numerous solutions but many of them probably inappropriate to the DM. Since there is no need to explore the entire search space, the computational overhead is reduced, which is pertinent to those applications where function evaluations involve expensive simulations (such as extrusion).

Formulating and integrating the DM preferences into MOEAs in order to direct the search is challenging and thus remains an active research topic. Preferences can be formulated as constraints that specify the limits for the objectives, or as a weight vector expressing how important the objectives are [16]. Most previous work on extrusion optimization adopted this approach, aggregating multiple objectives into a value or utility function [6]. The approach has three major limitations: (1) combining objectives referring to process parameters of different nature may not make sense; (2) small changes to the weights may produce quite different solutions without practical value, and (3) the method may not be applicable to certain shapes of the optimal Pareto front [11]. To overcome these difficulties, some authors proposed to provide reference points reflecting the aspiration level of the DM toward the objective values [17]. Others exploited a biased distribution of solutions by scaling differently the objectives [18], or mapped the solutions according to desirability functions that incorporate knowledge about target regions [19], or suggested a nonlinear transformation of the objectives into desirability functions [20] and the definition of a weighted function over the objective space [21]. Reviews of preference-based methods can be found in [22]. Integrating the preferences of the DM remains an open problem because the methods proposed so far are unable to clearly correlate the preferences defined by the DM (e.g., using weights or goals) with the solutions (or regions) on the Pareto front [23]. This is particularly delicate in problems where the solutions generate a Pareto front with a complex shape.

Simultaneously, when dealing with real-world extrusion, it is also important to consider the sensitivity of the solutions to small variations of the parameters of the problem. Solutions exhibiting little sensitivity to such variations are labelled as robust and are favored. Robustness is usually addressed either by optimizing the expectation and the variance [24], or by introducing additional constraints [17, 25]. The topic was reviewed by Jin and Branke [26] and Beyer and Sendhoff [27]. As in the case of DM, no existing method can address MO and robustness simultaneously in an efficient way, probably due to two main difficulties: (1) the likelihood of having to deal with various decision variables in robustness calculations; (2) the need to identify the neighbors of the point where robustness must be calculated [28].

Multiobjective Evolutionary Algorithm

The method starts with the identification of the problem characteristics, such as main objectives, decision variables and constraints of process parameters. Obviously, a process modeling tool must be available to predict process responses and thus evaluate the solutions.

A specific MOEA, the Reduced Pareto Set Genetic Algorithm (RPSGA), was used given its good performance when applied to various benchmark problems [29], including robustness studies [28, 30]. MOEAs are characterized by the use of an internal population of solutions that are progressively improved along various generations. The RPSGA maintains also an external population of the best solutions. In summary, at each generation the following operations are performed: (1) the internal population is evaluated using the results of the modeling routine; (2) a clustering technique is applied to reduce the number of solutions on the efficient frontier and to calculate the ranking of the individuals of the internal population; (3) the fitness of the individuals is calculated using a ranking function; (4) a fixed number of the best individuals are copied to the external population; (5) if the external population is not complete, genetic operators are applied to the internal population to generate a new population; (6) if the external population is complete, the clustering technique is applied to sort the individuals of the external population and a pre-defined number of the best individuals are incorporated in the internal population, replacing those individuals with the lowest fitness.

A second MOEA, the Non-dominated Sorting Genetic Algorithm (NSGA–II) [31], was used as reference. This algorithm is extensively used in optimization because it can be easily be adapted to real problems and converges quickly to a good approximation of the optimal Pareto front. It uses Pareto dominance and crowding distance measure concepts to evaluate the quality of the individuals of the population. A comprehensive presentation can be found in [31].

Each of the above two MOEAs yields a subset of the Pareto optimal solutions. As shown below, Decision Making and Robustness are taken into account when calculating the fitness of the individuals, which forces the MOEA to converge to specific regions (subsets) of the Pareto front.

