

# Mathematical Optimization models for Air Traffic Flow Management: A review<sup>1</sup>

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## Abstract

Congestion problems are becoming increasingly acute in many European and American airports and air sectors. To protect Air Traffic Control (ATC) from overload a planning activity called Air Traffic Flow Management (ATFM) tries to anticipate and prevent overload and limit resulting delays. When the traffic expects to exceed the airport arrival and departure capacities or the airsector capacity a delay in the flight arrival (so called-congestion) occurs. The casuistry to be considered in this field is very extensive. In general, most references to be found in the literature written some years ago refer to the simplest models, those which do not take into account airsector. This is so because this work was first studied in USA, where only the problems of congestion in airports basically occur. In the paper we present a state-of-the-art survey on the main optimization models encountered in the literature. They are classified as follows: (1) *Single-Airport Ground-Holding Problem (SAGHP)*. The simplest of the methodologies of planning modelling studied proposes solutions to the problem of deciding the optimal planning for an arrival airport. (2) *Multi-Airport Ground-Holding Problem (MAGHP)*. In this methodology the field of work is extended and the inter-relationship which exists between different airports is included. (3) *Air Traffic Flow Management Problem (ATFMP)*. This methodology attempts to solve real situations that are much more complex than those which can be dealt with using the previous methodologies, since the air sector capacity is also considered. (4) *Air Traffic Flow Management Rerouting Problem (ATFMRP)*. This methodology considers the more realistic situation where the flights can be diverted to alternative routes. (5) *Air Traffic Flow Management Rerouting Problem (ATFMRP) with uncertainty*. The ATFM problem is especially sensitive to changes in capacity. This leads to generalize the previous methodologies and to include generic uncertainty for these possible unforeseen changes in the parameters of the model, making way for stochastic methodologies. This type of problems are the most difficult ones, but alas the realistic ones.

## 1 Introduction

Congestion problems are becoming increasingly acute in many European and American airports and air sectors. To protect Air Traffic Control (ATC) from overload a planning activity called Air Traffic Flow Management (ATFM) tries to anticipate and prevent overload and limit resulting delays. When the traffic expects to exceed the airport arrival and departure capacities or the airsector capacity a delay in the flight arrival (so called-congestion) occurs.

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<sup>1</sup>This work is partially supported by the Spanish Ministry of Science and Innovation through the projects OPTIMOS MTM2006-14961-C05-05, OPTIMOS2 MTM2009-14039-C06-03, PLANIN MTM2009-14087-C04-01 and i-Math Ingenio Mathematica, and the project RIESGOS CM from Comunidad de Madrid.

Additional cost caused by delays are substantial, and they are quantified by a international organization: for European airlines the total yearly delay cost due to congestion was estimated to be Euro 5.73 billion/year in 1999 and USD 10 billion/year due to ATC actions which generate deviations from optimal aircraft flight profiles. 20 per cent of the flights in the European Space were delayed in September 2000 due to ATC capacity constraints. See Leal de Matos and Powell (2002) [29] and Dell'Olmo and Lulli (2003) [31].

So, the air traffic in Europe and the USA has experimented an spectacular growth during the last years (more than 50 per percent in the last 10 years according to recent estimations) and a 50 per cent traffic increase is expected by 2018. For example, Madrid is predicted to become the third busiest airport in Europe by the end of 2012. According to EUROCONTROL, the growth in air traffic expected during 2008 mean parts of Europe's air traffic control (ATC) system reached capacity, leading to on-ground delays at airports at peak times of activity in the busy summer months.

Air traffic delay due to difficult weather considerations has grown rapidly over the last few years. According to FAA (2002), flight delays have increased by more than 58 percent from 1995 until 2002, and cancellations by 68 percent, see Nilim and El Ghaoui (2004) [52]. Moreover, the successive deregulations in the air traffic and the prices cutting in the air flights will probably imply a sensible worsening of the current situation, as it was forsworn 15 years ago by Bianco (1995) [20].

To alleviate this problem, in the 1970's air traffic control in the USA was centralized and concentrated into one single organization with headquarters in Washington (The Air Traffic Control System Command Center) which has mitigated the impact of this increase in traffic taking into account the air traffic flow planning in that country. In Europe, where until the 90s, each country was responsible for the air traffic crossing its territory, the problem is much greater. While in the USA congestion problems are restricted to the airports, since there are no big problems in the airsector, in Europe the overcrowding of airsector is more important and in certain cases close to collapse. This congestion comes from the difficulty of coordinating air traffic control in each country. In order to solve these problems and avoid a worsening situation, a European organization has been set up to manage air traffic control in Europe (European Central Flow Management Unit, CFMU) which depends of EUROCONTROL. Leal de Matos and Ormerod (2000) [28] describe in some detail the working methodology of the CFMU.

The actions taken by the CMFU cover the medium and short term and do not require any extension of existing infrastructures. They only represent a small part of the action required when attempting to solve the problem. In general, the policies applicable to finding solutions to this problem can be divided into three main groups, according to the horizon under consideration (to which various methodologies apply):

1. Long term (several years) includes mainly the construction of new airports or expanding the existing ones, and an improvement in air traffic control technologies which lead to a reduction in the time needed to carry out different operations and ensure a safe distance between aircraft in flight. The increasing pressure exerted by the population living in the vicinity of the airport facilities and stricter demands on the environment, restrict to a large extent the relevant action which should be taken. See for example Stamatopoulos, Zografos and Odoni (2003) [62].

2. Medium term (up to 1 year) includes a modification in flight planning by means of a temporary redistribution of the same, diverting flights to off-peak times so as to avoid excessive demand periods. Included in this is the strategic and pre-tactical planning carried out by the CFMU.
3. Short term (24 hours) or tactical: in this case, the most effective actions are those which apply to aircraft delays. These policies which are used more and more, of delay on the ground, as opposed to those previously being applied regarding delays in the air, have been introduced as a result of their lower cost in economic and safety terms. So before the introduction of these policies, a flight received the authorization for take-off from an airport when there was sufficient capacity there, regardless of what might happen en route or at the destination of that flight. It was not therefore unusual that flights had to be diverted or kept in flight around the destination airport with the consequent increase in expenses and reduction in safety. With this new policy a flight does not take off until the progress throughout its journey is guaranteed and there is sufficient capacity at the destination airport. This type of short term problems is the subject of this paper.

The beginning of these policies goes back to 1973, when coinciding with the first oil crisis that air traffic management became aware of the scale of the problem. So, the Federal Aviation Agency (FAA) in USA adopted a policy consistent with delaying flights on the ground before departure in the event that their arrival could not be guaranteed. This change in policy, tending to stop the endless increase in the number of aircraft flying around the destination airport, came about through the rise in fuel costs which made delays in the air much more expensive. Initially, the development of this policy was left in the hands of the air traffic controllers who made the decision, based on their experience as to which flights should be delayed and which could depart. Later, advances in computer science and the application of Operational Research to this field have led to the development of assistance methodologies which make possible to find the ideal (or almost ideal) solution.

The objective of delays on the ground is to avoid delays expected in the air by transferring them to the ground. It is not always possible to calculate the total delay, especially in the case of long-haul flights, as changes in conditions en route can occur once the flight has begun its trip. That is why monitoring the occupation of each airspace and time period is necessary. Moreover, the US Federal Aviation Administration (FAA) deployed in June 2006 a tool known as Airspace Flow Programs. For the first time this tool gives the FAA the ability to control activity in congested airspace by issuing ground delays customized for each individual flight. Forty-four uses of this tool during the summers of 2006 and 2007 realized a benefit of approximately USD 118 million [41].

Furthermore, owing to the fact that many of the aircrafts are used for consecutive flights, whatever delay is decided for a particular flight affects all the following flights programmed for the same aircraft. What is more, commercial policies and alliances between companies guarantee passenger connection on different flights which further complicates the problem and multiplies the knock-on effect of a flight delay throughout the network. See in Ball et al. (2007) [12] a recent survey on air transportation irregular operations and control.

A High Level Group (HLG) on the Single European Sky (SES), established by the European Commission reported its conclusions in 2000. The work was undertaken against a background

of severe air traffic flow management (ATFM) delays. The main recommendation of the 2000 HLG, which is relevant for our purposes, is that the airspace should be managed for overall efficiency as a "single continuum" to optimize performance at a European level, optimizing the route and sector designs and flexible use of airspace, introducing concepts as "free routing". The European Commission SES regulation 529/2004 defines the Factual Airspace Blocks as an airspace block based on operational requirements, reflecting the need to ensure more integrated management of the airspace regardless of existing boundaries. See EUROCONTROL (2008) [32].

The purpose of this paper is to present a state-of-the-art survey on the relevant mathematical optimization models and solutions existing in the literature on the subject. There are different ways of classifying the papers. We choose to classify the works published in the open literature according to the following categories:

1. Single airport / multiple airport / Air traffic flow management problem. Solving single airport problems is easier than solving multiple airport problems and this is easier than solving air traffic flow management problems. However, the latter gives a better picture of the problem than the other two approaches.
2. Air traffic flow management rerouting problem. This problem is more difficult to solve than the three previous problems but it gives a better picture of the real-life problem to solve.
3. Flight cancellations.
4. Deterministic / stochastic planning. The latter type of problems are more difficult to solve than the deterministic problems but the capacity of the airspace (airports and airsectors) can be reduced along the time horizon due mainly to weather conditions, so, uncertainty is to be present in real-life problems.
5. Exact algorithms / heuristic algorithms. The exact algorithms give the mathematical optimal solution, but very frequently obtaining the solution requires more time than allowed in almost real-time circumstances. In these situations, heuristic algorithms are the preferable ones. They do not guarantee the optimality of the solution but usually the solution offered by the different approaches is the optimal one, or it is a quasi-optimal solution, at least.
6. Dependency in arrivals and departures.

