Product Differentiation in a Regulated Market: A Welfare Analysis

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Abstract

This article analyses both a circular and a linear market where consumers are distributed along the whole space, whilst firms are located in a region restricted by the regulator. We consider a three stage game in which in the first stage the regulator chooses the size of the space where firms will be located (the commercial area), in the second stage firms choose locations and in the third stage they compete in prices.

We find that with this type of market configuration independently of space considered under quadratic transportation costs, there exists price equilibrium for every possible firms' location. We also derive that the optimal size of the commercial area will depend on the welfare function of the regulator and, in particular, that once a regulator bias is considered, maximum differentiation, minimum differentiation or intermediate cases may be obtained.

Keywords: spatial competition, circular model, linear model, convex transportation costs, regulation, sequential equilibrium, social welfare.

JEL Classification: C72, D43, D63, L13, L51, R38

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1. Introduction

Models used on spatial competition consider consumers and firms distributed along a line[1] or a circle[2]. Price competition and product choice in a duopoly had been studied in the linear space, whereas the main interest of a circular space has been claimed to determine the number of firms in an industry[3]. In this context, various authors have used the circular city model and assumed that firms locate equidistantly around the circle[4]. Gupta et al (2004) showed dispersed location equilibrium where firms may choose distinct but not equidistant locations.

Nevertheless a regulated model as proposed allows for an analysis of product choice in both a circular and a linear space under duopoly. Such restrictions on firm and consumer location have seldom been considered in the literature, with the exception of the linear vertical differentiation model in Gabszewicz and Thisse (1986) and the circular model in Hamoudi and Risueño (2007). Present article instead develops two regulated market models, circular and linear where consumers are spread all over the whole market while firms are located only within a band established by the regulator. This model differs from the linear city model proposed by Gabszewicz and Thisse in which the market is divided into two regions in which consumers distribute uniformly in the linear-city [0,1] and two firms locate in interval $[1,+\infty)$. This is interpreted as an example of vertical differentiation, whereas both models proposed in this paper are an example of horizontal differentiation.

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[1] Linear model pioneered by Hotelling (1929)

[2] The first adaptation is due to Lerner and Singer (1937). After this early version, Vickrey (1964) applied a similar model which became widely disseminated with Salop's work (1979).

[3] The circular model is symmetric and no location is better than another a priori. (Tirole (1988), ch.7)

[4] For example, Noveshek (1980), Anderson (1986), Economides (1989 y 1993), Kats (1995), Junichiro and Noriaki (2004), Matsumura and Okamura (2006).

The configuration studied herein implies that firms are not able to offer all possible characteristics in goods and, specifically, the ideal consumer variety. Very often there is a limit in the product range. Not every product is offered and the spectrum of goods becomes incomplete. Pharmaceutical companies lacking technical knowledge or profit incentives to produce an ideal medicine for specific consumer needs; polluting firms in the geographic context of the environmental problem constrained to move away from victims (consumers), are two examples.

The main purpose of this paper will be to show how, consistently with his bias, a regulator by restricting the space allowed to firms can influence total welfare resulting from the industry and the consumers. Thus, the proposed model is built as a three-stage game. In the first stage, the regulator chooses the size of the commercial area, v; in the second stage, firms choose locations within v, and in the third stage they compete in prices.

In the location-then-price duopoly subgame, the cost of transport is assumed to be paid by consumers proportionally to the square of their distance as in horizontal product differentiation models *a la Hotelling*. The location problem of firms has been examined rather extensively with the convex transport cost function[5] and, particularly, with the quadratic function (see, D'Aspremont et al (1979), Gabszewicz and Thisse (1986), Economides (1986), (1989), Anderson (1986), (1988), (1997), Tabuchi and Thisse (1995), Junichiro and Noriaki (2004) and Brenner (2005), among others). Clearly, this assumption is made for mathematical convenience using the quadratic transportation costs, to ensure a perfect Nash equilibrium of a two-stage location-price non-cooperative game. This result holds in the linear and circular city model. In both

[5] Exceptionally, a few authors also consider a concave transport cost function. See, De Frutos et al (1999, 2002), Hamoudi and Moral (2005), Matsumura and Okamura (2006), Hamoudi and Risueño (2007).

cases, the location equilibrium involves maximum differentiation, in contrast with the principle of minimum differentiation claimed by Hotelling (1929). However, several studies have proven that such equilibria are characterized by intermediate levels of differentiation[6].

Transport cost function considered in this paper is convex quadratic[7]. In a circular market though, there exists a linear quadratic concave function for any linear quadratic convex function such that the location-then-price games induced by both functions are strategically equivalent (see De Frutos et al., 1999). Furthermore, the same authors prove that only two cost functions from the linear quadratic family: $C^{+}(x)=b^{+}x^{2}$ and $C^{-}(x)=b^{-}(x-x^{2})$ ensure existence of a perfect equilibrium in pure strategies in a two-stage game. Subsequently, in another article the equivalence result is generalised by the same authors to show the existence of a concave transport cost for any arbitrary convex transport cost such that the two games induced are equivalent (see Frutos et al, 2002). In the present article, it is not proved that the equivalent result could be extended to the model proposed.