Introducing Preference Based on Importance of Objectives

The Weighted Stress Function Method (WSFM) [23] is based on the idea that the solution that best meets the DM preferences must belong to the set of non-dominated solutions (i.e., the solutions on the Pareto surface), and that the difference between the ideal objective vector and each solution induces a "stress" on that solution that depends on the relative importance attributed to each objective. The concept was inspired by the stress–strain behavior of thermoplastic vulcanizates (TPV). These materials exhibit high variations of stress with strain at low or high strain values, while at intermediate strains the stress changes little. This behavior was described mathematically by Coran and Patel [32], taking into consideration the thermoplastic/rubber concentration, v_p. WSFM mimics this behavior. The weight w_j attributed to the jth objective and v_p range in the same interval [0,1], and play a role similar to that of increasing or decreasing the stress. The solution that best meets the DM preferences is the one having balanced stresses. WSFM can be used for both minimization and maximization problems. Given the weight vector $\boldsymbol{w}=(w_1, \ldots, w_m)$ specifying the relative importance of the m objectives and the set of N solutions $X=\{\boldsymbol{x}^1, \ldots, \boldsymbol{x}^N\}$, the solution that best meets the preferences can be found by solving:

$$\text{minimize}_{\boldsymbol{x}\in X} : T(\boldsymbol{x})= \max_{1\leq j\leq m} \quad \sigma_j(f(\boldsymbol{x}), \boldsymbol{w}) \tag{1}$$

where σ_j is a stress that is associated with the jth objective and computed as a function of its value and the value of weight w_j [23]. Thus, the fitness of the ith population individual can be formulated as:

$$F(i)=\text{Rank}(i)+\frac{T(i)}{T(i)+1} \tag{2}$$

where Rank(i) is the rank based on the Pareto dominance.

Introducing Preference Based on Robustness of Solutions

To address robustness, the following steps were introduced in the MOEAs:

(1) A variance-based measure of the ith individual with respect to the mth objective, R_m, is calculated with the following equation:

$$R_m(i)=\frac{1}{N'}\sum_{k=1}^{N'}\frac{|f_m(x_i)-f_m(x_k)|}{|x_i-x_k|} \tag{3}$$

where N' is the number of neighbors, k, whose distance in the decision space, d'_{ik}, in not greater than $d'_{\max}$. This distance can be determined using:

$$d'_{ik}=\sqrt{\sum_{l=1}^{L}\left(x_{li}-x_{lj}\right)^2} \tag{4}$$

Since multiple objectives are considered, the robustness measures of individual objectives are combined:

$$\text{R}(i)=\frac{1}{M}\sum_{m=1}^{M}R_m(i) \tag{5}$$

(2) The distance metric for diversity preservation, which is a measure of the density of neighbors, is computed from:

$$\text{I}(i)= \sum_{i=1}^{N} sh(d_{ik}) \tag{6}$$

where $d_{ik}$is the distance between the ith population member and all its k neighbors, $sh(d_{ik})$ is a sharing function that takes into account the distance in the objective space and is calculated as:

$$sh(d_{ik}) = \begin{cases} 1-\left(\dfrac{d_{ik}}{\sigma_{\text{share}}}\right)^2, & d_{ik} \leq \sigma_{\text{share}} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

with σ_{share} being an experimentally determined parameter.

(3) The global fitness value of the ith population individual, F$(i)$is given by:

$$\text{F}(i)=\text{Rank}(i)+\varepsilon\frac{I(i)}{I(i)+1}+(1-\varepsilon)\frac{R(i)}{R(i)+1} \tag{8}$$

where Rank(i) is the rank based on the Pareto dominance relation and ε is the dispersion parameter that determines the degree to which robustness influences global fitness. Smaller fitness values correspond to a better performance.

RESULTS AND DISCUSSION

Case Studies

The application of optimization methods to solve single screw extrusion problems will be discussed by addressing the 11 case studies presented in Table 2. Setting the most adequate operating conditions, defining the best screw geometry and the two together will be approached. The polymer properties as well as the die geometry will remain constant. For each case study, the table indicates the type of optimization to be performed, the decision variables (process parameters) and the objectives (process responses). The range of variation of the decision variables is defined between square brackets in Fig. 2, which schematizes a conventional small size single screw extruder. Objectives include maximize mass output, $Q \in [1, 20]$ kg/h and degree of distributive mixing, WATS $\in [0, 1300]$, and minimizing the length of screw required for melting, $L_{\text{melt}} \in [0.2, 0.9]$ m, melt

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TABLE 2. Optimization case studies.

