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Cost Efficient Equitable Water Distribution in Algeria: A Bi-criteria Fair Division Problem with Network Constraints

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Abstract

We describe a complex water distribution problem as a bi-criteria fair division problem over time with network constraints: we aim at distributing water fairly in a reliable and cost-efficient manner. The problem involves both the optimization of the pump operational schedules, as well as strategic planning. Complex rules establish energy fares depending on the daytime and the contractual issues of the pump facility. The problem is illustrated for the region of Kabylia, Algeria. We discuss the relevance and implementation of different solution concepts in this context, showing various alternatives which improve upon current management procedures.

Keywords: Water distribution, Fair division, Cost efficiency, Bi-criteria optimization, Pump scheduling.

1 Introduction

This study presents a real water distribution case in a rural area within the region of Kabylia, Algeria, supported by the Spanish Cooperation and Development Agency. Initially, the problem may be described as a relatively standard water distribution problem, with water coming from various wells, with several intermediate water deposits and pumping stations and consumption taking place in villages, with a very disperse population. However, specially in the summer, when less water is available and population is almost doubled, there is a considerable water scarcity, which is aggravated by significant water losses in the network. Unfortunately, the current management system in place seems to privilege some villages, in that water scarcity tends to affect certain villages for too long periods of time, and this has caused political unrest. A second feature of interest in this problem stems from the cultural traditions in the region. Traditionally, the Kabylians have preferred to live up in the mountains, and this creates important engineering problems, with a very high proportion of water distribution costs due to electricity consumption in pumping the water.

In a former paper, Udías et al. (2011), we performed a detailed data and design analysis of the current network. The inclusion of new pipes, tank sizing and placing, and pump operational scheduling were considered as design variables. We essentially concluded that the problem was not one of water availability but it was mainly produced by water losses, including water thefts, and inappropriate distribution schedules. Thus, unless willing to introduce important infrastructure investments, there was a need to distribute water in a much more equitable and economical fashion. We initially dealt with this bi-criteria problem in two phases, assuming a lexicographic approach, see e.g. Yu et al. (1985), as the first objective (maximize equity) seems much more important to the current management than the second one (minimize cost). Note that, as usual with water systems, see e.g de Neufville et al. (1971) or Walski et al. (1987), we face a multi-objective problem.

Therefore, in the first phase, we aim at determining an equitable water distribution schedule for the region, taking into account various distribution constraints. Somehow, equity is in the eye of the beholder. Indeed, our initial aim was to implement an objective function, required by the water distributors, which reflected the need that all inhabitants would receive the same volume of water. However, upon reflection, we realized that we could implement several other objective functions, many of them available from the bargaining literature, see e.g. Raiffa et al. (2007). The problem may be viewed, thus, as a fair division problem with network constraints: we need to divide fairly a good (the available water over a period of time), and distribute it efficiently. See Brams and Taylor (1996) for an introduction to fair division. There are also connections with the bankruptcy problem, see Aumann and Maschler (1985), although we need to operate over time with network constraints, as main differences.

Once we have obtained a fair scheme, which is, we insist, the most important objective by far for the management, we try to implement it in the less costly way, taking into account the time varying costs of pumping water, because of changes in electricity fares according to a daily schedule. This problem may be formulated as a mixed linear-integer optimization model.

In this way, we identify the tradeoffs between the total pumping cost and the satisfaction of water demand in the Kabylia water distribution system.

The structure of the paper is as follows. First, we formulate in Section 2 the problem as presented by the water company, with an objective function aimed at providing the same amount of water to each inhabitant. We then provide a discussion of several other potential objective functions, which might be used to obtain equitable water distribution schedules, and discuss their pros and cons. Once with our equitable schedule, we discuss in Section 3 how to provide it in the most cost effective manner. We discuss then how to generate Pareto efficient schedules which improve upon the lexicographic schedules identified and facilitate a powerful management tool to the water distributor. We illustrate these issues in Section 4 with our motivating case referring to the region of Kabylia, presenting the results of our multi-objective analysis. We end up with a discussion.