When maximizing the total welfare function, instead of analyzing alternative scenarios of the regulator bias (see Hamoudi and Risueño 2007) a continuum of political approach is considered. It is only at the conclusion of the effects in the welfare, by the weight attributed to the individuals that gives the political approach attached to the regulator.

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[6] For instance, Economides (1986) considers transport costs functions of the form d^{β} with 1≤β≤2. Lambertini (1994) considers a standard model in which firms are free to locate outside the linear city. Böckem (1994), generalizes the model on the demand side. Neven (1986), Tabuchi and Thisse (1995) consider general distributions of the consumers. Brenner (2005) extends the interval Hotelling model to the n-players case.

[7] The linear quadratic convex transport cost function was introduced by Gabszewicz and Thisse (1986), whereas Hamoudi (1990) introduced the linear quadratic concave case.

When maximizing the total welfare function, a weighed formula is used to combine the effects of firms' profit and consumers' disutility. Such weighed approach was defined by Baron and Myerson (1982) and used in welfare function formulation with several regulated models by Armstrong et al. (1994). There the weights 1 and π, $0 \le \pi \le 1$, applied by those authors, leads to asymmetric relative proportions varying $\left(\infty, \frac{1}{2}\right)$ for the first component and $\left(0, \frac{1}{2}\right)$ for the second. The approach included in this paper consists instead in using complementary weighing factors $\lambda, \quad 1-\lambda, \quad 0 \le \lambda \le 1$, thus providing a continuum of proportions $[0,1]$ for both components and allowing for the intuitive interpretation of relative percentage weight (0%, 100%).

The literature on spatial competition under regulation has mainly focused on the question of whether and how competition between firms will be modified with respect to previous results of equilibrium in a characteristic space or a geographic space. For example, Anderson and Merger (1994) demonstrate that for spatial duopoly with price–taking firms and a perfectly inelastic demand, the outcome corresponds to minimum differentiation. In a related article, Hinloopen (2002) studies the location choices of firms in a price-regulated spatial duopoly when demand is not completely inelastic. This analysis leads to three different equilibria, depending on the structure of the market: agglomerate at the market center, form two local monopolies or differentiate intermediately. Another example is found in Brekke et al (2006) where a three-stage game is considered. First, the regulator sets a price, secondly firms simultaneously choose locations and finally firms simultaneously choose the quality levels[8]. From

[8] The degree of horizontal differentiation is determined by the intensity of quality competition.

another perspective, Dijkstra and De Vries (2004), study the equilibrium that will occur when both firms (polluters) and households (victims of pollution) can choose their location in a two-region model. The authors, using evolutionary game theory, analyse three policy regimes: no environmental policy, taxation on pollution, and compensation of damage for the victims. Price regulation and ecological policy, however, are not an issue in the present paper.

The remainder of this paper is organized as follows. In Section 2, the circular model is first introduced, then the game is developed, to finalize with the optimization of the size of the commercial area and the comments thereof. In section 3, the linear model follows the same pattern. Section 4 is devoted to conclusions on both models.

2. The circular Market

2.1. The model

We consider a circular market of length 1 where the regulator chooses a commercial area given by the arc of circumference $[v_1, v_2]$. There are two firms selling a homogeneous product, with zero production costs, located at x_1 and x_2, such that $0 \le v_1 \le x_1 \le x_2 \le v_2 \le 1$. Prices p_1 and p_2 are chosen by firm 1 and firm 2, respectively, given their locations.

Consumers are uniformly distributed along the market, and each one of them purchases just one unit of the industry's product at the firm at which the delivered price (total price resulting from the addition of mill price and transport cost) is the lowest. They can travel along the whole circle and will always take the direction that implies the shorter distance to the chosen firm. The distance between consumer x and firm location x_i is given by $d_i = |x - x_i|, i = 1,2$. We will consider that transportation costs are quadratic. In particular, we will assume that $C(d_i) = b\, d_i^2$, b>0 (1).

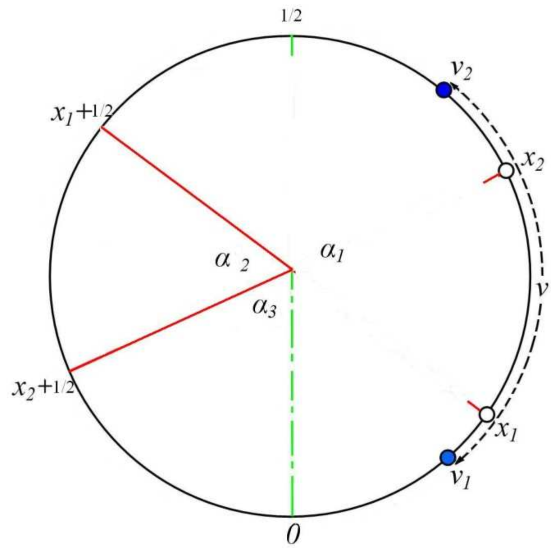

Fig. 1. The circular model

The model described above gives rise to a three-stage game in which in the first stage the regulator chooses the size of the commercial area, *v*, where $v=v_2-v_1$ and $x_2 - x_1 \leq v \leq 1$ (2).In the second stage firms decide simultaneously their location and in the third stage they simultaneously choose prices. The choice of the transportation costs function described in (1) is a technical assumption that avoids non-existence of equilibrium problems in the second and third stages of the game.