Case	Problem type	Decision Variable										Objectives
1	Operating											Q, L_{melt}
2	conditions											Q, T_{melt}
3					N	T_{b1}	T_{b2}	T_{b3}				Q, Power
4												Q, $WATS$
5												All
6	Screw											Q, Power
7	geometry			L_1	L_2	H_1	H_3	P	e			Q, WATS
8												All
9	Operating	N	T_{b1}	T_{b2}	T_{b3}	L_1	L_2	H_1	H_3	P	e	Q, Power
10	conditions and	N	T_{b1}	T_{b2}	T_{b3}	L_1	L_2	H_1	H_3	P	e	Q, WATS
11	Screw geometry	N	T_{b1}	T_{b2}	T_{b3}	L_1	L_2	H_1	H_3	P	e	All

temperature at die exit, $T_{melt} \in [150, 210]\,°C$, and mechanical power consumption, Power $\in [0, 9200]$ W.

Decision Making Based on Importance of Objectives

The influence of the relative importance of the objectives on the results of an optimization run can be more easily understood for those case studies of Table 2 involving only two objectives. The numerical experiments were performed for combinations of weights $w_1, w_2 \in \{0.1, 0.2, 0.5, 0.8, 0.9 | w_1+w_2=1\}$.

Figure 3 displays the results for case studies 1–4 referring to operating conditions and Table 3 presents the best solutions in terms of the values of the decision variables (operating parameters) and objectives (process responses) proposed by RPSGA. The graphs on the left column were obtained using RPSGA, those on the right resulted from using NSGA–II. Table 2 also includes the results for case study 5 involving the simultaneous optimization of 5 objectives (thus, a 5-dimensional Pareto front exists). The correlations between output and the remaining objectives were expected, at least qualitatively. Higher outputs require longer length of screw channel to complete melting, induce higher viscous dissipation (higher melt temperature at die exit) and higher mechanical power consumption. The effect on distributive mixing (WATS) is more complex, but generally WATS decreases with increasing Q due to the joint effect of shorter screw length fully filled with melt and lower residence time. From an optimization point of view, for a fixed weight vector the solutions converge to a specific Pareto optimal region, its location in the objective space depending on the value of w_1, w_2. For $w_1=0.9$, the algorithm focus mainly on maximizing output, whereas for $w_2=0.9$, Q is somewhat neglected. When optimizing Q and WATS, the solutions

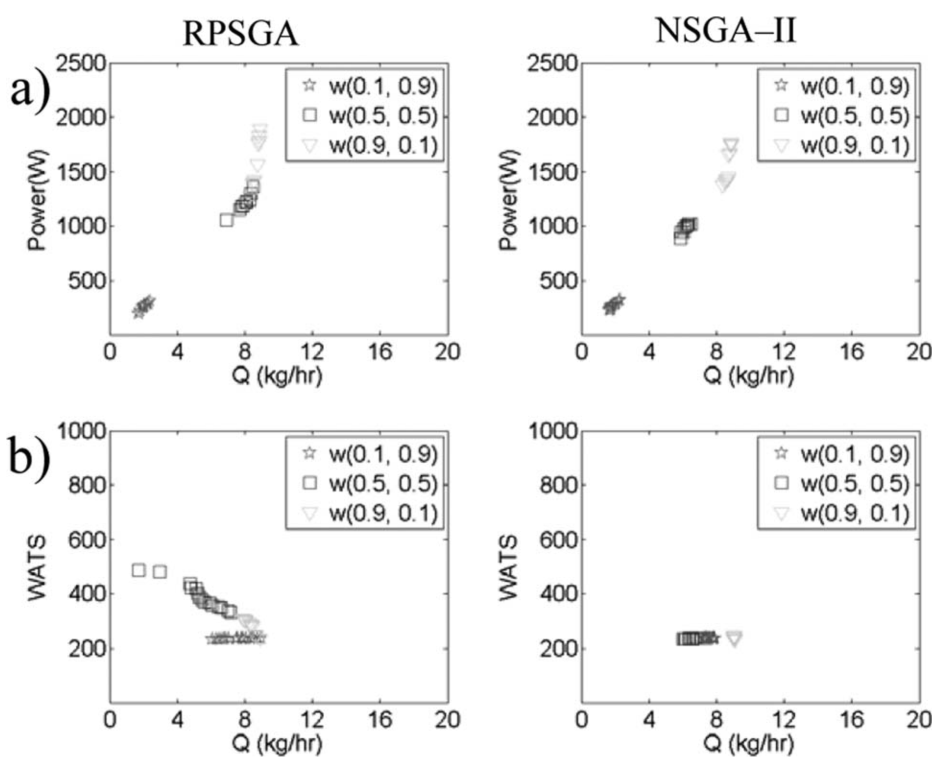

FIG. 3. Pareto frontiers for decision making considering the optimization of operating conditions and using RPSGA and NSGA-II: (a) case study 3; (b) case study 4.