The remainder of the paper is organized as follows: Section 2 is devoted to the main strategies for air traffic flow management, section 2.1 is devoted to present the basic model for Single Airport Ground Holding Problem (SAGHP), section 2.2 reviews the basic model for Multi Airport Ground Holding Problem (MAGHP), section 2.3 presents the state-of-the art in Air Traffic Flow Management Problem (ATFMP) studying the seminal work of Bertsimas and Stock (1998) [18], and section 2.4 studies the Air Traffic Flow Management Rerouting Problem (ATFMRP), presenting the basic features of the BLO model (see Bertsimas, Lulli and Odoni (2009) [16]) and the Deterministic Air Traffic Flow Management (DATFM) (see Agustin et al. (2009) [3]). (We will notice when the paper is treating flight cancellations, stochastic planning

and heuristic algorithms). Section 3 is devoted to study the uncertainty in ATFMRP, section 3.1 gives the motivation, section 3.2 presents the main concepts of stochastic optimization, section 3.3 gives the features of the stochastic problem to model and section 3.4 introduces our Stochastic Air Traffic Flow Management (SATFM) model (see Agustin et al. (2010) [2]). Finally, section 4 concludes.

## 2 Strategies for Air Traffic Flow Management

The casuistry to be considered in this field is very extensive. In general, most references to be found in the literature refer to the simplest models, those which do not take into account airsector. This is so, basically because the problem was first studied in USA, where, as previously mentioned, basically only the problems of congestion in airports occur. Odoni (1987) [53] presents a pioneer work on flight planning in real time to minimize congestion costs. The conceptualization of this first methodology in airport planning modelling is fundamental in the methodologies developed subsequently. A taxonomy of these methodologies distinguishes between Airport planning considering one single airport (SAGHP) and a group of airports (MAGHP), airport planning which considers also various airsectors (ATFMP) and in addition provides alternative routes for flights where there is congestion (ATFMRP).

From another point of view, the methodologies for Airport Planning modelling are divided into Static methodologies and Dynamic methodologies, according to how the airport capacity is viewed for individual periods or in groups of periods for a given planning horizon, respectively. Finally, the methodologies can be classified as Deterministic methodologies (where it is assumed that all the data referring to take-off and landing capacity in the airports and airsector capacity are known, that is, variable weather conditions are not taken into account) and Stochastic methodologies (where variable weather conditions are considered and therefore the variability of the above mentioned capacities is considered too.)

### 2.1 Single-Airport Ground-Holding Problem (SAGHP)

The simplest of the methodologies of planning modelling studied proposes solutions to the problem of deciding the optimal planning for an airport, taking into account the limitations with regard to the number of landing and take-off operations that can be carried out within the time units. This strategy has been applied at some Italian airports, following studies by the team lead by Bianco (1987 and 1995) [20, 22], the Boston Logan airport by Andreatta and Romain-Jacur (1987) [10], Andreatta, Odoni and Richetta (1993) [9] and Richetta (1995) [57], and the Frankfurt airport by Platz and Brokof (1994) [55]. Other applications have been made by the Institute of Flight Guidance for airports in Germany, whose results can be seen in Völkers and Bohme (1995) [67]. The problem is partially solved by space-time networks in Zenios (1991) [73]. Arrival sequencing and scheduling for the static case is modeled in Bianco et al. (1997) [21]. Deterministic formulations which have been efficiently implemented in USA airspace are proposed in Hoffman and Ball (2000) [39]. See also Richetta and Odoni (1993 and 1994), [58, 59], Terrab and Odoni (1993) [63] and Ball et al. (1999) [13].

In most of the optimization models developed to manage airport operations, arrivals and

departures are treated as independent variables, i.e., the number of flights to take off does not depend on the number of arrivals at the same time period. In fact, it is a strong assumption in most of the congested airports, where many interactions between arrivals and departures take place. Dell’Olmo and Lulli (2003) [31] face the problem of finding the optimal trade-off between the number of arrivals and the number of departures in order to reduce a delay function in SAGHP, by using a more realistic representation of the airport capacity.

Since it is safer and less expensive to absorb delays in the ground as we have commented above, many models have been developed to assign ground holdings delays optimally. Still, the undeniable fact remains that airborne delays cannot be totally avoided. Ma, Cui and Cheng (2004) [49] present a model based on multicommodity dynamic network flow for short-term air traffic management, where ground and airborne holds are minimized under sudden airport capacity reduction.

A dynamic stochastic optimization based approach is presented by Mukherjee and Hansen (2007) [51] for SAGHP, where ground delays assigned to flights can be revised during different decision stages, based on weather forecasts.

A Constrained Satisfaction Problem algorithm is proposed by Idrissi and Li (2006) [42] for the capacity allocation problem.

The basic SAGHP model [9, 63] assumes that the capacity of the given arrival airport, say  $k$ , is a deterministic function of time, known in advance with certainty. Besides this deterministic characteristic, an unlimited capacity in the departure airports and airsectors is assumed, so, no alternative routes are considered, nor the flight speed is taken into consideration. Arrival advances in the schedule are not allowed. Additionally, no continued flights are permitted. The time horizon consists of  $T$  time periods, and an extra time period  $T + 1$ , whose capacity is large enough to allow the arrival of any number of flights (e.g., a night period where any number of arrivals can be accommodated); it is the way to treat cancellation flights. No airlines preferences are considered on how to allocate the ground holding of the flights.

### Sets

$\mathcal{F}$ , set of flights

$\mathcal{T}$ , set of time periods  $\{1, \dots, T\}$ , where  $\mathcal{T}^+ = \mathcal{T} \cup \{T + 1\}$ .

### Parameters

$r_f$ , scheduled arrival to airport  $k$  for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$c_f^d$ , ground holding delay time unit cost of flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$R^t$ , arrival capacity of airport  $k$  at time period  $t$ ,  $\forall t \in \mathcal{T}$  for the given scenario.

### Variables

$x_f^t$ , 0-1 variable such that its value is 1 if flight  $f$  is planned to arrive to airport  $k$  at time period  $t$  and, otherwise, it is zero,  $\forall f \in \mathcal{F}, t \in \mathcal{T}^+$ .

## Model

The pure 0-1 model to obtain the planned arrivals of the flights at airport  $k$  to minimize the total ground holding delay cost is as follows:

Objective function

$$\min \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}^+ | r_f \leq t} c_f^d x_f^t \quad (1)$$

Constraints

$$\sum_{t \in \mathcal{T}^+ | r_f \leq t} x_f^t = 1 \quad \forall f \in \mathcal{F} \quad (2)$$

$$\sum_{f \in \mathcal{F}} x_f^t \leq R^t \quad \forall t \in \mathcal{T} \quad (3)$$

$$x_f^t \in \{0, 1\} \quad \forall f \in \mathcal{F}, t \in \mathcal{T}^+ | r_f \leq t. \quad (4)$$

The model is the typical Generalized Assignment Problem. Numerical problems can be solved by using standard GAP and Min Cost Flow algorithms. The reported computational results in [63] show that, even for this simplest model, large savings in the total delay cost could be achieved by assigning the available arrival capacity according to the solution provided by the model.

## 2.2 Multi-Airport Ground-Holding Problem (MAGHP)

In this methodology the field of work is extended and the inter-relationship which exists between different airports is included. The objective consists of finding a planning adapted to the limitations of the capacity imposed by the infrastructures available at each airport (simulating various alternatives of the said infrastructures). A dynamic optimization taking multiple airports and flight connectivity into account is presented in Wang (1991) [70]. Several applications have been made, so we find works by Andreatta, Brunetta and Guastalla (1994) [8], and Brunetta, Guastalla and Navazio (1996) [23] for different problems simulated at the Pseudo Official Air Guide Generator at the Drapper laboratory; and Burlingame et al. (1994) [24] from MITRE Corporation for FAA data. See the works of Vranas, Bertsimas and Odoni (1994a) [69] for the static case and Vranas, Bertsimas and Odoni (1994b) [68] for the dynamic case at Boston Logan airport.

See in Gilbo (1993) [35] a methodology that considers the interdependence between take-off and landing capacity in airports.

The performance of some of these mentioned models are evaluated on a set of seven test instances in Andreatta and Brunetta (1998) [7]. Three algorithms proposed in the literature for the multi-airport ground holding problem are compared. They analyze the model presented in Vranas, Bertsimas and Odoni (1994a) [69] where congestion is only caused by insufficient arrival capacity, since infinite departure capacity is assumed. The second model to consider is due to Andreatta and Tidona (1994) [11] that has no longer a need for continuity constraints since include this factor on the number of variables (it then becomes larger than the other models).

Finally, they consider the model due to Bertsimas and Stock (1994) [17]. The main advantage of this model is that the 0-1 variables are interpreted in a different way, they take value 1 if and only if flight  $f$  has arrived **by** time  $t$ , i.e., it arrives at time  $t$  or earlier. These tree models have considered airport capacities and continued flights, and the objective function consists of minimizing cost delay time units.

Zhang et al. (2007) [74] have developed a co-evolutionary Genetic Algorithm for MAGHP and validated it with real data from Beijing, Shanghai and Guangzhou ATC Centers of the Civil Aviation Administration of China.

The basic MAGHP model [68] assumes that the departure and arrival capacity of the airports are deterministic functions of the time, known in advance with certainty. Besides these characteristics, it is assumed an unlimited capacity in the air sector, so, no scheduled or alternative routes are considered, nor the flight speed is taken into consideration. The upper bounds on the ground holding and air delay are unlimited and then it paves the way for considering even partially flight cancellations. Notice that a flight is continued e.g. if the related aircraft will also perform the continuation flight along the time horizon. It is assumed that the *slack* time for a continued flight is known, such that if the flight arrives at its destination at most slack time periods late, then the departure of the continuation flights not affected; otherwise, the ground holding delay of the continuation flight is the total (ground holding plus air) delay of the continued flight minus the slack time, at least.

Let us use the same notation as before, except when otherwise it is explicitly stated.