In order to determine the market boundaries and derive the demands faced by each firm, we will have to find the marginal consumers. A consumer is indifferent to buying from one firm or the other if and only if: $p_1 + C(d_1) = p_2 + C(d_2)$ (3).

Since consumers will travel to firms taking the direction that minimizes the distance to the chosen firm, there are three possible indifferent consumers each one belonging to a different segment of the circumference given by $\alpha_1 \in [0, x_1+½]$, $\alpha_2 \in [x_1+½, x_2+½]$, and $\alpha_3 \in [x_2+½, 1]$, respectively (see *Figure 1*). In order to calculate the location of the indifferent consumer, let

$q = x_1 + x_2$ and $z = x_2 - x_1$. Substituting (1) into (3) and computing the expression for the marginal consumers, it turns out that:

$$\alpha_1 = \frac{p_1 - p_2}{2bz} + \frac{q}{2}, \ \alpha_2 = \frac{p_1 - p_2}{2b(1-z)} + \frac{q}{2} + \frac{1}{2} \quad \alpha_3 = \frac{p_1 - p_2}{2bz} + \frac{q}{2} + 1 \qquad (4)$$

In order to analyze the model we start seeking equilibrium in the prices stage. Then we look for equilibrium locations given equilibrium prices found in the previous stage. Finally we will analyze the optimal market size.

2.2. The Price sub game in the circle model

The first step is to derive the demands for the two firms The demand function of firm 1 can be expressed as follows:[9]

$$D_1(p_1, p_2, x_1, x_2, v) = \begin{cases} 1 & p_1 - p_2 \leq -bz(1-z) \\ \alpha_3 - \alpha_2 & -bz(1-z) \leq p_1 - p_2 \leq bz(1-z) \\ 0 & bz(1-z) \leq p_1 - p_2 \end{cases}$$

Note that in this representation total market demands as well as firms' individual demands are independent of v. However it must be remembered that $v=v_1-v_2$ and $v<z$.

Since production costs are zero, the profit function for firm i, $i=1, 2$, is given by:

$B_i(p_1, p_2, x_1, x_2, v) = D_i(p_1, p_2, x_1, x_2, v)\, p_i$.

Substituting in the demand functions the expressions (4) for the indifferent consumers, we obtain the following profit functions:

[9] The corresponding demand of firm 2 is simply $D_2 = 1 - D_1$

$$B_1(p_1, p_2, x_1, x_2, v) = \begin{cases} p_1 & p_1 - p_2 \le -bz(1-z) \\ p_1\left[\dfrac{p_2 - p_1}{2bz(1-z)} + \dfrac{1}{2}\right] & -bz(1-z) \le p_1 - p_2 \le bz(1-z) \\ 0 & bz(1-z) \le p_1 - p_2 \end{cases}$$

$$B_2(p_1, p_2, x_1, x_2, v) = \begin{cases} 0 & p_1 - p_2 \le -bz(1-z) \\ p_2\left[\dfrac{p_1 - p_2}{2bz(1-z)} + \dfrac{1}{2}\right] & -bz(1-z) \le p_1 - p_2 \le bz(1-z) \\ p_2 & bz(1-q) \le p_1 - p_2 \end{cases}$$

Given that the profit functions are quasi-concave in prices, the existence of price equilibrium is assured [10]. Consequently, from the first order conditions we obtain the equilibrium prices:

$$p_1^* = p_2^* = b\, z\, (1-z) \qquad (5)$$

Substituting the equilibrium prices (4), into the profit expression, we obtain:

$$B_1^* = B_2^* = (1/2)\, b\, z\, (1-z) \qquad (6)$$

We now characterize the equilibrium in the following proposition.

Proposition 1.

Independent of the value of v where $v=v_2-v_1$, and for any given location pair (x_1, x_2) such that $0 \le v_1 \le x_1 \le x_2 \le v_2 \le 1$, *there exist a unique price equilibrium given by the expression* (5).

Proof: Since both profits are independent of *v*, it is clear that prices equilibrium does not depend on *v*.

This concludes the analysis of the price subgame

2.3. The location subgame in the circle model

We turn now to the second stage of the game. We will compute equilibrium locations, given, the size of the commercial area, *v*, and taking into account the equilibrium prices p_1*, p_2*, obtained above. Price-location equilibrium is defined as a location pair $\left[x_1^*(v), x_2^*(v)\right]$, and a price pair $\left[p_1^*(x_1^*, x_2^*, v), \quad p_2^*(x_1^*, x_2^*, v)\right]$such that:

$$B_i\left[v, x_i^*, x_j^*, p_i^*(v, x_i^*, x_j^*), p_j^*(v, x_i^*, x_j^*)\right] \geq B_i\left[v, x_i\ , x_j^*, p_i^*(v, x_i^*, x_j^*), p_j^*(v, x_i^*, x_j^*)\right]$$

For $\forall i, j = 1{,}2;\ i \neq j$ and $\forall x_i \in \left[v_1, v_2\right]$.