TABLE 3. Best results presented in the decision variables and objectives domains using RPSGA for the 11 case studies of Table 2: W1 (0.1,0.9); W2 (0.5,0.5); W3 (0.9,0.1); W4 (0.1,0.225,0.225,0.225,0.225); W5 (0.5,0.125,0.125,0.125,0.125); W6 (0.9,0.025,0.025,0.025,0.025).

		Decision variables										Objectives				
		N	T_{b1}	T_{b2}	T_{b3}	L_1	L_2	H_1	H_2	P	e	Q	L_{melt}	T_{melt}	Power	WATS
Case	Weights	rpm	°C	°C	°C	mm	mm	mm	mm	mm	mm	kg/h	mm	°C	W	
1	W1	24.4	208.8	159.1	199.1							3.7	0.2	199.6	1047.3	423.4
	W2	53.4	209.6	184.6	188.6							7.7	0.3	197.3	2069.2	310.6
	W3	60.0	152.4	190.5	209.5							9.0	0.7	209.8	1765.7	235.9
2	W1	16.8	150.1	187.8	150.0							2.4	0.4	153.4	535.2	283.1
	W2	50.2	150.0	174.1	150.0							7.2	0.6	162.0	1798.9	239.9
	W3	60.0	150.1	190.2	199.9							8.9	0.6	195.0	1816.6	236.6
3	W1	14.4	209.9	193.0	208.4							2.4	0.2	208.5	310.8	427.5
	W2	52.0	209.8	167.7	206.0							7.8	0.5	203.5	1177.0	242.3
	W3	60.0	170.1	201.6	209.9							8.8	0.6	213.2	1786.8	237.3
4	W1	42.6	153.2	179.5	209.1							6.4	0.6	206.2	1263.9	235.1
	W2	13.2	209.7	168.9	150.1							1.7	0.1	156.8	510.1	486.9
	W3	58.2	210.0	177.8	190.6							8.2	0.5	196.5	1731.3	295.5
5	W4	36.8	205.8	171.0	151.8							5.1	0.2	170.3	1449.3	419.7
	W5	51.7	207.9	182.0	151.0							7.2	0.4	175.7	1564.1	275.0
	W6	44.1	204.4	186.8	152.3							6.0	0.4	172.9	1351.0	280.3
6	W1					382.2	392.1	20.5	26.5	37.0	3.0	10.3	0.8	178.4	1254.2	49.0
	W2					117.1	171.9	24.1	28.4	41.5	3.0	12.2	0.7	181.8	1434.0	91.2
	W3					118.2	204.0	21.2	27.1	41.1	3.0	11.0	0.6	181.8	1468.0	114.6
7	W1					104.6	177.9	21.5	31.9	39.9	3.3	6.4	0.2	179.0	2058.2	691.9
	W2					113.8	172.5	21.7	31.7	40.3	3.5	6.7	0.2	179.3	2130.2	639.8
	W3					119.8	175.0	22.6	27.7	41.0	3.0	10.5	0.7	181.4	1476.7	99.3
8	W4					299.4	307.5	20.6	31.9	41.3	3.3	7.5	0.6	179.1	1554.4	266.9
	W5					110.0	293.2	24.8	28.8	39.0	3.4	10.7	0.5	181.5	1846.5	224.2
	W6					141.7	238.9	20.6	27.4	41.7	3.5	10.7	0.6	181.9	1642.2	135.0
9	W1	16.0	209.7	158.2	207.1	381.0	285.0	22.8	26.1	39.9	3.0	4.0	0.5	194.6	431.7	153.0
	W2	37.1	208.4	153.9	209.2	264.6	330.2	25.9	26.4	40.6	3.2	8.8	0.5	195.1	1236.9	151.8
	W3	55.2	208.6	193.4	193.2	169.7	185.5	24.2	26.1	40.8	3.1	12.6	0.7	200.2	1138.9	48.2
10	W1	59.9	183.1	188.8	155.6	105.1	179.4	23.3	31.4	41.1	3.5	8.4	0.3	176.6	2362.7	606.8
	W2	59.1	164.7	197.8	161.8	104.1	179.4	24.8	31.5	38.9	3.4	8.1	0.4	182.2	1990.5	515.2
	W3	59.4	208.3	206.7	188.9	125.2	181.0	23.0	26.6	41.9	3.6	14.2	0.7	204.2	1434.4	79.8
11	W4	46.0	178.6	199.0	151.9	105.2	186.7	23.3	31.5	39.5	3.2	6.2	0.3	168.1	1809.9	575.5
	W5	52.2	157.0	194.0	150.4	144.3	194.9	20.8	31.4	34.5	3.5	6.6	0.3	164.4	2642.7	511.0
	W6	58.3	179.6	169.5	155.5	107.9	187.1	22.5	27.7	41.1	3.3	11.8	0.7	179.1	1793.6	131.7