The mixed 0-1 model (that obviously can be converted in a pure 0-1 model) is as follows:

#### Input sets

$K$ , set of airports, by independency  $K = K^d \cup K^a$ , where  $K^d$  is the set of departure airports and  $K^a$  is the set of arrival airports. Note: An airport usually belongs to both sets.

#### Parameters

$d_f \in \mathcal{T}$ , scheduled departure time for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$r_f \in \mathcal{T}$ , scheduled arrival time to its destination for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$c_f^a$ , air delay time unit cost of flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$c_f^g$ , ground holding delay time unit cost of flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$k_f^d$ , departure airport for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$k_f^a$ , arrival airport for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$G_f$ , maximum ground holding time units delay for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$A_f$ , maximum air time units delay for flight  $f$ ,  $\forall f \in \mathcal{F}$ . Notice that the maximum time allowed for flight  $f$  to be in the air has the expression  $r_f - d_f + A_f$ .

$s_f$ , slack time of the aircraft after flight  $f$ ,  $\forall f \in \mathcal{F}$ .



$D_k^t$ , departure capacity of airport  $k$  at time period  $t$ ,  $\forall k \in \mathcal{K}, t \in \mathcal{T}$  for the given scenario.

$R_k^t$ , arrival capacity of airport  $k$  at time period  $t$ ,  $\forall k \in \mathcal{K}, t \in \mathcal{T}$  for the given scenario.

### Calculated sets in preprocessing

$\mathcal{T}_f^d$ , set of feasible time periods for the departure of flight  $f$ , such that  $T_f^d = \{t \in \mathcal{T} | d_f \leq t \leq \min\{d_f + G_f, T\}\}, \forall f \in \mathcal{F}$ .

$\mathcal{T}_f^a$ , set of feasible time periods for arrival of flight  $f$ , such that  $T_f^a = \{t \in \mathcal{T} | r_f \leq t \leq \min\{r_f + G_f + A_f, T\}\}, \forall f \in \mathcal{F}$ .

### Variables

$u_f^t$ , 0-1 assignment decision variable, such that its value is 1 if the departure time period from airport  $k_f^d$  is time period  $t$  and, otherwise, it is zero,  $\forall f \in \mathcal{F}, t \in \mathcal{T}$ .

$v_f^t$ , 0-1 assignment decision variable, such that its value is 1 if the arrival time period to airport  $k_f^a$  is time period  $t$  and, otherwise, it is zero,  $\forall f \in \mathcal{F}, t \in \mathcal{T}$ .

$g_f$ , delay decision variable that gives the number of time periods delay in ground holding for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$a_f$ , delay decision variable that gives the number of time periods delay in the air for flight  $f$ ,  $\forall f \in \mathcal{F}$ .

### **Model**

#### Objective function

$$\min \sum_{f \in \mathcal{F}} (c_f^g u_f + c_f^a v_f) \quad (5)$$

#### Constraints

$$\sum_{f \in \mathcal{F} | k_f^d = k, t \in \mathcal{T}_f^d} u_f^t \leq D_k^t \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (6)$$

$$\sum_{f \in \mathcal{F} | k_f^a = k, t \in \mathcal{T}_f^a} v_f^t \leq R_k^t \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (7)$$

$$\sum_{t \in \mathcal{T}_f^d} u_f^t = 1 \quad \forall f \in \mathcal{F} \quad (8)$$

$$\sum_{t \in \mathcal{T}_f^a} v_f^t = 1 \quad \forall f \in \mathcal{F} \quad (9)$$

$$g_f = \sum_{t \in \mathcal{T}_f^d} t u_f^t - d_f \quad \forall f \in \mathcal{F} \quad (10)$$

$$a_f = \sum_{t \in \mathcal{T}_f^a} tv_f^t - r_f - g_f \geq 0 \quad \forall f \in \mathcal{F} \quad (11)$$

$$g_{f'} + a_{f'} - s_{f'} \leq g_g \quad \forall (f', f) \in \mathcal{C} \quad (12)$$

$$u_f^t, v_f^t \in \{0, 1\} \quad \forall f \in \mathcal{F}, t \in \mathcal{T} \quad (13)$$

The objective function (5) gives the minimization of the total ground holding and air delay for the flights. Constraints (6) and (7) force the satisfaction of the capacity of the departure and arrivals, respectively. Constraints (8) and (9) ensure that exactly one departure and one arrival will occur for each flight. Expressions (10) and (11) define the ground holding and air delays, respectively. Constraints (12) are the coupling constraints. They transfer an excessive delay of flight  $f'$  to its next flight  $f$  for  $(f', f) \in \mathcal{C}$ . Notice that if flight  $f'$  arrives at its destination with a total delay  $g_{f'} + a_{f'}$  which is greater than the slack  $s_{f'}$ , then the next flight will have to be delayed in the ground  $g_{f'} + a_{f'} - s_{f'}$  time periods, at least.

### 2.3 Air Traffic Flow Management Problem (ATFMP)

This methodology attempts to solve real situations that are much more complex than those which can be dealt with using the previous methodologies. This is due to the fact that the previous strategies are only applicable in situations that arise frequently in USA where the problems of congestions are confined to airports. In Europe the situation is critical in the airsectors too and this has motivated some research teams to consider models which provide acceptable solutions. The literature contains few works on the study of this model, Odoni (1994) [54] defines and identifies some of the major issues for this problem and suggest decision support needs, mostly based on the USA situation. Also we draw attention to the work by Bertsimas and Stock (1998) [18] for the great impact it has made and the results obtained, in which a mathematical model is proposed for the study of air traffic flow management problem. Both the theoretical results of this work and the practical application to real cases point to the advantages of these models as opposed to others used in the literature. Specifically, they use genuine data which derives from Boston Logan, NY La Guardia, Washington National airports and a fictitious airport which represents the outside world. For problems involving more than 1000 flights, optimal planning is obtained in really impressive computing times (less than 10 minutes, with 1994 optimization algorithmic and computer technologies). We should also note the work by Alonso (1997) [4] in which the goodness of this model is also proven and where, for a series of simulated cases, solutions were obtained in less than 3 minutes, using a personal computer. See also Koepke et al. (2008) [44].

Solutions to the slot allocation problem are provided by a different approach in Barnier et al. (2001) [15]. See also the methodologies proposed by Helme (1992) [38], Lindsay, Boyd and Burlingame (1995) [47], Hoffman and Ball (2001) [40], and Flener et al. (2007) [33], among others. Tošić and Babić (1995) [64] and (Tošić et al. (1995) [66] dealt with this problem, where they presented a heuristic approach.

Geng and Cheng (2007) [34] present an integer programming based methodology to determine the routes open to certain users during given time periods.

In 2003, Dell'Olmo and Lulli [31] treat the ATFMP in a free flight scenario. In this sense,

they also consider a network but with no fix routes, i.e., the flight can follow any link of the network where times is the only important issue, i.e., arrive just in time, not after neither before. Each aircraft has a minimum, a maximum and a preferred flight time to travel in an arc. The model offers the possibility of considering airport capacity. Once again, the objective function is to minimize a cost function, included by the cost for flight arrival lately and early, and cost for the flight deviation from its preferred speed. Since this function is a nonlinear one, the computational time is very high, even if it does not consider ATFMP features as sectors capacity, continued flights, etc.

Ball et al. (2003) [14] present a stochastic approach for its application to the ground holding problem in air traffic flow management under Collaborative Decision Making, see also Kotnyek and Richetta (2006) [45]. There is much research on this problem in recent years, let us mention a few works: first, Dell’Olmo and Lulli (2002) [30] present a new methodology for solving the ATFMP, based on a two-level hierarchical architecture, and that goes a step forward letting the introduction of some aspects of the so-called Free Flight; Lulli and Odoni (2007) [48] presented a model with ground and airborne holding, and show the complex nature of European ATFM solutions, and the benefits that can be obtained by purposely assigning a more expensive airborne holding delay to certain flights. In a different context, see Waslander, Raffard and Tomlin (2007) [72] to explicitly incorporating airline preferences in ground holding policies due to weather disruptions. And, in the end, Krozel, Jakobovits and Penny (2006) present in [46] a routing and scheduling algorithm for ATFMP including ground delays, route selection and airborne holding, aligning with a Collaborative Decision Making philosophy.

The basic ATFMP model [18] assumes that the airport capacity for departure and arrivals as well as the airsector capacity are deterministic functions of time, known in advance with certainty. No alternative routes are assumed, nor the flight speed is taken into consideration. Cancellation is implicitly considered, but no big decisions on it can be made. Flight continuation is allowed, such that a turnaround time is given for the continued flight (i.e., the time slack between the arrival of a flight to an airport and the departure of given flights from the same airport). Let us use the notation as above, but the following additional notation is needed.

#### Input sets

$J$ , set of sectors under consideration, where the departure and arrival airports are included.

$\mathcal{P}_f$ , path of flight  $f$  given by a set of ordered sectors,  $\forall f \in \mathcal{F}$

#### Parameters

$N_f$ , number of sectors in the path of flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$P(f, i)$ , the  $i$ th sector in the path of flight  $f$  and, so,  $\mathcal{P}_f$  is the ordered set of sectors in the path of flight  $f$ , such that  $\mathcal{P}_f = \{P(f, i), 1 \leq i \leq N_f\}$ ,  $\forall f \in \mathcal{F}$ . Note:  $k_f^d = P(f, 1)$  and  $k_f^a = P(f, N_f)$ .

$\ell_{fj}$ , number of time periods that flight  $f$  must spend in sector  $j$ ,  $\forall f \in \mathcal{F}, j \in \mathcal{J}$ .

$S_j^t$ , capacity of sector  $j$  at time period  $t$ ,  $\forall j \in \mathcal{J}, t \in \mathcal{T}$  for the given scenario.