Given the commercial area, *v*, and using the expression for profits derived in the previous section, Eq. (6), after taking first-order conditions we obtain the following result

P**roposition 2:**

For $0 \leq v \leq 1$, there exists a unique location equilibrium given by:

$$\begin{cases} x_1^* = v_1, \quad x_2^* = v_2 & if \quad v \leq \dfrac{1}{2} \quad (7a) \\ x_1^* = v_1, \quad x_2^* = v_1 + \dfrac{1}{2} & if \quad v \geq \dfrac{1}{2} \quad (7b) \end{cases}$$

The result is the well-known principle of maximum differentiation obtained in the literature when quadratic transportation costs functions are assumed. Firms choose the two extremes of the commercial area to keep price competition as low as possible. This result is relevant since, as mentioned above, when regulated markets are considered, other costs functions that give good results in terms of equilibrium existence in the free model have been proven to show equilibrium-existence-problems when regulation is introduced.

Given that the optimal locations are x_1^*, x_2^* and substituting in the previous price equilibrium functions Eq. (5) we obtain :

$$\begin{cases} p_1^*(v) = p_2^*(v) = bv(1-v) \quad if \quad v \leq \dfrac{1}{2} \quad (8a) \\ p_1^*(v) = p_2^*(v) = \dfrac{b}{4} \qquad\qquad if \quad v \geq \dfrac{1}{2} \quad (8b) \end{cases}$$

thus the corresponding demand functions and prices are:

$$D_1^{**}(x_1^*, x_2^*) = D_2^{**}(x_1^*, x_2^*) = \frac{1}{2} \quad (9)$$

and next, using Eq (6) we compute profits as:

$$\begin{cases} B_1^* = B_2^* = \dfrac{1}{2} bv(1-v) \quad if \quad v \leq \dfrac{1}{2} \quad (10a) \\ B_1^* = B_2^* = \dfrac{b}{8} \qquad\qquad if \quad v \geq \dfrac{1}{2} \quad (10b) \end{cases}$$

From the best -response price functions given by the expression (9), it is obvious that when the value of *v* increases, prices increase; $\dfrac{dp_1}{dv} = \dfrac{dp_2}{dv} = b(1-2v) \geq 0 \quad if \;\; v \leq \dfrac{1}{2}$; This means that a low value for the commercial region *v* implies increased price competition. However, $if \;\; v \in \left(\dfrac{1}{2}, 1\right]$ the prices are independent of the value of *v*. In this case, the role of parameter b of the transportation costs must be considered as a measure of the value attached by each consumer to his/her favourite version of the product. When b is large, price competition among firms relaxes. On the other hand, when b is relatively small, price competition to capture market share is very intense so that firms will have the incentive to locate far from each other, *ceteris paribus*. Notice that as the transportation cost increases price competition for market share becomes less intensified and firms would locate to each other.

Turning to the equilibrium locations, from expressions (7a) and (7b), it is apparent that locations x_1, x_2 are functions of the commercial area, *v*, and are intricately dependent on the regulator decisions.

We can then proceed to the analysis of the optimal size of the commercial area.

2.4. Optimal size of the commercial area in the circle model

To end the analysis of our model, we will focus our interest in search of the optimal size of the commercial area, v^*, given the results obtained above. More precisely we will denote by v^* the optimal strategy of the regulator such that $v^* = \underset{v}{ArgMax}\, W(v) \quad s.t. \quad 0 \le v \le 1$, where $W(v)$ is the regulator's objective function:

$$W(v) = \lambda B^* + (1-\lambda)\,(p^* D^* + C_T)\,, \qquad (11)$$

λ being the weight given by the regulator to firms' profit, in accordance with his political bias, whereas $(1-\lambda)$ is the weight given to consumers disutility, $0 \le \lambda \le 1$.

Where $p^* = p_1^* = p_2^*$ is the price given by the expressions (8a) or (8b), and $B^* = B_1^* + B_2^*$, is the total profit such that B_1^*, B_2^* are given by the expressions (10a) or (10b), which depends on v and $D^* = D_1^* + D_2^* = 1$ is the market demand.

Substituting B^* and D^*, into the expression (11) for the regulator's welfare objective function, we obtain:

$$W(v) = (2\lambda - 1)\, p^* - (1-\lambda)\, C_T\ , \qquad (12)$$

$C_T\ (v)$ is the overall transportation costs incurred by consumers, given the optimal locations obtained above, Eqs (7) and (8) and defined as: $C_T(v) = I_1 + I_2$

$$I_1 = \int_0^{\alpha_1^*} \left[b(x - x_1^*)^2\right]dx + \int_{\alpha_2^*}^{1} b\left[(1-x) + x_1^*\right]^2 dx\,, \qquad I_2 = \int_{\alpha_1^*}^{\alpha_2^*} \left[b(x_2^* - x)^2\right]dx$$

where the first integral I_1 corresponds to the overall transportation costs paid by those consumers that address firm 1 and the second integral I_2 to the overall transportation costs paid by those consumers that address firm 2; while α_1^* , α_2^* are indifferent consumers under location equilibrium.