2 Equitable Water Distribution

Water distribution problems may be described in relatively simple terms, see e.g. Soncini-Sessa et al. (2007). We have a number of water sources, usually wells and reservoirs, which together provide the water offer. We also have a number of water consumption points, which, depending on the granularity of the problem, might refer to houses or villages. We aim at satisfying water demand, given the water offer, taking into account a number of physical and engineering constraints, reflecting the structure of the distribution network, typically with intermediate deposits and pumping stations.

It could be the case that, given the demand, the offer and the distribution network features, we are not able to satisfy completely the demand, and we, thus, look for ways of making this unsatisfied demand as balanced as possible among consumers. We call this problem *equitable water distribution*. Figure 1 provides a simple scheme to which we shall refer later on, which reflects the structure of the problem used in Section 4 with only wells as source points and deposits as intermediate points.

2.1 Decision Variables and Constraints

We shall consider the operation of the water distribution system over a time T. The operating time will be designated t, with $t \in \{1, 2, \ldots, T\}$. Assume there are N_p source points, designated by $i \in \mathcal{P}$; N_d intermediate points, designated by $j \in \mathcal{D}$; N_v consumption points, designated by $k \in \mathcal{V}$; N_w^i well pumps (pumping water from the source point i to the network), designated

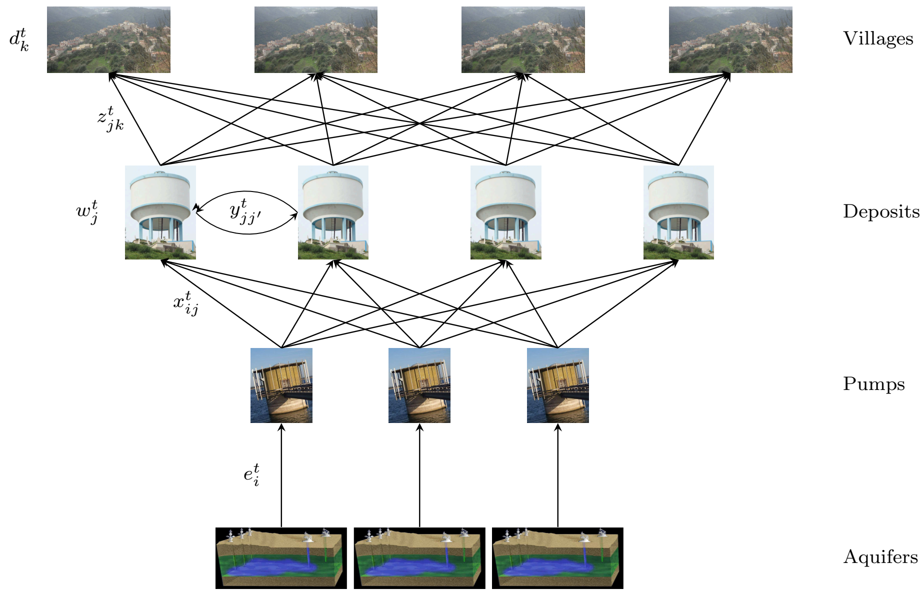

Figure 1: Water distribution network

by $w \in \mathcal{W}^i,\ i \in \mathcal{P}$; and, finally, N_s^j station pumps (pumping water from the initial pump station j to the deposits), designated by $s \in \mathcal{S}^j,\ j \in \mathcal{D}$. At each time t:

- We define integer variables I_w^t and I_s^t to describe the state of the pumps,

$$I_w^t = \begin{cases} 1, & \text{if the pump } w \in \mathcal{W}^i,\ i \in \mathcal{P} \text{ is functioning at time } t, \\ 0, & \text{otherwise,} \end{cases}$$

  and similarly for $I_s^t, s \in \mathcal{S}^j,\ j \in \mathcal{D}$.