The bold values indicate that the corresponding objectives were used during optimization.

obtained by NSGA–II are less dispersed around the preferred region than those attained by RPSGA. This suggests that NSGA–II applies higher selection pressure to the regions of interest, while RPSGA yields higher diversity of solutions and thus capture more complex process responses.

The Pareto fronts for case studies 6 and 7 are shown in Fig. 4 while Table 3 contains the related best solutions, as well as those for case study 8 (considering simultaneously Q, Power, and WATS). The purpose is to design a screw operating at constant conditions ($N = 60$ rpm and $T_{\mathrm{bi}} = 190$°C) in view of different combinations of objectives. Figure 4(a) demonstrates the difficulty in controlling Q and Power exclusively by means of the screw geometry, as the best solutions are located within a small region of the objective space regardless of the weights. As discussed above, Q and Power are conflicting and considerably dependent on screw speed and barrel temperature. The solutions found by NSGA–II for case study 6 (incorporating Q and WATS) are biased toward high Q while WATS remains small. Conversely, the solutions proposed by RPSGA are distributed along most of the Pareto optimal region.

Figure 5 deals with the joint optimization of operating conditions and screw geometry (case studies 9 and 10 in Table 2). As before, Table 3 presents the associated best solutions both for these two case studies and case study 11 (all objectives together). The two optimization algorithms locate distinct Pareto optimal regions for Q and Power that depend on the relative importance of each objective. Higher Q and lower Power are attained in comparison to the previous case studies. Specifically, for RPSGA, in comparison to the case studies involving screw geometry and operation conditions alone, the maximum value of Q increased by approximately 80% and 35%, while the minimum value of Power decreased by nearly 20% and 87%, respectively. This obviously results from the possibility of manipulating more process parameters, offers more control over the process. Similar conclusions can be taken with regards to Q and WATS. Again, RPSGA performs better than NSGA-II.

Table 3 summarizes the practical best operating conditions, screw geometry or the two together for the 11 case studies of Table 2. They correspond to the individuals of the final population that have the highest fitness values (*Eq. 2*). For cases