Substituting Eqs (7) and (9) or (8) and (10) into the expression (4), we compute marginal consumers as:

$$\begin{cases} \alpha_1^* = \dfrac{v_1 + v_2}{2}, \alpha_2^* = \dfrac{1 + v_1 + v_2}{2} & \text{if} \quad v = v_2 - v_1 \leq \dfrac{1}{2} \quad (13a) \\ \alpha_1^* = \dfrac{4v_1 + 1}{4}, \alpha_2^* = \dfrac{4v_1 + 3}{4} & \text{if} \quad v = v_2 - v_1 \geq \dfrac{1}{2} \quad (13b) \end{cases}$$

We now analyze the optimal size of the commercial area depending on the welfare objective function of the regulator:

Case 1. $0 \leq v \leq \dfrac{1}{2}$

Using the expression (13a) we compute the total transportations cost C_T *(v) as:*

$$C_T(v) = \frac{b}{12}(1 - 3v + 3v^2)$$

thus, using the expression (12), the regulator's objective function is given by :

$$W(v) = (2\lambda - 1)\, bv\ (1 - v) - \frac{b}{12}(1 - \lambda)\,(1 - 3v + 3v^2)$$

P**roposition 3:**

For $0 \leq v \leq 1/2$, the optimal size of the commercial area is *given by:*

$$\begin{cases} v_{C1}^* = \dfrac{1}{2}, & \text{if} \quad \lambda \geq \dfrac{3}{7} \quad (14a) \\ v_{C2}^* \in \left[0, \dfrac{1}{2}\right], & \text{if} \quad \lambda = \dfrac{3}{7} \quad (14b) \\ v_{C3}^* = 0, & \text{if} \quad \lambda \leq \dfrac{3}{7} \quad (14c) \end{cases}$$

Proof:

Using the first order conditions, $\frac{\partial W}{\partial v} = \frac{b}{4}(1-2v)(7\lambda-3) = 0$, we find that $v = \frac{1}{2}$.

Taking into account the second order condition: $\frac{\partial^2 W}{\partial v^2} = -\frac{b}{2}(7\lambda-3)$, we find the following solution:

$$\frac{\partial^2 W}{\partial v^2} \le 0 \quad if \;\; \lambda \ge \frac{3}{7} \quad \Rightarrow \text{ the maximum is reached at } v = \frac{1}{2}$$

$$\frac{\partial^2 W}{\partial v^2} \le 0 \quad if \;\; \lambda = \frac{3}{7} \quad \Rightarrow \text{ the maximum is reached at } 0 \le v \le \frac{1}{2}$$

$$\frac{\partial^2 W}{\partial v^2} \le 0 \quad if \;\; \lambda \le \frac{3}{7} \quad \Rightarrow \text{ the maximum is reached at } v = 0$$

■

With these results (14a), (14b) and (14c), the value of the welfare function for each of them is:

$$W(\frac{1}{2},\lambda) = \frac{b}{48}(25\lambda-13), \qquad if \;\; \lambda \ge \frac{3}{7}$$

$$W(v,\frac{3}{7}) = -\frac{b}{21} \qquad , \qquad if \;\; \lambda = \frac{3}{7}$$

$$W(0,\lambda) = -\frac{b}{12}(1-\lambda) \qquad , \qquad if \;\; \lambda \le \frac{3}{7}$$

Of these three values it is noteworthy that the maximum value of social welfare is reached for $\lambda \ge \frac{3}{7}$, when the bias of the regulator is inclined towards the profit of firms.

Case . $\frac{1}{2} \le v \le 1$

Using the expression (13b) we compute the total transportations cost $C_T(v)$ *as:*

$$C_T(v) = \frac{b}{12}(7-24v+24v^2)$$

then, using the expression (12), the regulator's objective function is given by :

$$W(v,\lambda) = \frac{b}{2}(2\lambda - 1) - \frac{b}{48}(1-\lambda)\,(7 - 24v + 24v^2)$$

Proposition 4:

For 1/2≤ v ≤ 1, the optimal size of the commercial area is given by: $v = \frac{1}{2}$.

Proof:

After taking first order-conditions, $\frac{\partial W}{\partial v} = \frac{b}{4}(1-2v)(7\lambda - 3) = 0$, we obtain the best-response functions: $v = \frac{1}{2}$, taking into account the second order condition: $\frac{\partial^2 W}{\partial v^2} \leq 0$.

2.5. Welfare analysis on the circle model

Figure 2 provides an indication of the values that the regulator must choose to obtain your aim as the relative weight that you want to give the profit of firms according to their political vision of social welfare.