- We decide to extract a volume of water e_i^t from the i-th source point, with $0 \leq e_i^t \leq \varepsilon_i^t$, where $\varepsilon_i^t > 0$ is the maximum water extraction capacity from the i-th source point at time t.

- The water stored at the j-th intermediate point is w_j^t, with $0 \leq w_j^t \leq \omega_j$, where $\omega_j > 0$ is the maximum water storage at j. Note, though, that the cost of supplying water from storages is always cheaper than pumping it from aquifers. We should, however, be careful as we could be tempted to extract too much water from the deposits, rendering them impossible to be refilled for the next period. We must ensure that jeopardizing the supplies for the future, in exchange for a short-term gain, will not occur. To do so, we should impose an operational constraint on each deposit, requiring them to have a minimum prescribed water level $\varpi_j \geq 0$ at the beginning of each period, leading actually to the constraint

$$\varpi_j \leq w_j^t \leq \omega_j. \tag{1}$$

- We decide to derive a volume of water x_{ij}^t from the i-th source point towards the j-th intermediate point, with

  $$0 \leq x_{ij}^t \leq \phi_{ij}, \tag{2}$$

  being ϕ_{ij} the maximum water flow capacity for the conduction between the source point i and the intermediate point j.

- The capacities Q_w of the N_w well pumps do not vary with time. Thus, the following condition must hold

  $$\sum_{j \in \mathcal{D}} x_{ij}^t = \sum_{w \in \mathcal{W}^i} I_w^t \cdot Q_w, \quad \forall i \in \mathcal{P}, \ \forall t. \tag{3}$$

- We decide to release a volume of water $y_{jj'}^t$ from the intermediate point j towards the intermediate point j', with

  $$0 \leq y_{jj'}^t \leq \xi_{jj'}, \tag{4}$$

  being $\xi_{jj'}$ the maximum water flow capacity between j and j'.

- The capacities Q_s of the N_s station pumps do not vary with time. Thus, the following condition must hold

  $$\sum_{j' \in \mathcal{D}} y_{jj'}^t = \sum_{s \in \mathcal{S}^j} I_s^t \cdot Q_s, \quad \forall j \in \mathcal{D}, \ \forall t. \tag{5}$$

- We decide to release a volume of water z_{jk}^t from the intermediate point j to the consumption point k, with

  $$0 \leq z_{jk}^t \leq \zeta_{jk}, \tag{6}$$

  being ζ_{jk} the maximum water flow capacity for the conduction between j and k;

- The consumption point k will demand a volume d_k^t of water, which will be assumed known in this paper.

Note that, for the generality of notation, we have described a fully connected distribution network with links between all intermediate points; links between each source point and every intermediate point and, finally, links between each intermediate point and every consumption point. Links that are not available may be described through a zero upper bound.

The following additional constraints will hold in our problem:

- The amount of water pumped from each source point cannot exceed its maximum offered value

$$e_i^t = \sum_{j \in \mathcal{D}} x_{ij}^t \leq \varepsilon_i, \quad \forall i \in \mathcal{P},\ \forall t. \tag{7}$$

- Mass continuity condition for intermediate points, describing that the variation in the water stored at the j-th intermediate point between times $t-1$ and t equals the balance between the inflows coming from source points and the releases towards consumption points

$$w_j^t = w_j^{t-1} + \sum_{i \in \mathcal{P}} x_{ij}^t + \sum_{j' \in \mathcal{D}} y_{j'j}^t - \sum_{j' \in \mathcal{D}} y_{jj'}^t - \sum_{k \in \mathcal{V}} z_{jk}^t, \quad \forall j \in \mathcal{D},\ \forall t. \tag{8}$$

- Demand satisfied at each consumption point. We aim at achieving the target demand d_k^t. For this, we introduce slacks

$$d_k^t = \sum_{j \in \mathcal{D}} z_{jk}^t - s_k^t + \delta_k^t, \quad \forall k \in \mathcal{V},\ \forall t. \tag{9}$$

  where $s_k^t, \delta_k^t \geq 0$ represent, respectively, the water surplus and deficit at consumption point k and time t. Both slacks cannot be positive simultaneously. To achieve this, we may introduce the constraint $s_k^t \cdot \delta_k^t = 0$, although this constraint may be removed, depending on the objective function introduced.