#### Calculated sets in preprocessing

$\mathcal{T}_f^j$ , set of feasible time periods for entering flight  $f$  in sector  $j$ , such that

$$\mathcal{T}_f^j = \{t \in \mathcal{T} | \underline{T}_f^j \leq t \leq \min\{\underline{T}_f^j + G_f + A_f, T\}\} \quad \forall f \in \mathcal{F}, j = P(f, i), 1 < P(f, i) < N_f,$$

where  $\underline{T}_f^j$  denote the scheduled arrival time period for flight  $f$  to sector  $j$ , such that it can be expressed

$$\underline{T}_f^j = d_f + \sum_{j'=P(f,i'), i' < j} \ell_{fj'} \quad \forall f \in \mathcal{F}, j \in \mathcal{P}_f.$$

Let  $\overline{T}_f^j$  denote the latest time period in set  $\mathcal{T}_f^j$ .

### Variables

$x_{fj}^t$ , 0-1 assignment decision variable, such that its value is 1 if flight  $f$  arrives at sector  $j$  by time period  $t$  and, otherwise, it is zero,  $\forall f \in \mathcal{F}, j \in \mathcal{J}, t \in \mathcal{T}_f^j$ . Notice that  $x_{fj}^t = 1$  means that the flight arrives to the sector at time period  $t$  or earlier. This type of variable makes the model much tighter than the variable where it takes the arrival of the flight at time period  $t$ .

### **Model**

The pure 0-1 model is as follows:

#### Objective function

$$\min \sum_{f \in \mathcal{F}} [(c_f^g - c_f^a) \sum_{t \in \mathcal{T}_f^d} t(x_{fk_f^d}^t - x_{fk_f^d}^{t-1}) + c_j^a \sum_{t \in \mathcal{T}_f^{k_f^a}} t(x_{fk_f^a}^t - x_{fk_f^a}^{t-1}) + (c_f^a - c_f^g)d_f - c_f^a r_f] \quad (14)$$

#### Constraints

$$\sum_{f \in \mathcal{F} | k_f^d = k, t \in \mathcal{T}_f^d} (x_{fk}^t - x_{fk}^{t-1}) \leq D_k^t \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (15)$$

$$\sum_{f \in \mathcal{F} | k_f^a = k, t \in \mathcal{T}_f^a} (x_{fk}^t - x_{fk}^{t-1}) \leq R_k^t \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (16)$$

$$\sum_{f \in \mathcal{F} | P(f,i)=j, 1 < i < N_j, P(f,i+1)=j'} (x_{fj}^t - x_{fj'}^t) \quad \forall j \in \mathcal{J} \setminus K^d \setminus K^a, t \in \mathcal{T} \quad (17)$$

$$x_{fj'}^{t+\ell_{fj}} - x_{fj}^t \leq 0 \quad \forall f \in \mathcal{F}, i < N_f, j = P(f, i), j' = P(f, i + 1), t \in \mathcal{T}_k^j \quad (18)$$

$$x_{fj}^t - x_{fj'}^{t+(r_f-d_f)+A_f} \leq 0 \quad \forall f \in \mathcal{F}, j = P(f, 1), j' = P(f, N_f), t \in \mathcal{T}_f^j \quad (19)$$

$$x_{fk}^t - x_{f'k}^{t-s_{f'}-1} \leq 0 \quad \forall (f', f) \in \mathcal{C}, t \in \mathcal{T}_f^k, \text{ where } k = k_f^d = k_{f'}^a \quad (20)$$

$$x_{fj}^t - x_{fj}^{t-1} \geq 0 \quad \forall f \in \mathcal{F}, j \in \mathcal{P}_f, t \in \mathcal{T}_f^j \quad (21)$$

$$x_{fj}^{\overline{T}_f^j} = 1 \quad \forall f \in \mathcal{F}, j \in \mathcal{P}_f \quad (22)$$

$$x_{fj}^t \in \{0, 1\} \quad \forall f \in \mathcal{F}, j \in \mathcal{P}_f, t \in \mathcal{T}_f^j \quad (23)$$

Objective function (14) minimizes the total (ground holding plus air) delay. Constraints (15) ensures that the number of flight departures from each airport does not exceed the departure capacity at each time period. Likewise, constraint (16) ensure that the number of flight arrivals to each airport will no exceed its arrival capacity at each time period. Constraints (17) ensure that the number of flights at each time period in each air sector will not exceed its capacity. Constraints (18) forces connectivity between sectors, such that if a flight arrives at sector, say  $j'$  by time period  $t + \ell_{fj}$ , then it must arrive at sector  $j$  by time period  $t$ , at most, where  $j$  and  $j'$  are consecutive sectors in the path of flight  $f$ . Constraints (19) ensure that the air delay for flight  $f$  does not exceed its upper bound  $A_f$ . Constraints (20) forces connectivity between airports to handle the cases where a flight is continued. Constraints (21) forces time continuity such that if a flight arrives at a sector by time period say  $t$  (i.e.,  $x_{fj}^t = 1$ ) then  $x_{fj}^{t'} = 1$  for all later periods ( $t' > t$ ). Constraints (22) forces that all flights depart from the departure airport, go along the sector path and arrive to their arrival airport by the latest admissible time period.

Model (14)-(22) has been a breakthrough in the open literature of air traffic flow management.

## 2.4 Air Traffic Flow Management Rerouting Problem (ATFMRP)

A very common application in daily operations is to divert flights. Given the implications that these decisions can have for the entire airport network, the design of new methodologies has been necessary, whereby the impact of diverting flights is taken into account. Within this field of study, we draw attention to two works by Bertsimas and Stock, the first one in (1998) [18] on the Boston Logan, NY La Guardia and Washington National airports, and the second one in (2000) [19] where they present a dynamic network flow approach and address the problem of dynamically rerouting aircrafts; and those by Tošić et al. (1995) [65], and Leal de Matos, Chen and Ormerod (2001) [27] for a selection of European routes (Iberian Peninsula - Germany, Iberian Peninsula - Italy, Balearic Islands - Germany ,etc.), see also Leal de Matos and Powell (2002) [29]. More recently, Ma, Cui and Cheng (2004) based their model on multicommodity dynamic network flow for short-term air traffic flow management, in [49]. Obstacle avoidance using the linearized constrained Uninhabited Aerial Vehicle dynamic has been modelled by Richards and How (2002) [56].

Dell'Olmo and Lulli (2003) [31] present a two-level hierarchical architecture for air traffic management problems to be solved by mathematical models. The first level represents the air route network, and its solution provides the air traffic flows on each arc of the network. This level interacts with the second one which represent a single airway and its own air traffic flows. This later model allows to assign air traffic route to each flight and to optimize the airsector capacity. Both exact and heuristic algorithms are provided.

Whatever policy is applied, the possibility of flight cancellation should explicitly be considered. The methodologies including this strategy have scarcely been studied in the literature. Alonso (1997) [4] presents some approximations to the problem. Slot-exchange mechanism for inclusion within the Airspace Planning and Collaborative Decision-Making Model is proposed in Sherali et al. (2006) [60]. Soomer and Franx (2008) [61] present an heuristic taking the pref-

erences of the airlines in collaborative decision making. An evaluation between heuristic and classical optimization algorithms for managing traffic is presented in Jakobovits et al. (2007) [43]. See also Lindsay, Boyd and Burlingame (1995) [47].

Lulli and Odoni in 2007 [48] propose this problem in the European environment, since they assume critical differences with the current systems in USA. And so, they get a harder problem since in Europe sector capacities have to be considered. Therefore they get a deterministic and dynamic model to compute the cost of ground and air delays of the flights. Also all these flights are scheduled as the sector they visit in the route for equality speed travel. And then, they will get results of the model when consider just one sector or just one destination airport. See the nonseparable ground holding and air delay cost function.

Recently, Bertsimas, Lulli and Odoni 2009 [16] propose a model that covers all the phases of each flight defined above, i.e., ground and air delay, rerouting, continued flights, cancellation, etc. As in Bertsimas and Stock work [18] routes are given by the order of cross sectors and the travel time units in a sector is only consider as a lower bound. They address problems of size comparable to the entire USA, and their computational times are very good. They use the objective function from [48] to minimize the ground holding and air delay that in a clever way prioritizes the first one over the second one. Also, Churchill, Lovell and Ball in 2009 [25] propose a more complete work based on the work [16]. They include airspace volumes in place of sectors and a more extensive method of enforcing aircraft connectivity. However, they find additional questions about defining the capacity of such volumes, as well as the entry and exit points used by the flows. Unfortunately, aircraft connectivity adds additional set of decision variables on the base model. Therefore, the proposed extension introduces several difficulties.

The model proposed in Agustin et al. (2009) and (2010) [1, 3] additionally considers minimum and maximum travel time unit between each two waypoints of the route. And, so, we force that the air delay will be share along the complete route and not just in a short distance. The basic ATFMRP model that we will denote it as DATFM (Deterministic Air Traffic Flow Management) is as follows:

The airspace can be represented as a collection of directed subgraphs  $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{A}_f)$  where  $\mathcal{N}_f$  are the nodes (airports and waypoints) for flight  $f$  and  $\mathcal{A}_f$  are the directed arcs interconnecting the nodes for flight  $f$ . Since we are taking into account rerouting, i.e., flight  $f$  could have more than one route to choose, we will denote the nodes and arcs in the scheduled route as  $\mathcal{N}_f^* \subseteq \mathcal{N}_f$  and  $\mathcal{A}_f^* \subseteq \mathcal{A}_f$ , respectively. Note that both  $\mathcal{N}_f^* = \mathcal{N}_f$  and  $\mathcal{A}_f^* = \mathcal{A}_f$  when the flight  $f$  has just one route to follow. And since we deal with subgraphs we can define the sets of incidence arcs for a given node  $n$  and a flight  $f$  as follows:

$$\begin{aligned} \text{In-going incidence set: } \Gamma_f^-(n) &= \{m / (m, n) \in \mathcal{A}_f\} \\ \text{Out-going incidence set: } \Gamma_f^+(n) &= \{m / (n, m) \in \mathcal{A}_f\} \end{aligned}$$