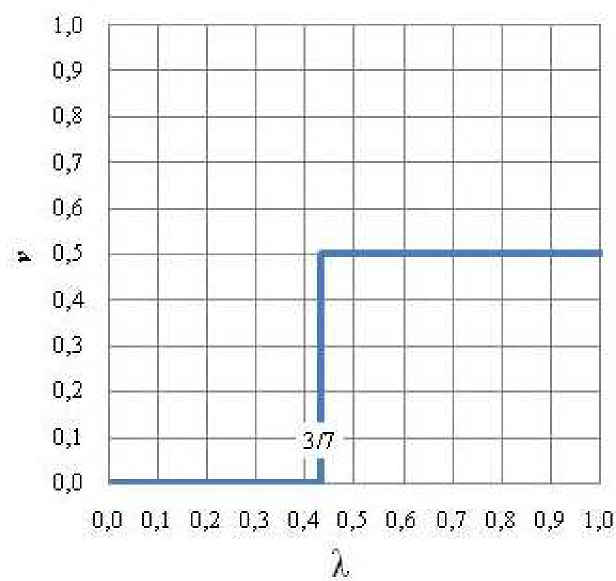

Fig. 2. Commercial region v upon λ chosen by the regulator. Circular Model

It is noted that giving the benefit of the companies weighing less than 3 / 7 (approx. 42%) is particularly surprising because it would force them to a fixed location with no differentiation would mean an equilibrium "*á la Bertrand*". On the other hand give the benefit of the companies weighing more than (3 / 7) would result in having to fix the area commercial $v=1/2$. However, setting the area commercial v such that $1/2 \leq v \leq 1$, which would mean giving a weight to firms (or consumers, respectively) from 3 / 7 (4 / 7) of the global welfare.

3. The linear model

3.1. The model

It is considered the same features as the previous model except that the space is linear instead of circular (See Fig 3).

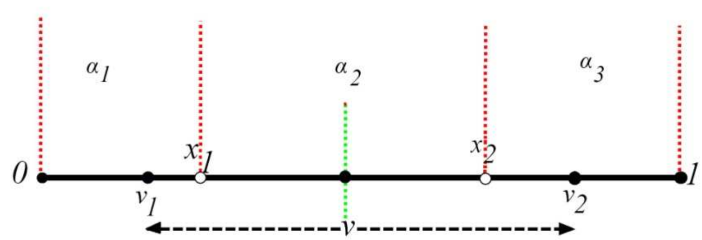

Fig. 3. The linear model

The regulator chooses, a commercial area, given by the arc interval $[v_1, v_2]$, of length v such that, $v=v_1-v_2$ and $0\leq v\leq 1$. This area is occupied by two firms which sell a homogeneous commodity with zero production costs. We denote by x_i the location of firm i in this area, such that $0 \leq v_1 \leq x_1 \leq x_2 \leq v_2 \leq 1$. There is a continuum of consumer uniformly distributed on a unit-length interval, with a mass normalized to unity without loss of generality. Each consumer buys only one unit of the goods at the firm with the lowest total cost, that is, the mill price plus the transportation cost.[10] The distance between the consumer and firm i is defined by $di = |x - x_i|$, $i = 1, 2$. We assume that transportation costs are quadratic, as follows: $C(d_i) = b\, d_i^2$, b>0.

In this context there is only one indifferent consumer whose location is given by the following expression:

$$\alpha = \alpha_1 = \alpha_2 = \alpha_3 = \frac{p_1 - p_2}{2bz} + \frac{q}{2} \qquad (15)$$

Remember that z denote the distance between the two firms, that is, $z = x_2 - x_1$. Also let q be the sum of the two firms locations, $q = x_1 + x_2$. Given firms' locations, $q/2$ represents the equidistant point between the two firms and constitutes a useful symmetry measure, as we will see later on.

We consider a sequential game where firstly a regulator choose the commercial area then firms select their locations in the industrial region and next, they decide prices which maximize profits given the selected locations. Thus, we solve backwards, first finding the Nash equilibrium prices for given locations.

In the next section, we analyze the existence of the sequential equilibrium in the linear model of spatial competition.

3.2. Location-price subgame in the linear model

Consider the linear model as described in the previous section (Figure 1). We first derive firms' demands and then we analyze the existence of the price equilibrium.

By the construction of the model, all consumers located to the left of α (See expression (15)) select firm 1, while the remaining consumers choose firm 2. Therefore,

[10] For simplicity, we assume that all consumers have enough willingness to pay. This assumption is common in all the literature on spatial differentiation.

depending on the location of the indifferent consumer on the line, the demand of firm 1 is the following:[11]

$$D_1(p_1, p_2, x_1, x_2, v) = \begin{cases} 1 & , \quad p_1 - p_2 \le b(2 - zq) \\ \alpha & , \quad b(2 - zq) \le p_1 - p_2 \le bzq \\ 0 & , \quad bzq \le p_1 - p_2 \end{cases}$$

It is also noted here that the demand function does not depend on the limits set by the regulator. However there is a link through which the restriction of the regulator, given by the relation $z \le v$.

We now analyze the existence of a Nash-price equilibrium for given locations.

It is well known that a Nash price equilibrium exists under quadratic transportation costs when there are no restrictions on firms and consumers' locations, see D'Aspremont et al. (1979). With the introduction of the regulator, the result is exactly the same and is given by:

$$p_1^* = (1/3)\, b\, z\, (2+ q), \quad p_2^* = (1/3)\, b\, z\, (4-q) \qquad (16)$$

Given this price equilibrium pair p_1^*, p_2^* we compute the location equilibrium, considering the regulator restriction $z \le v$ obtaining one single solution: $x_i^* = v_1 \quad , x_2^* = v_2$.

First evaluation of this result is that the maximum differentiation principle is not altered by the intervention of the regulator.