A distribution schedule will be defined through the vector

$$\left\{(x_{ij}^t, y_{jj'}^t, z_{jk}^t)\right\}_{t=1}^T = (\boldsymbol{x}_{ij}, \boldsymbol{y}_{jj'}, \boldsymbol{z}_{jk}) = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}).$$

We shall denote the above constraints through $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}$.

2.2 Objective Function

We describe now various potential objective functions referring to equity in water distribution. We shall start by considering the objective function originally proposed by the water distribution company and, then, several alternatives. We shall assume that, for each village, its utility is minus the per capita water deficit inflicted by our water distribution policy. Each village aims at minimizing its per capita water deficit and, collectively, we aim at balancing such deficit.

2.2.1 Egalitarian Solution

The initial objective function suggested by the water distribution company aimed at inflicting the same water deficit to all inhabitants and minimizing such deficit. The deficit for the k-th consumption point over the whole planning period is $\Delta_k = \sum_{t=1}^{T} \delta_k^t$. If N_k is the population of the k-th consumption point, the per capita water deficit is $L_k = \Delta_k / N_k$. As required by the water distribution company, we would aim at solving

$$\begin{array}{ll} \min & L_k \\ \text{s.t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \\ & L_k = L_{k'}, \quad \forall k, k' \in \mathcal{V}. \end{array} \tag{10}$$

This corresponds to the egalitarian solution in arbitration, see e.g. Raiffa et al. (2007). This leads us to think about considering other arbitration solution concepts.

2.2.2 Smorodinsky-Kalai Solution

Intimately related with the above concept is the Smorondinsky-Kalai solution, see e.g. Kalai (1977) or Alexander (1992). The problem has a min-max formulation which is reformulated to a problem closely related to (10). Indeed, we formulate it as

$$\begin{array}{ll} \min \max_k & L_k \\ \text{s.t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \end{array}$$

which we reformulate as

$$\begin{array}{ll} \min & \mu \\ \text{s.t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \\ & L_k \leq \mu, \quad \forall k \in \mathcal{V}. \end{array} \tag{11}$$

2.2.3 Nash Solution

Another important concept is Nash solution, see e.g. Nydegger and Owen (1974), Alexander and Ledermann (1994) or Mariotti (1999), which in our context would be formulated in two equivalent ways, the second one being more efficient from a computational point of view:

$$\begin{aligned} \max \quad & \prod_{k \in \mathcal{V}} (-\alpha_k L_k + \beta_k) \\ \text{s.t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \end{aligned}$$

or, equivalently,

$$\begin{aligned} \max \quad & \sum_{k \in \mathcal{V}} \log(-\alpha_k L_k + \beta_k) \\ \text{s.t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}. \end{aligned} \tag{12}$$

α_k and β_k are appropriate constants making positive the utilities.

2.2.4 Utilitarian Solution

Finally, a fourth important arbitration concept is the utilitarian one, see Ponsati and Watson (1997), which aims at maximizing the sum of attained utilities (in this case, minimizing the sum of deficits)

$$\begin{aligned} \min \quad & \sum_{k \in \mathcal{V}} L_k \\ \text{s.t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}. \end{aligned}$$

Equivalently, we would aim at maximizing the average utility, i.e., minimizing the average water deficit received at the consumption points

$$\begin{aligned} \min \quad & \frac{1}{N_v} \sum_{k \in \mathcal{V}} L_k \\ \text{s.t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}. \end{aligned} \tag{13}$$