Since the airspace is divided into sector we can also define  $\mathcal{N}_f^j$  as the nodes in sector  $j$  for flight  $f$ . Also note that there is always, at least, one border node in-between two consecutive sectors, say  $j'$  and  $j$ . Therefore, we will define  $\mathcal{N}_f^{j'+}$  and  $\mathcal{N}_f^{j-}$  the incoming border nodes to sector  $j$  and the outgoing border nodes from sector  $j$ , respectively, for flight  $f$  as follows:

$$\begin{aligned} \text{In-coming border node set: } \mathcal{N}_f^{j'+} &= \{n / n \in \mathcal{N}_f^j, \Gamma_f^-(n) \notin \mathcal{N}_f^j\} \\ \text{Out-going border node set: } \mathcal{N}_f^{j-} &= \{n / n \in \mathcal{N}_f^j, \Gamma_f^+(n) \notin \mathcal{N}_f^j\} \end{aligned}$$

Notice that  $\Gamma_f^-(n) \notin \mathcal{N}_f^j$  means the set of arcs  $(m, n)$  whose ending node is node  $n \in \mathcal{N}_f^j$  and the origin nodes do not belong to the same sector  $j$ , i.e., the set of in-going arcs in the sector to whom node  $n$  belongs to; in other words,  $n$  is a border entering node in sector  $j$  and  $(m, n)$  is an in-coming arc to sector  $j$ . Similarly, notice also that  $\Gamma_f^+(n) \notin \mathcal{N}_f^j$  means the set of arcs  $(n, m')$  whose starting node is node  $n$  and the ending nodes do not belong to the same sector  $j$ , where by the other statement  $n \in \mathcal{N}_f^j$ , i.e., the set of in-going arcs in a different sector to whom node  $n$  belongs to; in other words,  $n$  is a border exit node from sector  $j$ .

Let the following additional notation to the notation used in the previous models and above.

### Input parameters

$k_f^d$ , departure airport for flight  $f, \forall f \in \mathcal{F}$ .

$q(k)$ , node from the boundary of airport  $k$  for the departure of any flight,  $\forall k \in \mathcal{K}^d$ .

$k_f^a$ , arrival airport for flight  $f, \forall f \in \mathcal{F}$ .

$p(k)$ , node from the boundary of airport  $k$  for the arrival of any flight,  $\forall k \in \mathcal{K}^a$ .

$\ell_{f,m,n} : (m, n) \in \mathcal{A}_f$ , scheduled travel time (i.e., number of time periods) for flight  $f$  to arrive at node  $n$  throw arc  $(m, n)$  in the air sector,  $\forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f$ . (Note:  $\ell_{f,k_f^d,q(k_f^d)} = 1, \ell_{f,p(k_f^a),k_f^a} = 1$  and  $r_f = d_f + \sum_{(m,n) \in \mathcal{A}_f^*} \ell_{f,m,n}$ ).

$\underline{\ell}_{f,m,n}, \bar{\ell}_{f,m,n}$  minimum and maximum travel time (i.e., number of time periods) for flight  $f$  to arrive at node  $n$  throw the arc  $(m, n)$  in the air sector,  $\forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f$ . Note that  $\underline{\ell}_{f,m,n} \leq \ell_{f,m,n} \leq \bar{\ell}_{f,m,n}$ , but also  $\underline{\ell}_{f,k_f^d,q(k_f^d)} = \ell_{f,k_f^d,q(k_f^d)} = \bar{\ell}_{f,k_f^d,q(k_f^d)} = 1$  and  $\underline{\ell}_{f,p(k_f^a),k_f^a} = \ell_{f,p(k_f^a),k_f^a} = \bar{\ell}_{f,p(k_f^a),k_f^a} = 1$ .

$r_f^n \in \mathcal{T}$ , scheduled arrival time to node  $n$  for flight  $f$  throw the initial route where  $(m, n) \in \mathcal{A}_f^*$ , i.e.,  $m, n \in \mathcal{N}_f^*$ ,  $\forall f \in \mathcal{F}$ . (Note:  $r_f^{k_f^d} = d_f, r_f^n = r_f^m + \ell_{f,m,n}$  and  $r_f^{k_f^a} = r_f$ ).

$c_{ft}^n$ , cost of flight  $f$  to arrive at node  $n$  at time period  $t, \forall f \in \mathcal{F}, n \in \mathcal{N}_f \setminus \{k_f^d\}, t \in \mathcal{T}_f^n$ . Note: It is zero if the arrival is on schedule. Additionally, it can accommodate the objective functions coefficients from Bertsimas, Lulli and Odoni (2009) [16], since  $c_{ft}^{k_f^a}$  can denote the delay cost of flight  $f$  for the total number  $(t - r_f)$  of time units (ground, holding, air, rerouting) delay. A scheme is proposed for obtaining this cost, such that it is  $(t - r_f)^{1+\epsilon_2}$  (where  $\epsilon_2 > 0$  is a given parameter), being  $c_{ft}^{k_f^a} = 0$  for  $t \leq r_f$ . Additionally,  $c_{ft}^{k_f^d}$  can denote the reduction cost of ground holding delay of flight  $f$  for the number  $(t - d_f)$  of time units delay. A scheme is proposed for obtaining this reduction cost, such that it is  $(t - d_f)^{1+\epsilon_2} - (t - d_f)^{1+\epsilon_1}$  (where  $0 < \epsilon_1 < \epsilon_2$  is a given parameter), being  $c_{ft}^{k_f^d}$  for  $t \leq d_f$ . Notice that, by combining both costs, the resulting cost is the total delay time units minus the reduction time units due to the the ground holding both at the total delay price, plus the ground holding cost at its own price. In this way, both cost are not accounted for separately as it happens in alternative schemes. See below the objective function 07).

$c_f^{a-}$ , time unit cost of flight  $f$  in case it is on the air fewer time periods than scheduled,  $\forall f \in \mathcal{F}$ .  
It can be negative.

$c_f^{a+}$ , time unit cost of flight  $f$  in case it is on air more time periods than scheduled,  $\forall f \in \mathcal{F}$ .

$c_f^c$ , cancelation cost of flight  $f$ ,  $\forall f \in \mathcal{F}$ .

$c_f^{m,n}$ , cost due to the use of arc  $(m, n)$  in a route of flight  $f$ ,  $\forall f \in \mathcal{F}$ ,  $(m, n) \in \mathcal{A}_f$ . Note:  
 $c_f^{m,n} = 0$  for the scheduled route  $(m, n) \in \mathcal{A}_f^*$ .

$c_{f,m,n}^-$  and  $c_{f,m,n}^+$ , time unit costs of flight  $f$  if it spends fewer time periods and more time periods, respectively, than  $\ell_{f,m,n}$  time periods on arc  $(m, n)$ ,  $\forall f \in \mathcal{F}$ ,  $(m, n) \in \mathcal{A}_f$ .

### Calculated sets in preprocessing

- $\mathcal{T}_{*f}^{m,n}$ , set of feasible time periods for flight  $f$  to arrive at node  $n$  through arc  $(m, n)$ :
- $\underline{\mathcal{T}}_f^{m,n}$ , the smallest interval containing  $\mathcal{T}_{*f}^{m,n}$ :

$$\mathcal{T}_f^{m,n} = \{\underline{\mathcal{T}}_f^{m,n}, \underline{\mathcal{T}}_f^{m,n} + 1, \dots, T_f^{m,n}\}, \quad \forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f$$

where  $\underline{\mathcal{T}}_f^{m,n} = \min\{t \mid t \in \mathcal{T}_{*f}^{m,n}\}$  and  $T_f^{m,n} = \max\{t \mid t \in \mathcal{T}_{*f}^{m,n}\}$ .

- $\mathcal{T}_{*f}^n$ , set of feasible time periods for flight  $f$  to arrive at node  $n$  through any arc:
- $\underline{\mathcal{T}}_f^n$ , the smallest interval containing  $\mathcal{T}_{*f}^n$ :

$$\mathcal{T}_f^n = \{\underline{\mathcal{T}}_f^n, \underline{\mathcal{T}}_f^n + 1, \dots, T_f^n\}, \quad \forall f \in \mathcal{F}, n \in \mathcal{N}_f \setminus \{k_f^d\}$$

where

$$\underline{\mathcal{T}}_f^n = \min\{t \mid t \in \mathcal{T}_{*f}^n\} = \min\{\underline{\mathcal{T}}_f^{m,n} \mid m \in \Gamma_f^-(n)\},$$

$$T_f^n = \max\{t \mid t \in \mathcal{T}_{*f}^n\} = \max\{T_f^{m,n} \mid m \in \Gamma_f^-(n)\}.$$

Note that, since  $\mathcal{G}_f$  is an acyclic graph, it is possible to numerate the nodes such that  $m < n$  if  $(m, n) \in \mathcal{A}_f$  and the previous expressions can be calculated recursively.

### Variables

$$x_{f,m,n}^t = \begin{cases} 1, & \text{if flight } f \text{ is planned to arrive at node } n \\ & \text{on its flight path through the arc } (m, n) \\ & \text{by the end of time period } t \\ 0, & \text{otherwise} \end{cases} \quad \forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f, t \in \mathcal{T}_f^{m,n}$$

Notice that  $x_{f,m,n}^t = 1$  means that a la Bertsimas-Stock flight  $f$  arrives at node  $n$  through arc  $(m, n)$  at time period  $t$  or earlier. Note:  $x_{f,m,n}^t = 0, \forall t \in \mathcal{T} : t < \underline{\mathcal{T}}_f^{m,n}$  and  $x_{f,m,n}^t = x_{f,m,n}^{\underline{\mathcal{T}}_f^{m,n}}, \forall t \in \mathcal{T} : t > T_f^{m,n}$ . Notice also that the models introduced in [16, 18, 19] do only consider the nodes, since the  $x$  variable is represented by  $x_{fn}^t$ .