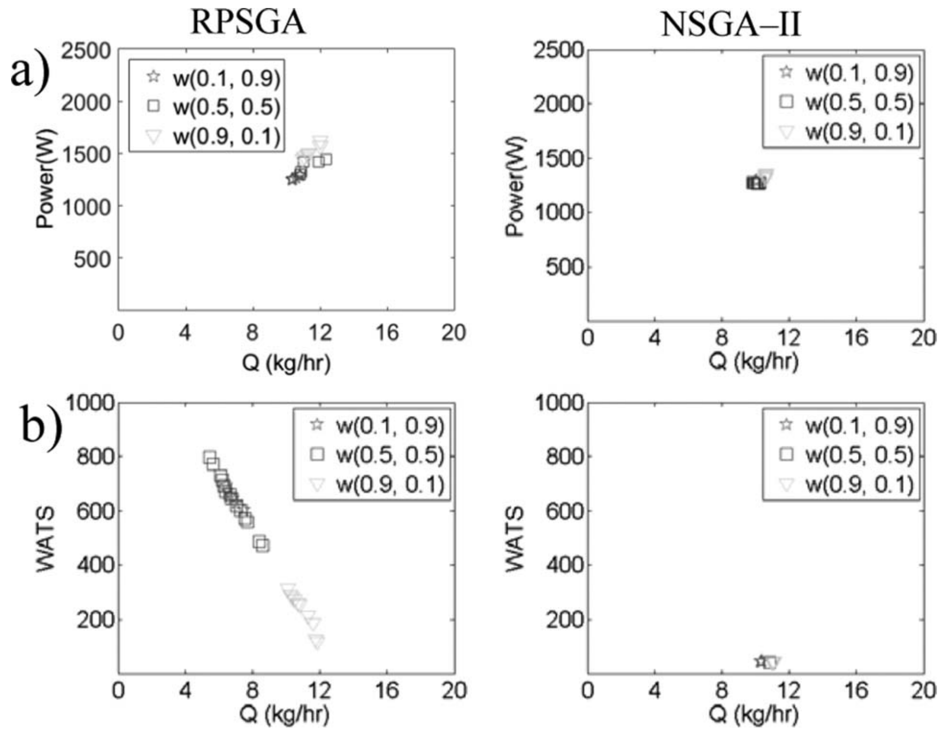

FIG. 4. Pareto frontiers for decision making considering the optimization of screw geometry and using RPSGA and NSGA-II: (a) case study 6; (b) case study 7.

studies 1–5, where only the operating conditions need to be defined, when Q is important the best results are attained for high screw speeds; L_{melt} is smaller when T_{b1} is higher; T_{melt} is lower when the last heating zone downstream is set to the lowest value; Power is lower for reduced screw speeds and higher barrel temperatures. If all objectives are taken simultaneously, a compromise solution is suggested. When designing a screw that will work under constant operating conditions (case

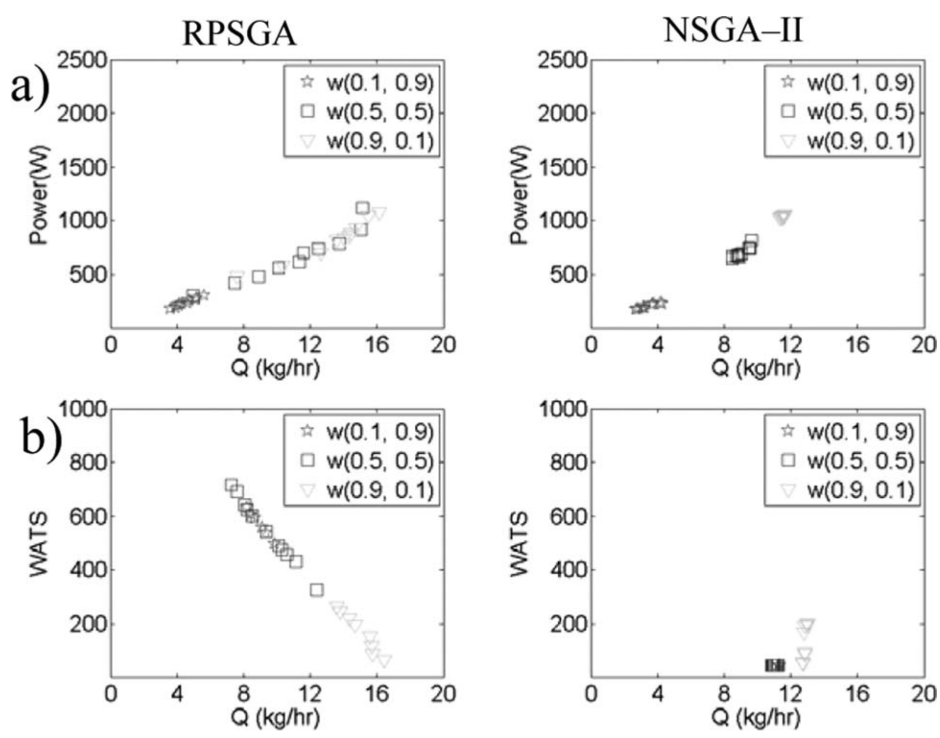

FIG. 5. Pareto frontiers for decision making considering the optimization of both operating conditions and screw geometry using RPSGA and NSGA-II: (a) case study 9; (b) case study 10.