2.2.5 Variance Solution

Additionally, though not usually included as an arbitration solution, we could aim at minimizing the standard deviation of the water deficits to the villages over time. Thus, we would be solving the problem:

$$\begin{aligned} \min \quad & IV = \sqrt{\frac{1}{N_v} \sum_{k \in \mathcal{V}} L_k^2 - \left(\frac{1}{N_v} \sum_{k \in \mathcal{V}} L_K \right)^2} \\ \text{s.t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}. \end{aligned} \tag{14}$$

2.3 Inter and Intra Equity

The above approaches provide schedules which aim at reflecting equity among consumption points, taking a global view over the period T. However, it could be the case that deficits for various villages are balanced, i.e., deficits among consumers are somehow similar, but the deficit at, at least one of the consumers, is unbalanced over time. As an example, for a given consumption point k, the per capita water deficits would be

$$\frac{1}{N_k}\left(\delta_k^1, \ldots, \delta_k^T\right),$$

which, compared with the values received by other consumption points might seem equitable. However, it could be the case that the above quantities are very unequal, thus leaving the consumption point with little water for some days, with much bigger volumes of water for others. Therefore, we need to consider both the intervariability, defined above through IV, and the intravariability, defined as follows:

$$V_k = \frac{1}{N_k}\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\delta_k^t\right)^2 - \left(\frac{1}{T}\sum_{t=1}^{T}\delta_k^t\right)^2}.$$

The average intervariability would be

$$iv = \frac{1}{N_v}\sum_{k\in\mathcal{V}} V_k.$$

Then, we could aim at maximizing the water delivered, or minimizing the deficit, and minimizing both the intra and intervariability, by considering the problem

$$\begin{array}{ll} \min & \alpha \sum L_k + \beta IV + \gamma iv \\ \text{s.t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \end{array}$$

for appropriate weights α, β, γ reflecting the importance we give to various objectives.

3 Cost Effective Equitable Water Distribution

The previous section described various models leading to equitable distribution schedules in that they minimize deficits in a balanced way among consumers over time. Once we have chosen the schedule, we discuss now how to distribute it in the most cost effective manner, taking into account that pumping costs vary according to a daily schedule. This issue is specially important in our incumbent case study, as pumping costs amount to the largest share of the distribution costs.

To do so, we divide a given period t in m parts. We assume that, at the l-th part of the period, the distribution costs are $c_l,\ l = 1, \ldots, m$. The schedule at a given period t is then $\left(x^t_{ij}, y^t_{jj'}, z^t_{jk}\right)$, which can be expressed, when considered the various parts of each period through

$$\left(x^{tl}_{ij}, y^{tl}_{jj'}, z^{tl}_{jk}\right)^m_{l=1}.$$

The pumping cost to be minimized at each period t would then be

$$\Phi(x, y, z) \equiv \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{D}} \sum_{l=1}^{m} x^{tl}_{ij} c_l + \sum_{j,j'\in\mathcal{D}} \sum_{l=1}^{m} y^{tl}_{jj'} c_l + \sum_{j\in\mathcal{D}} \sum_{k\in\mathcal{V}} \sum_{l=1}^{m} z^{tl}_{jk} c_l,$$

subject to the constraints

$$\sum_{l=1}^{m} x^{tl}_{ij} = x^t_{ij},$$
$$\sum_{l=1}^{m} y^{tl}_{jj'} = y^t_{jj'},$$
$$\sum_{l=1}^{m} z^{tl}_{jk} = z^t_{jk},$$

$\forall i \in \mathcal{P},\ \forall j, j' \in \mathcal{D},\ \forall k \in \mathcal{V}$, together with the pertinent constraints in relation to maximum storage and maximum pumping capacities and continuity conditions, mentioned above. Specifically, conditions (1)– (9) must hold at each time period. Solving this optimization problem provides the cheapest pumping schedule fulfilling the equity requirements.

3.1 Pareto Efficient Water Distribution Schedules

Through an example we have seen that various equitable concepts may lead to very different schedules in terms of cost and equity. Moreover, the one chosen by the water distribution company was easily beaten by other criteria.