$\alpha_f^+, \alpha_f^-$ , positive and negative time units difference, respectively, between the actual and scheduled air trip lengths for flight  $f$ ,  $f \in \mathcal{F}$ .

$\delta_{f,m,n}^+, \delta_{f,m,n}^-$  positive and negative time units difference, respectively, between the actual and scheduled air trip lengths in arc  $(m, n)$  for flight  $f$ ,  $f \in \mathcal{F}$ ,  $(m, n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\}$ .

## Model

### Objective functions

The objective function to be optimized can be included by different terms, depending the goal of the decision maker. In the following list some of these terms are represented. They can be combined, with an appropriate weight to create a multi-objective function to be optimized:

o1) Total cancellation cost:

$$\sum_{f \in \mathcal{F}} c_f^c \left( 1 - x_{f, k_f^d, q(k_f^d)}^{T_f^{k_f^d, q(k_f^d)}} \right)$$

For  $c_f^c = 1, \forall f \in \mathcal{F}$ , this expression represent the number of cancelled flights.

o2) Cost of using each route.

$$\sum_{f \in \mathcal{F}} \sum_{(m,n) \in \mathcal{A}_f} c_f^{m,n} x_{f,m,n}^{T_f^{m,n}}$$

Notice that, since the scheduled routes have a zero penalization, this expression represent the cost of using alternative flight routes.

o3) Cost of the deviation of the actual arrival time from the scheduled one at each node in the flight route.

$$\sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f \setminus \{k_f^d\}} \sum_{m \in \Gamma_f^-(n)} \sum_{t \in \mathcal{T}_f^{m,n}} c_{ft}^n (x_{f,m,n}^t - x_{f,m,n}^{t-1})$$

Note that the deviation at node  $q(k_f^d)$  gives the ground delay, and the deviation at node  $k_f^a$  gives the total delay (at ground and air) for flight  $f$ .

o4) Cost of the deviation of the actual flight duration from the scheduled one at each arc in the flight route.

$$\sum_{f \in \mathcal{F}} \sum_{(m,n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\}} (c_{f,m,n}^+ \delta_{f,m,n}^+ + c_{f,m,n}^- \delta_{f,m,n}^-)$$

o5) Cost of the deviation of the actual flight duration from the scheduled one.

$$\sum_{f \in \mathcal{F}} (c_f^{a+} \alpha_f^+ + c_f^{a-} \alpha_f^-)$$

- o6) Number of flights exceeding the maximum allowed number of time units of arrival delay (due to ground and air delay or the use of alternative routes), so-called type 1 delay.

$$\sum_{f \in \mathcal{F}} \left( x_{f,p(k_f^a),k_f^a}^{T_f^{p(k_f^a),k_f^a}} - x_{f,p(k_f^a),k_f^a}^{r_f+M_f} \right)$$

Note that for each flight this difference takes the value 1 only if flight is not cancelled and it arrives at its destination at least  $M_f$  time periods later than it is scheduled in its initial route.

- o7) Non-separable cost of delaying flight  $f$  for  $(t - r_f)$  time units and the reduced cost obtained by holding flight  $f$  in the ground for  $(t - d_f)$  time units.

$$\sum_{f \in \mathcal{F}} \left[ \sum_{t \in \mathcal{T}_{*f}} c_{ft}^{k_f^a} \left( x_{f,p(k_f^a),k_f^a}^t - x_{f,p(k_f^a),k_f^a}^{t-1} \right) - \sum_{t \in \mathcal{T}_{*f}} c_{ft}^{k_f^d} \left( x_{f,k_f^d,q(k_f^d)}^t - x_{f,k_f^d,q(k_f^d)}^{t-1} \right) \right]$$

Note that objective function o7) can be easily derived from objective function o3).

### Capacity constraints

$$\sum_{f \in \mathcal{F}: k_f^d=k, t \in \mathcal{T}_f^{k,q(k)}} \left( x_{f,k,q(k)}^t - x_{f,k,q(k)}^{t-1} \right) \leq D_k^{t-1} \quad \forall t \in \mathcal{T}, k \in \mathcal{K}^d \quad (24)$$

$$\sum_{f \in \mathcal{F}: k_f^a=k, t \in \mathcal{T}_f^{p(k),k}} \left( x_{f,p(k),k}^t - x_{f,p(k),k}^{t-1} \right) \leq R_k^{t-1} \quad \forall t \in \mathcal{T}, k \in \mathcal{K}^a \quad (25)$$

$$\sum_{f \in \mathcal{F}} \left( \sum_{n \in \mathcal{N}_f^{j+}} \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^t - \sum_{n \in \mathcal{N}_f^{j-}} \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^t \right) \leq S_j^t \quad \forall t \in \mathcal{T}, j \in \mathcal{J} \quad (26)$$

### Flight structure constraints

$$\sum_{m \in \Gamma_f^+(n)} x_{f,n,m}^{t+\underline{\ell}_{f,n,m}} \leq \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^t \leq \sum_{m \in \Gamma_f^+(n)} x_{f,n,m}^{t+\bar{\ell}_{f,n,m}} \quad \forall f \in \mathcal{F}, n \in \mathcal{N}_f \setminus \{k_f^d, k_f^a\}, t \in \mathcal{T}_f^n \quad (27)$$

$$x_{f,k_f^d,q(k_f^d)}^{T_f^{k_f^d,q(k_f^d)}} - x_{f,p(k_f^a),k_f^a}^{T_f^{p(k_f^a),k_f^a}} = 0 \quad \forall f \in \mathcal{F} \quad (28)$$

$$x_{f,m,n}^{t-1} - x_{f,m,n}^t \leq 0 \quad \forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f, t \in \mathcal{T}_{*f}^{m,n} \quad (29)$$

$$x_{f,m,n}^{t-1} - x_{f,m,n}^t = 0 \quad \forall f \in \mathcal{F}, (m, n) \in \mathcal{A}_f, t \in \mathcal{T}_f^{m,n} \setminus \mathcal{T}_{*f}^{m,n} \quad (30)$$

$$x_{f,k_f^d,q(k_f^d)}^t - x_{f',p(k_f^a),k_f^a}^{t-1-s_{f'}} \leq 0 \quad \forall (f', f) \in \mathcal{C}, t \in \mathcal{T}_f^{k_f^d,q(k_f^d)} \quad (31)$$

$$x_{f,k_f^d,q(k_f^d)}^t - x_{f,p(k_f^a),k_f^a}^{t+r_f-d_f+A_f-1} \leq 0 \quad \forall f \in \mathcal{F}, t \in \mathcal{T}_f^{k_f^d,q(k_f^d)} \quad (32)$$

### Delay constraints

$$\begin{aligned} & \sum_{t \in \mathcal{T}_f^{m,n}} t(x_{f,m,n}^t - x_{f,m,n}^{t-1}) - \\ & \sum_{m' \in \Gamma_f^-(m)} \sum_{t \in \mathcal{T}_f^{m',m}} t(x_{f,m',m}^t - x_{f,m',m}^{t-1}) - \\ & \ell_{f,m,n} x_{f,m,n}^+ - \delta_{f,m,n}^+ + \delta_{f,m,n}^- = 0 \quad \forall f \in \mathcal{F}, (m,n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\} \end{aligned} \quad (33)$$

$$\begin{aligned} & \sum_{t \in \mathcal{T}_f^{p(k_f^a),k_f^a}} t(x_{f,p(k_f^a),k_f^a}^t - x_{f,p(k_f^a),k_f^a}^{t-1}) - \\ & \sum_{t \in \mathcal{T}_f^{k_f^d,q(k_f^d)}} t(x_{f,k_f^d,q(k_f^d)}^t - x_{f,k_f^d,q(k_f^d)}^{t-1}) - \\ & (r_f - d_f - 1)x_{f,p(k_f^a),k_f^a}^+ - \alpha_f^+ + \alpha_f^- = 0 \quad \forall f \in \mathcal{F} \end{aligned} \quad (34)$$

### Bounds of the variables

$$\delta_{f,m,n}^+ \in [0, \bar{\ell}_{f,m,n} - \ell_{f,m,n}] \quad \forall f \in \mathcal{F}, (m,n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\} \quad (35)$$

$$\delta_{f,m,n}^- \in [0, \ell_{f,m,n} - \underline{\ell}_{f,m,n}] \quad \forall f \in \mathcal{F}, (m,n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\} \quad (36)$$

$$\alpha_f^+, \alpha_f^- \geq 0 \quad \forall f \in \mathcal{F} \quad (37)$$

$$x_{f,m,n}^t \in \{0, 1\} \quad \forall f \in \mathcal{F}, (m,n) \in \mathcal{A}_f, t \in \mathcal{T}_f^{m,n} \quad (38)$$

Constraints (24) ensure that the number of flights departures from airport  $k$  at time period  $t$  will not exceed its departure capacity at that period. Constraints (25) ensure that the number of flights arrivals to airport  $k$  at time period  $t$  will not exceed its arrival capacity at that period. Constraints (26) ensure that the number of flights in air sector  $j$  at time period  $t$  will not exceed its sector capacity at that period. Constraints (27) force connectivity between nodes. Constraints (28) ensure that a flight arrives to its destination if it departs from its departure airport and vice versa; otherwise, the flight is cancelled. Constraints (29) and (30) force time continuity. Constraints (31) force connectivity between airports to handle the cases where a flight is continued. Constraints (32) ensure that a flight arrives to its destination not exceeding its maximum time on the air; otherwise, the flight is cancelled. Constraints (33) ensure that the difference between the actual time length in a given arc along the flight route and the scheduled one ( $\ell_{f,m,n}$ ) gives the positive or negative value, if the flight is not cancelled. Constraints (34) ensure the the difference between the actual flight arrival time and the actual departure time is equal, if the flight is not cancelled, to the scheduled flight duration plus the positive or negative deviation on the scheduled time. Constraints (35) and (36) give the range for the difference between the actual time length and the scheduled one in any arc of the flight route. Constraints (37) ensure the  $\alpha$  variables to be non-negative. Constraints (38) ensure that the  $x$  variables are  $\{0, 1\}$ .