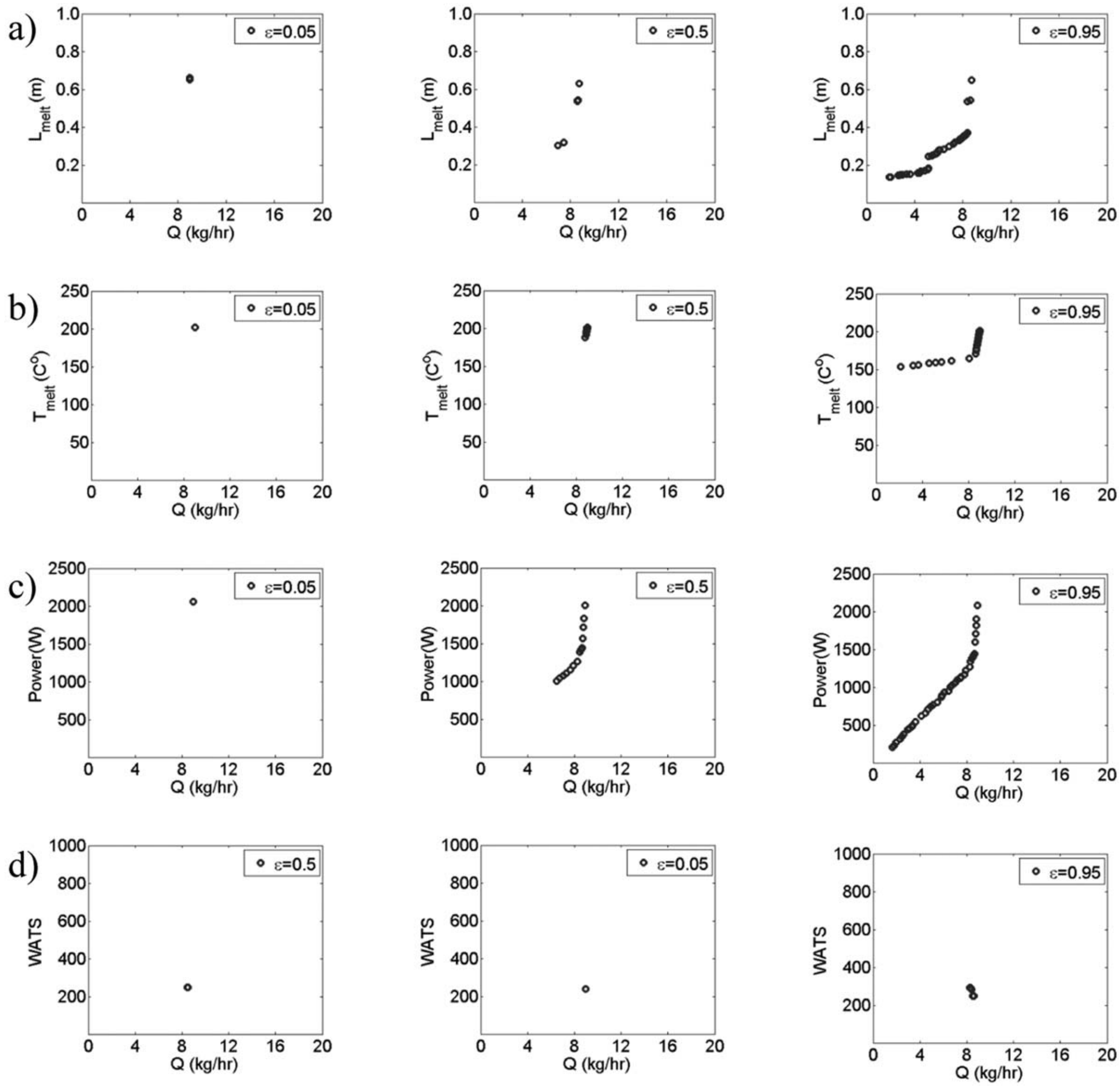

FIG. 6. Pareto frontiers for robustness considering the optimization of operating conditions using RPSGA for different values of the dispersion parameter: (a) case study 1; (b) case study 2; (c) case study 3; (d) case study 4.

studies 6–8), the most important parameter governing output is the channel depth in the metering zone. When multiple objectives are included (case study 8) a balance is again found. When more process variables can be set (i.e., operating conditions and screw design), better performances can be reached. For example, the highest output is obtained for high screw speeds and for deeper channels in the metering zone. Therefore, the proposed methodology offers to the process engineer a practical and efficient decision tool.