This suggests the possibility of approximating a Pareto frontier with two criteria, one referring to equity, the other by cost. We shall do this with an ϵ-constraint approach, by adjusting the level of the equity measure, finding then the most equitable schedule, and, then, the least expensive implementation of such schedule. As an example, if we consider the Smorodinsky-Kalai solution, we would solve for several ρ_k values the $P(\rho_k)$ problem

$$\begin{array}{ll} \min & \mu \\ \text{s.t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{C}, \\ & L_k \leq \mu, \quad \forall k \in \mathcal{V}, \\ & \mu \geq \rho_k, \end{array}$$

which would provide an optimal schedule $(x^{\rho_k}, y^{\rho_k}, z^{\rho_k})$ with equity measure μ_{ρ_k}, which would then lead to the optimal cost $\Phi(x^{\rho_k}, y^{\rho_k}, z^{\rho_k}) = \Phi(\rho_k)$. The use of such Pareto frontier is a powerful management tool for the water distributor in that, for a given distribution cost, the company may find the most equitable distribution schedule, and viceversa.

4 Case Study: Cost-Efficient Equitable Water Distribution in Kabylia

Kabylia is in the North-east part of Algeria, bounded in the north by the Mediterranean Sea, in the east by the region of Bedjaia, in the West by the region of Boumerdes, and in the South by the region of Bouira. The total area of the region is 2957 km^2, 80% of which lies in slopes with inclinations greater than 12%. It is composed of 67 municipalities (1380 villages) with a total population of more than 1.1 million people.

We have developed a model for a portion of the total distribution network in Kabylia, the so-called "Chaine de Tassadort", with 110,000 people living in 90 villages. Most of the population is located in mountainous areas with altitudes over 900 m in some cases. The average annual rainfall is around 900 mm/m^2. The Company *L'Algerienne des Eaux* (ADE) supplies water in the region. In 2009 the reported volume of water supplied amounted to 6,500,559 m^3, but only 1,696,294 m^3 (26% !!) was billed, a huge loss which results in a significant shortfall. These losses are mainly due to leakages, thefts and outdated pipes and components in the networks. Figure 2 shows the general layout of the Tassadort water distribution network. Given the current distribution system and the scarcity of water, it is frequently the case that several villages end up having no water for an extended period, say of a week, specially over the summer. This creates many inconveniences, including the need to transport water by truck to certain villages, social unrest, etc.

The network is mainly fed (72%) by groundwater through several wells that extract water from the Sebaou aquifer, specifically from:

- 9 wells in the field of Bouaid with a total pumping capacity of 1,040 m^3 h^{-1};
- 5 wells in the field of Takhoukht with a total pumping capacity of 140 m^3 h^{-1}; and
- Surface water drawn from the Taksebt reservoir, an alternative source, which provided 295,347 m^3 in the first quarter of 2010.

The network also includes 38 deposits (with a total storage capacity of 28,000 m^3), 6 pumping stations (with 22 fixed rate pumps and a total pumping capacity of 5,400 m^3 h^{-1}), and a main network of pipes spreading over various hundreds of kilometers, communicating all points in the network. The daily average consumption is, approximately, 19.000 m^3. The peak demand within a one hour period is estimated to be around a 6% of the daily global demand.

4.1 Outline of Methodology Used

The choice of the time-step used in the optimization problem is crucial for the computational burden of the problem. The optimization period is denoted by T, and we have discretized it into n steps. Different step lengths $t = T/n$ may be considered, but we have found a fairly conservative time-step of 1 hour to be a reasonable choice, as a trade-off between what would be desirable in real-time scheduling and the need of completing computations before the next update.

While it is possible to envisage a rolling operating horizon longer than 24 hours in those places where the storage available is exceptionally large, most water-distribution networks operate on a 24 hours-cycle basis, refilling the tanks at night, and pumping water from them during daytime. However, we have set a 48 hours simulation period in springtime to consider two consecutive days with significant variations in demand (with an average demand of 150 liters per day per person), assuming losses in the network. This would imply that for every decision variable, e.g. a transportation arc, we have to define 48 additional variables corresponding to the water volume transported at each time period. Smaller step lengths would increase in a prohibitive manner the computational burden, due to the presence of integer variables and nonlinear terms in the objective function of the Nash solution.