## 3 Uncertainty in Air Traffic Control

### 3.1 Motivation

The Air Traffic Management problem is especially sensitive to changes in capacity. Goi et al. (2009) [36] have clearly noted an impact from take off into the sectors capacity and on delays. But one common feature of all the presented methodologies is that they consider that the capacity of the elements of the problem (airports and airsectors) is perfectly well-established at the start of the time planning horizon under consideration. But a more realistic view of the problem leads us to believe that this is not so, and that throughout the planning horizon variations can arise in the capacity of the airports or air sectors, thus invalidating the solution proposed initially. These variations are usually due to changes in weather conditions and on some occasions involve a decrease of more than 50 per cent in the capacity of certain airports. Richetta and Odoni (1993) [58], for the static case and Richetta and Odoni (1994) [59] for the dynamic case, present methodologies which deal with the stochastic case in SAGHP. A heuristic simulation model dealing with stochastic regards is presented in Wanke and Greenbaum (2007) [71]. See also Grignon (2002) [37], Nilim and El Ghaoui (2004) [52] and d'Aspremont et al. (2006) [26], among others.

This leads us to generalize the previous methodologies and to include generic uncertainty for these possible unforeseen changes in the parameters of the model, making way for stochastic methodologies. These can also be static or dynamic. They are considered to be dynamic if the solution is updated throughout the planning horizon, according to the situations that arise and static if the solution holds firm from the start of the planning horizon. Clearly, in this latter case stable solutions should be provided whatever the possible results may be.

In order to model this uncertainty, we can resort to stochastic optimization. This methodology allows us to represent the uncertainty by means of a set of possible scenarios that can arise. Each of these scenarios has a weight according to the certainty that the decision-maker has of its occurrence, see [6]. In stochastic methodology, the solution proposed must be such that all scenarios are considered and that it does not depend on any one of them in particular. This kind of solutions can consider different types of objective functions such as maximizing the expected objective function value over the scenarios (*neutral risk*), maximizing a composite mean-risk function included by the expected objective function and the weighted probability of maximizing the occurrence of scenarios whose objective function value is greater than a given threshold (*risk aversion*) and minimizing the difference between the value of the proposed solution and the air traffic management solution value for each scenario (*robust optimization*).

As previously shown, in this case the uncertainty is caused by possible variations that can occur in the maximum capacity for take-off and landing at airports, in the maximum amount of movement in the air sectors contemplated in the model and the density of flight traffic between several airports, see Alonso, Escudero and Ortuño (2000) [5].

For each airport or air sector and each period of time, the set of all possible representative scenarios that can arise is considered; this is characterized by the different values that the capacity of the sector can take during the period of time considered. A given weight is assigned to each of these scenarios, representing the likelihood which the decision-maker associates with the occurrence of the scenario that corresponds to a time-varying airport capacity profile. See

also Ball et al. (2003) [14].

A serious problem which arises is that the number of scenarios considered grows exponentially with the number of time periods. But even for a reduced number of periods, if the number of scenarios considered in each time period is high, the final number of scenarios will be excessively large. The problem considered in this way cannot be dealt with and it is necessary to reduce the number of scenarios. Therefore, it becomes necessary to find at a compromise solution, so that without considering an excessive number of scenarios, neither very representative scenarios nor those very likely to occur, will be eliminated, see Möeller, Römisch and Weber (2004) [50].

**Definition 1.** A *stage* of a given time horizon is a set of time periods where the realization of the uncertain parameters takes place.

**Definition 2.** A *scenario* is one realization of the uncertain parameters along the stages of the given time horizon.

**Definition 3.** A *scenario group* for a given stage is the set of scenarios with the same realization of the uncertain parameters up to the stage.

Let the following notation related to the scenario tree to be built from the set of scenario groups:

$\mathcal{T}$ , set of consecutive stages along the time horizon.  $\mathcal{T}^- \equiv \mathcal{T} - \{|\mathcal{T}|\}$ , being  $T$  the last stage in the time horizon.

$\Omega$ , set of scenarios.

$\mathcal{R}$ , set of scenario groups, so that we have a tree where  $\mathcal{R}$  is the set of nodes.

$\mathcal{R}_t$ , set of scenario groups in stage  $t$ , for  $t \in \mathcal{T}$  ( $\mathcal{R}_t \subseteq \mathcal{R}$ ).

$\Omega_r$ , set of scenarios in group  $r$ , for  $r \in \mathcal{R}$  ( $\Omega_r \subseteq \Omega$ ).

$t(r)$ , time period to whom scenario group  $r$  belongs to, for  $r \in \mathcal{R}$ .

$a(r)$ , immediate ancestor node of node  $r$ , for  $r \in \mathcal{R}$ .

$\mathcal{V}_r$ , set of ancestor scenario groups to group  $r$ , including itself, for  $r \in \mathcal{R}$ .

$\mathcal{V}^r$ , set of successors of scenario group  $r$ .

$w_r$ , weight factor representing the likelihood that is associated with scenario group  $r$ , for  $r \in \mathcal{R}$ . Note:  $w_r = \sum_{\omega \in \Omega_r} w^\omega$ , where  $w^\omega$  gives the likelihood that the modeler associates with scenario  $\omega$ , for  $\omega \in \Omega$ , and  $\sum_{\omega \in \Omega} w^\omega = 1$  and  $\sum_{r \in \mathcal{R}_t} w_r = 1 \forall t \in \mathcal{T}$ .

### 3.2 Features of the type of stochastic problem to model

The type of problems that is proposed to solve is the air traffic flow management problem (multi airports and multi airsectors) at the level of aircraft with rerouting, flight cancellations and continuations, with uncertainty in the airspace capacity along the given time horizon, for optimizing the expected value of the objective function.

A summary of the main features of the model that we have introduced in Agustin et al (2010b) [2] that we will denote as SATFM (Stochastic Air Traffic Flow Management) is as follows:

- Optimizing the Air Traffic Flow Management with Rerouting Problem (ATFMRP).
- Considering the uncertainty in the airport and sector capacity as well as the flight demand from the airlines, via scenario analysis.
- Allowing to advance in time the schedule of any flight, besides to delay it. Other models only have this last feature.
- Allowing to know in which waypoints the flight has suffered some delay.
- Allowing lower and upper bounds on the time an aircraft can be flying within two specific waypoints. This feature avoids that the model allows an excessive delay in between both waypoints that physically would be impossible to realize. The current models in the literature do not consider this feature and, then, they may allow an excessive delay in a small flight segment, something that perhaps the allowed speed of the aircrafts does not permit.
- Allowing a dynamic configuration of the sectors along the time horizon.

### 3.3 Stochastic Air Traffic Flow Management (SATFM)

Let the following additional notation to the one used in the previous models.

#### Input set

$\mathcal{F}^r$ , set of flights operating under scenario group  $r$ , such that  $\mathcal{F}^r \subseteq \mathcal{F}$ . This set is related to the flight demand by the air companies, since they can cancel flights after the schedule is presented for reasons outside the own scheduling proposal.

#### Uncertain input parameters

$D_k^r$ , departure capacity of airport  $k$  at time period  $t(r)$  under scenario group  $r$ ,  $\forall k \in \mathcal{K}, r \in \mathcal{R}$ .

$R_k^r$ , arrival capacity of airport  $k$  at time period  $t(r)$  under scenario group  $r$ ,  $\forall k \in \mathcal{K}, r \in \mathcal{R}$ .

$S_j^r$ , capacity of sector  $j$  at time period  $t(r)$  under scenario group  $r$ ,  $\forall j \in \mathcal{J}, r \in \mathcal{R}$ .

#### Variables

$x_{f,m,n}^r$ , 0-1 variable a la Bertsimas-Stock [18], such that its value is 1 if flight  $f$  is planned to arrive at node  $n$  on its flight path through the arc  $(m, n)$  by the end of time period  $t(r)$  under scenario group  $r$ .

$\alpha^{r+}, \alpha^{r-}$ , positive and negative time units difference, respectively, between the actual and scheduled air trip lengths for flight  $f$  under scenario group  $r$ ,  $\forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) = T_f^{p(k_f^a), k_f^a}$ .

$\delta_f^{r+}, \delta_f^{r-}$ , positive and negative time units difference, respectively, between the actual and scheduled air trip lengths in arc  $(m, n)$  for flight  $f$  under scenario group  $r, \forall r \in \mathcal{R}, f \in \mathcal{F}^r$  :  
 $t(r) = T_f^{m,n}, (m, n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\}$ .

## Model

### Objectives functions

The objective function to be minimized can be included by different terms, depending the goal of the decision maker. They can be combined, with an appropriate weight to create a multi-objective function to be optimized.

o1) Total expected cancelation cost.

$$\sum_{r \in \mathcal{R}} w_r \sum_{\substack{f \in \mathcal{F}^r: \\ t(r) = T_f^{k_f^d, q(k_f^d)}}} p_f^c (1 - x_{f, k_f^d, q(k_f^d)}^r)$$

For  $p_f^c = 1, \forall f \in \mathcal{F}$ , this expression represent the number of flights cancelled.

o2) Expected cost for using the arcs in the flights route for each time period.

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_f^{m,n}} \sum_{(m,n) \in \mathcal{A}_f} c_f^{m,n} x_{f,m,n}^r$$

Notice that if the scheduled route has a zero penalization, this expression represents the expected cost of using alternative flight routes

o3) Expected cost of the deviation of the actual arrival time from the scheduled one at each node in the flight route.

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r} \sum_{n \in \mathcal{N}_f \setminus k_f^d} \sum_{m \in \Gamma_f^-(n)} \sum_{t(r) \in T_f^{m,n}} c_{ft(r)}^n (x_{f,m,n} - x_{f,m,n}^{a(r)})$$

o4) Expected cost of the deviation of the actual flight duration at each arc from the scheduled one  $(\ell_{f,m,n})$  in the flights route

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_f^{m,n}} \sum_{(m,n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\}} (c_{f,m,n}^+ \delta_{f,m,n}^{r+} + c_{f,m,n}^- \delta_{f,m,n}^{r-})$$

o5) Expected cost of the deviation of the actual flight duration from the scheduled one.