The corresponding profits of this equilibrium are obtained

$$B_1^* = \frac{1}{18} bv(2 + v_2 + v_1)^2, \qquad B_2^* = \frac{1}{18} bv(4 - v_2 - v_1)^2 \qquad (17)$$

[11] The demand of firm 2 is simply $D_2 = 1 - v - D_1$

3.3. Optimal size of the commercial area in the linear model

As said above, in the first stage the regulator chooses the commercial region, *v*, anticipating how this choice will affect the firms' decision about locations and prices. We concentrate on symmetric equilibria[12] outcomes to obtain analytical solutions – in view of the complexity of solving this three-stage game – and facilitate comparisons with the circular model studied herein, as all location pairs in the circular model are symmetrical by definition. On the other hand an asymmetrical choice by the regulator could be considered discriminatory and arbitrary as it would mean favouring part of the consumers (the nearest) against the other.

Under such symmetric location condition an given that firms choose the maximum differentiation within the allowed region i.e. $z = v$ or in other terms:

$$x_1^* = v_1 = \frac{1}{2} - \frac{v}{2}, \quad x_2^* = v_2 = \frac{1}{2} + \frac{v}{2} \qquad (18)$$

To analyze the optimum size of the commercial region v we remember the regulators welfare function given by Eq. (11) :

$$W(v) = \lambda B^* + (1-\lambda)\,(p^* D^* + C_T)$$

λ being the weight given by the regulator to firms' profit, in accordance with his political bias, whereas $(1-\lambda)$ is the weight given to consumers disutility, $0 \le \lambda \le 1$.

Where $p^* = p_1^* = p_2^*$ is the price given by the expressions (8a) or (8b), and $B^* = B_1^* + B_2^*$, is the total profit such that B_1^*, B_2^* are given by the expressions (10a) or (10b), which depends on v and $D^* = D_1^* + D_2^* = 1$ is the market demand.

Substituting the equilibrium locations, Eq. (18), into profit functions Eq.(17) we obtain:

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[12] v centered thus, $v_1 + v_2 = 1, \quad x_1^* = v_1, \quad x_2^* = v_2$

$B_1^* = B_2^* = \frac{1}{2}bv$ and therefore the total profits function results $B^* = bv$ (19)

Now we compute the global transportation costs incurred by consumers *C_T (v)*, given the optimal locations obtained above, Eq (18) and defined as: $C_T(v) = I_1 + I_2$

$$I_1 = \int_0^{\alpha^*} b\left[x - \frac{1}{2} + \frac{v}{2}\right]^2 dx, \qquad I_2 = \int_{\alpha^*}^{1} b\left[x - \frac{1}{2} - \frac{v}{2}\right]^2 dx$$

where the first integral I_1 corresponds to the overall transportation costs paid by those consumers that address firm 1 and the second integral I_2 to the overall transportation costs paid by those consumers that address firm 2; while, $\alpha^* = \frac{1}{2}$ is the indifferent consumer under location equilibrium.

Finally we compute the total transportations cost as*:*

$$C_T(v) = \frac{b}{12}(1 - 3v + 3v^2) \qquad (20)$$

Next using Eqs. (19) (20) we compute the welfare function as follows:

$$W(v,\lambda) = (2\lambda - 1)bv - (1-\lambda)\frac{b}{12}(1 - 3v + 3v^2) \qquad (21)$$

P**roposition 5:**

For 0 ≤ v ≤ 1, the optimal size of the commercial area is *given by:*

$$\begin{cases} v_{L1}^* = 0, & if \quad \lambda \le \frac{3}{7} \qquad (22a) \\ v_{L2}^* = \dfrac{7\lambda - 3}{2(1-\lambda)}, & if \quad \frac{3}{7} \le \lambda \le \frac{5}{9} \qquad (22b) \\ v_{L3}^* = 1, & if \quad \frac{5}{9} \le \lambda \qquad (22c) \end{cases}$$

Proof:

Using the first order conditions, $\frac{\partial W}{\partial v} = \frac{b}{4}(7\lambda - 3 - 2v - 2\lambda v) = 0$, and for $\lambda \neq 1$ we find that $v^* = \frac{7\lambda - 3}{2(1-\lambda)}$.

Taking into account the second order condition: $\frac{\partial^2 W}{\partial v^2} = -\frac{b}{2}(1-\lambda) < 0$, consequently v^* could be the solution, if and only if: $0 \le v^* \le 1$ equivalent to[13]: $\frac{3}{7} \le \lambda \le \frac{5}{9}$

While if $\frac{3}{7} \ge \lambda \Rightarrow \frac{\partial W}{\partial v} \le 0$ so in this case the maximum is reached at $v^*=0$, and if $\lambda \ge \frac{5}{9}$ implies that in this case the maximum is reached at $v=1$.■