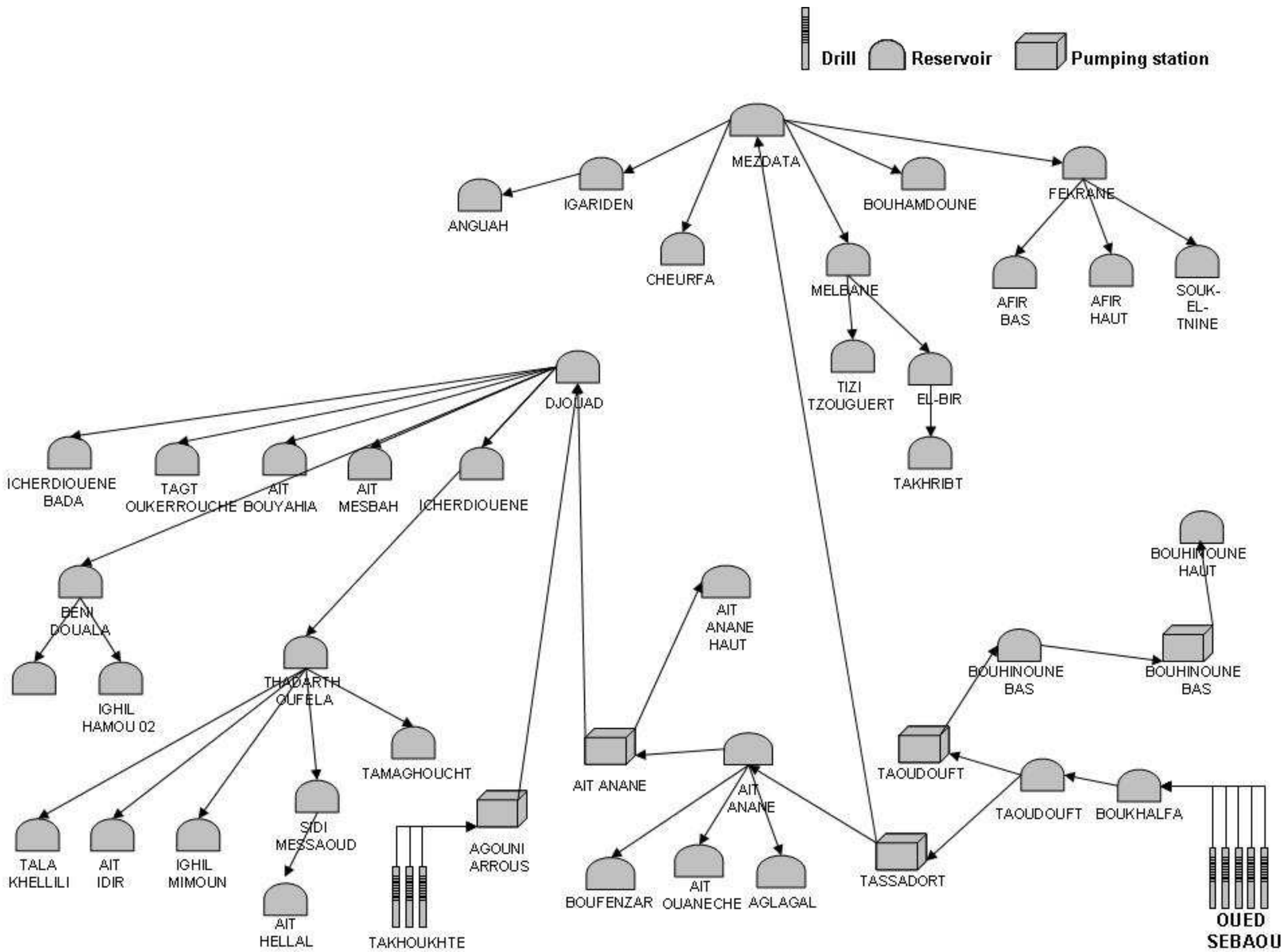

Figure 2: Water distribution network at Tassadort

Water demand variations occur on a daily basis, and are modeled through a demand curve, which plots the percentage of daily per capita demand versus time (Figure 3). In this approach, we assume the same daily water demand at every village. Different scenarios were generated varying the water demand and the infrastructure available on the network to check for robustness.