Decision Making Based on Robustness of Solutions

The decision making based on robustness is especially relevant when defining operating conditions, since in practical

TABLE 4. Most robust solutions in the decision variables domain and objectives considering the optimization of operating conditions for RPSGA and $\varepsilon = 0.05$.

	Decision variables							Objectives	
Case	N	T_{b1}	T_{b2}	T_{b3}	Q	L_{melt}	T_{melt}	Power	WATS
	rpm	°C	°C	°C	kg/hr	mm	°C	W	
1	60.0	152.3	198.3	209.8	9.0	0.6	211.8	1704.3	236.1
2	60.0	152.4	208.2	209.9	9.0	0.5	214.2	1836.8	243.6
3	60.0	152.4	158.9	210.0	9.0	0.6	203.2	2005.1	237.1
4	60.0	150.0	168.5	210.0	9.0	0.6	205.2	1934.6	237.1

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extrusion fluctuations of screw speed and barrel temperatures are unavoidable. These disturb thermal exchanges and flow, possibly causing variations in the process responses.

The solutions of case studies 1–4 created by RPSGA are shown in Fig. 6 and Table 4. In Fig. 6, each column of graphs corresponds to a given value of ε, the dispersion parameter that sets the influence of robustness on the global fitness. As the value of ε increases, the solutions become better distributed along the Pareto optimal region. The plots reveal a few useful patterns to the decision maker. For example, the mass output of a robust solution is limited to approximately 9 kg/h, but most robust solutions (i.e., with a small ε) are located in regions of high Q. However, this relates to poor values for the other objectives. If the DM wishes to avoid such an outcome, a higher value of ε can be selected, potentially leading to a higher grained resolution of a wider region of the Pareto front. Table 4 presents the values of the variables and of the objectives corresponding to the solutions with the best fitness values for case studies 1–4. Simultaneously, an analysis of the most robust solutions presented in Table 4 shows that they are obtained for the higher screw speed, for the lowest barrel temperature in the feed zone (T_{b1}) and for the highest barrel temperature in the metering zone (T_{b3}), while the results for T_{b2} are more irregular.

It should be noted that the features of the Pareto front remain unchanged in the present and previous section. What changes is the focus of the search, which is determined either by the importance of the objectives or by the robustness of solutions. Also, it is important to note that the best solution with respect to robustness is the same for different values of ε. By changing ε, the user can widen or reduce the range of robust region, whereas the location of the region and its best solution remain unchanged.

CONCLUSIONS

A design optimization approach was proposed for the multi-objective optimization of plasticating single screw extrusion. The methodology couples MOEAs with decision-making preferences, in order to support the identification of the solutions with the most desirable characteristics. The two MOEAs used, RPSGA and NSGA-II, showed distinct performances, RPSGA being globally was able to suggest better results. Formulating the DM preferences was addressed in two ways. First, preference information was quantified by attributing weights expressing the relative importance of individual objectives. In addition, the robustness of solutions against small perturbations in the decision variables was taken into consideration. In this case, the DM expressed his or her preferences by means of a dispersion parameter controlling the extension of the solution according to their robustness. The smaller the dispersion parameter, the more robust solutions are obtained.

The method was used to tackle various case studies involving the definition of the extruder operating conditions, screw design and both together. It was demonstrated that reaching the objectives is greatly affected by the choice of the design variables, thus highlighting the importance of using effective tools to support technical decisions concerning extrusion. Both the relative importance of the objectives and the level of robustness of the solutions can be used as part of decision making. Depending on the preferences, different solutions are suggested. Simultaneously, the trade-offs between objectives provide quantitative knowledge on major process responses and can also contribute to achieving a higher extrusion performance. The use of decision making strategies such as the proposed here offers the plastic engineer an effective tool to solve practical extrusion problems.

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