The corresponding welfares of these values are:

$$W(v_{L1}^*, \lambda) = -(1-\lambda)\frac{b}{12},$$

$$W(v_{L2}^*, \lambda) = \frac{b}{48(1-\lambda)}\left[-95\lambda^2 + 145\lambda + 46\right],$$

$$W(v_{L3}^*, \lambda) = \frac{b}{12}(25\lambda - 13).$$

3.4. Welfare analysis on the linear model

From the findings above (see Fig.4) we observe that giving to the firms' profits a weight $\frac{3}{7} \le \lambda \le \frac{5}{9}$ (approximately between 42% and 55%) result in v values represented by Eq.(22b). I.e. an unbiased regulator giving a weight 50/50 to firms and consumers would be therefore be inclined to restrict the commercial zone to

[13] $v^* \ge 0 \quad if \ \lambda \ge \frac{3}{7}$, $v^* \le 1 \quad if \ \lambda \le \frac{5}{9}$

$v^* = \frac{1}{2}, \quad x_1^* = v_1^* = \frac{1}{4}, \quad x_2^* = v_2^* = \frac{3}{4}$. Such situation reflects the political approach of a centrist regulator.

Analyzing weights below $\frac{3}{7}$ (42%) we observe they yield a totally restricted commercial zone *v=0*, which means $v_1^* = v_2^* = x_1^* = x_2^* = \frac{1}{2}$. This result forces the firms to compete "á la Bertrand" and surprisingly restores the Hotelling solution of minimum differentiation in the center of the market. This could define a typical socialist regulator.

Finally firms' weights above $\frac{5}{9}$ (55%) render a totally unrestricted space and firms are allowed to locate freely along the whole market space. More than possibly such approach enters the profile of a liberal regulator.

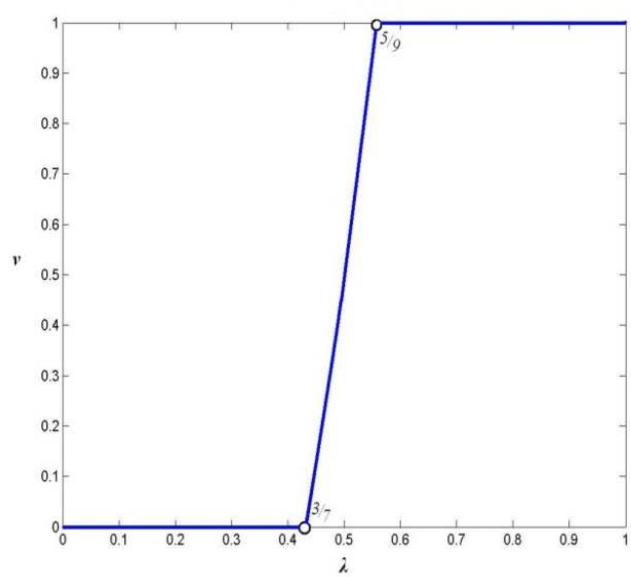

Fig. 4. Commercial region v upon λ chosen by the regulator. Linear model

4. Conclusions

One common denominator of both models is that a regulator bias below (respectively above) 42% (58%) favorable to the firms' profits (consumers' utility) implies to establish a restriction of the commercial zone to a single point and force them to compete intensively in prices. Such could be characterized as a socialist approach. Nevertheless as it is well known, such fierce competition exclusively on prices, leads to a forecast of death or collusion. The welfare results become negative as it can be observed in Fig. 5, where the maximum welfare in equilibrium is plotted for both models, assuming b=1, without loss of generality.

Another common denominator at the opposite end, for a liberal regulator favorable to the firms (respectively consumers) above 55% (below 45%), yields to not to establish any restriction and allow the firms to locate with the maximum differentiation. Surprisingly enough such approach yields the maximum welfare in all cases.

The possibilities for a regulator to establish a certain zone, other than all or nil, result exclusively reserved to the centrist approach in the vicinity of an unbiased regulator who could consistently sustain that his is not favoring any of the market actors. The welfare results however are intermediate between the other two described regulator bias.

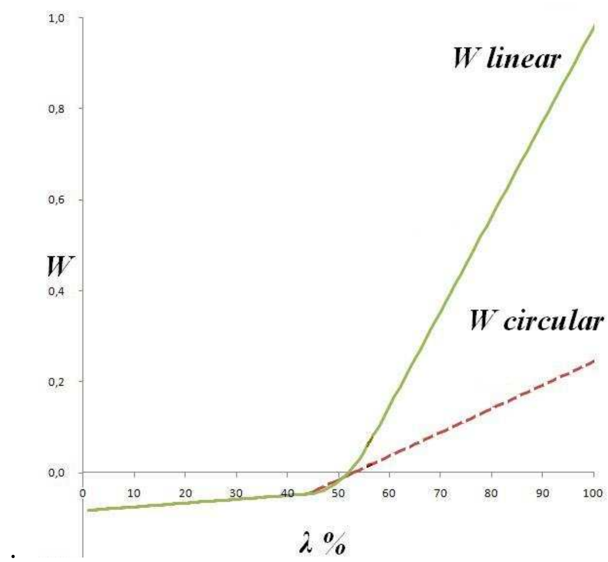

Fig.5. Welfare comparison upon model morphology (b=1)

The welfare absolute maximum is thus obtained when firms obtain the maximum freedom to choose both reaching their maximum profits in a perfect Nash equilibrium.

The welfare of the circle model being always below the linear one is explained by the fact that the last allows double differentiation and four fold welfare.

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