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_f^{p(k_f^a), k_f^a}} (c_f^{a+} \alpha_f^{r+} + c_f^{a-} \alpha_f^{r-})$$

o6) Total expected number of flights exceeding the maximum time units of ground and air delay type 1:

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_f^{p(k_f^a), k_f^a}} (x_{f, p(k_f^a), k_f^a}^r - x_{f, p(k_f^a), k_f^a}^{r'})$$

where  $r' = \mathcal{V}_r \cap \mathcal{R}_{r_f + M_f}$

o7) Expected non-separable cost of delaying flight  $f$  for  $(t(r) - r_f)$  units and the reduced cost obtained by holding flight  $f$  in the ground for  $(t(r) - d_f)$  time units.

$$\sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_{*f}^{p(k_f^a), k_f^a}} c_{ft(r)}^{k_f^a} (x_{f,p(k_f^a), k_f^a}^r - x_{f,p(k_f^a), k_f^a}^{a(r)}) - \sum_{r \in \mathcal{R}} w_r \sum_{f \in \mathcal{F}^r: t(r) = T_{*f}^{k_f^d, q(k_f^d)}} c_{ft(r)}^{k_f^d} (x_{f,k_f^d, q(k_f^d)}^r - x_{f,k_f^d, q(k_f^d)}^{a(r)})$$

### Capacity constraints

$$\sum_{f \in \mathcal{F}^r: k_f^d = k, t(r) \in \mathcal{T}_f^{k, q(k)}} (x_{f,k, q(k)}^r - x_{f,k, q(k)}^{a(r)}) \leq D_k^{a(r)} \quad \forall r \in \mathcal{R}, k \in \mathcal{K}^d \quad (39)$$

$$\sum_{f \in \mathcal{F}^r: k_f^a = k, t(r) \in \mathcal{T}_f^{p(k), k}} (x_{f,p(k), k}^r - x_{f,p(k), k}^{a(r)}) \leq R_k^{a(r)} \quad \forall r \in \mathcal{R}, k \in \mathcal{K}^a \quad (40)$$

$$\sum_{f \in \mathcal{F}^r} \left( \sum_{\substack{n \in \mathcal{N}_f \cap \mathcal{N}^j: \\ \Gamma_f^-(n) \notin \mathcal{N}^j}} \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^r - \sum_{\substack{n \in \mathcal{N}_f \cap \mathcal{N}^j: \\ \Gamma_f^+(n) \notin \mathcal{N}^j}} \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^r \right) \leq S_j^r \quad \forall r \in \mathcal{R}, j \in \mathcal{J} \quad (41)$$

### Flight structure constraints

$$\sum_{\substack{m \in \Gamma_f^+(n) \\ r'' = \mathcal{V}_{r'} \cap \mathcal{R}_{t(r) + \underline{\ell}_{f,n,m}}} x_{f,n,m}^{r''} - \sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^r \leq 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r, n \in \mathcal{N}_f \setminus \{k_f^d, k_f^a\} : t(r) \in \mathcal{T}_f^n, r' \in \mathcal{V}^r \cap \mathcal{R}_{t'} \quad (42)$$

where  $t' = t(r) + \max_{m \in \Gamma_f^+(n)} \underline{\ell}_{f,n,m}$

$$\sum_{m \in \Gamma_f^-(n)} x_{f,m,n}^r - \sum_{\substack{m \in \Gamma_f^+(n) \\ r'' = \mathcal{V}_{r'} \cap \mathcal{R}_{t(r) + \bar{\ell}_{f,n,m}}} x_{f,n,m}^{r''} \leq 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r, n \in \mathcal{N}_f \setminus \{k_f^d, k_f^a\} : t(r) \in \mathcal{T}_f^n, r' \in \mathcal{V}^r \cap \mathcal{R}_{t'} \quad (43)$$

where  $t' = t(r) + \max_{m \in \Gamma_f^+(n)} \bar{\ell}_{f,n,m}$

$$x_{f,k_f^d, q(k_f^d)}^{r'} - x_{f,p(k_f^a), k_f^a}^r = 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) = T_f^{p(k_f^a), k_f^a} \text{ where } r' = \mathcal{V}_r \cap \mathcal{R}_{T_f^{k_f^d, q(k_f^d)}} \quad (44)$$

$$x_{f,m,n}^{a(r)} - x_{f,m,n}^r \leq 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r, (m, n) \in \mathcal{A}_f : t(r) \in \mathcal{T}_{f*}^{m,n} \quad (45)$$

$$x_{f,m,n}^{a(r)} - x_{f,m,n}^r = 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r, (m, n) \in \mathcal{A}_f : t(r) \in \mathcal{T}_f^{m,n} \setminus \mathcal{T}_{f*}^{m,n} \quad (46)$$

$$x_{f,k_f^d, q(k_f^d)}^r - x_{f',p(k_f^a), k_f^a}^{r'} \leq 0 \quad \forall r \in \mathcal{R}, (f', f) \in \mathcal{C} : \{f \in \mathcal{F}^r, f \in \mathcal{F}^{r'} \text{ and } t(r) \in \mathcal{T}_f^{k_f^d, q(k_f^d)}\} \quad (47)$$

where  $r' = \mathcal{V}_r \cap \mathcal{R}_{t(r) - 1 - s_{f'}}$

$$x_{f,k_f^d, q(k_f^d)}^{r'} - x_{f,p(k_f^a), k_f^a}^r \leq 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : \{t(r) \in \mathcal{T}_f^{p(k_f^a), k_f^a}, t(r') \in \mathcal{T}_f^{k_f^d, q(k_f^d)}\} \text{ where } r' = \mathcal{V}_r \cap \mathcal{R}_{t(r) - (r_f - d_f + A_f - 1)} \quad (48)$$



## Delay constraints

$$\begin{aligned}
& \sum_{r' \in \mathcal{R}: t(r') \in \mathcal{T}_f^{m,n}} t(r')(x_{f,m,n}^{r'} - x_{f,m,n}^{a(r')}) - \\
& \sum_{m' \in \Gamma_f^-(m)} \sum_{r' \in \mathcal{R}: t(r') \in \mathcal{T}_f^{m',m}} t(r')(x_{f,m',m}^{r'} - x_{f,m',m}^{a(r')}) - \\
& \quad \ell_{f,m,n} x_{f,m,n}^r - \delta_{f,m,n}^{r+} + \delta_{f,m,n}^{r-} = 0 \\
\forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) = \mathcal{T}_f^{m,n}, (m, n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\}
\end{aligned} \tag{49}$$

$$\begin{aligned}
& \sum_{r' \in \mathcal{R}: t(r') \in \mathcal{T}_f^{p(k_f^a), k_f^a}} t(r')(x_{f,p(k_f^a), k_f^a}^{t(r')} - x_{f,p(k_f^a), k_f^a}^{a(r)}) - \\
& \sum_{r' \in \mathcal{R}: t(r') \in \mathcal{T}_f^{k_f^d, q(k_f^d)}} t(r')(x_{f,k_f^d, q(k_f^d)}^{t(r')} - x_{f,k_f^d, q(k_f^d)}^{a(r)}) - \\
& (r_f - d_f - 1)x_{f,p(k_f^a), k_f^a}^r - \alpha_f^{r+} + \alpha_f^{r-} = 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) = \mathcal{T}_f^{p(k_f^a), k_f^a}
\end{aligned} \tag{50}$$

## Bounds of the variables

$$\delta_{f,m,n}^{r+} \in [0, \bar{\ell}_{f,m,n} - \ell_{f,m,n}] \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) \in \mathcal{T}_f^{m,n}, (m, n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\} \tag{51}$$

$$\delta_{f,m,n}^{r-} \in [0, \ell_{f,m,n} - \underline{\ell}_{f,m,n}] \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) \in \mathcal{T}_f^{m,n}, (m, n) \in \mathcal{A}_f \setminus \{(k_f^d, q(k_f^d)), (p(k_f^a), k_f^a)\} \tag{52}$$

$$\alpha_f^{r+}, \alpha_f^{r-} \geq 0 \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r : t(r) = \mathcal{T}_f^{p(k_f^a), k_f^a} \tag{53}$$

$$x_{f,m,n}^r \in \{0, 1\} \quad \forall r \in \mathcal{R}, f \in \mathcal{F}^r, (m, n) \in \mathcal{A}_f \tag{54}$$

Constraints (39)-(41) ensure that the capacity of the flight departures from airports, flight arrivals to airports and flights arrivals to each air sector do not exceed the related capacities at each scenario group. Constraints (42)-(47) force the connectivity between nodes, airports for continued flights and time periods of a given flight, respectively. Constraints (44) ensure that the flight will land if it has taken off, otherwise it is cancelled. Constraints (48) ensure that a flight arrives to its destination not exceeding its maximum allowed time in the air, if it is not cancelled. Constraints (49)-(50) define the differences between the actual duration in a given arc and the trip duration, respectively, versus the scheduled ones for each scenario group. Constraints (51)-(52) ensure the range of the  $\delta$  variables.

## 4 Conclusions

A state-of-the-art survey for Air Traffic Flow Management optimization models has been presented. Simplest but useful models to more realistic ones are considered, which also happens to present them in a chronological order. Most of the research has been carried out during the last

20 years, which coincides with the explosion of computer power. This explosion has been well utilized since the magnitude of the size of the models is very big. The large scale size of the models for air traffic flow management with rerouting under uncertainty is to be pointed out. The uncertainty is due to the stochasticity in the capacity of the departure and arrival airports and the air sectors as well as the flight availability along a given daily time horizon. In summary, we can say that today modelling and algorithmic developments together with the current computer power make this discipline to be ready for using it in real air operations.

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