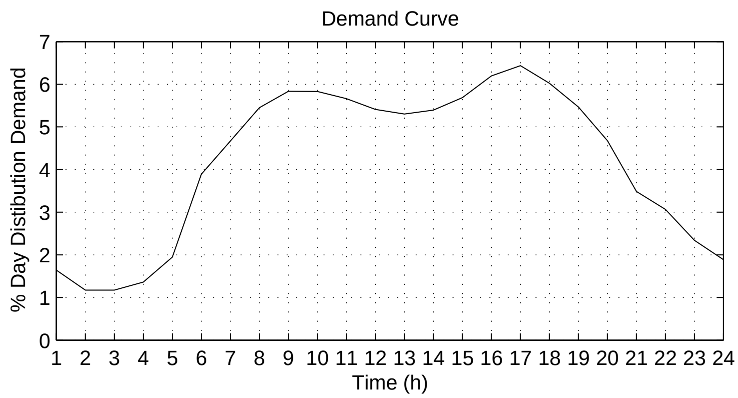

Figure 3: Average daily demand

There are additional considerations to make about the energy fare policies existing in Tizi Ouzou that we must take into account in our problem. Table 1 shows the energy costs in Algerian Dinars per kWh (DA/kWh), for each of the $m = 3$ time periods in which the energy fares are structured (night, flat, peak), and on the specific plant from which the water is pumped (E41 and E42 standing for Tassadort and Sebaou pumping fields, respectively). We assume that electricity prices do not vary within the planning time.

Table 1: Pumping cost for different day periods and facilities

Facility fare	Day period	Cost (DA/kWh)
E41	Night	85.33
E41	Flat	161.47
E41	Peak	726.28
E42	Night	150.53
E42	Flat	150.53
E42	Peak	126.68

In this regard, the initial simulation time is set at midnight, local time, when the night energy fare applies and there is lower demand.

In our analysis, as mentioned, we have considered two objectives: one refers to minimizing the total pumping cost, whereas the other refers to maximizing equity, through some of the equitable solutions described in Section 2.2. We compare various solutions with our measures and then provide their optimal cost schedules. All computations were performed on a PC with an Intel Core II Duo T7200 processor, with 2 GHz and 2Gb of RAM, running under Windows XP. The water network was modeled with the aid of the OPL Studio Library v 3.7, using ILOG CPLEX 9.0 as the embedded optimizer.

Our network models comprise around 7,000 variables and 60,000 constraints. Each of the situations considered (each single point in the Pareto frontiers) is the result of a run of the model, lasting around 20 minutes. However, in hardly any of the Pareto points, the optimizer found the optimal solution within these 20 minutes. Due to the presence of integer variables, the optimizer usually needed more than 24 hours to explore the whole searching space and find the optimal solution. But as we checked, in practice the estimated solution obtained after 20 minutes was generally within 99% of accuracy with respect to the optimal one.

We should mention that due to the discrete nature of the pumping variables I_w^t and I_s^t, the results have a stepwise profile, as the pumps have two possible states for each planning period (hours in our case): ON or OFF. Therefore, whenever a pipe changes its status, there is a "jump" in the cost and in the volume of water pumped into the network. Otherwise, the cost and quality of the water supplied would remain constant.

In what follows, we will only show graphical results of three of the solutions discussed in Section 2.2, the egalitarian solution, formulated in (10), the Smorodinsky-Kalai solution, given by (11), and the utilitarian solution, as expressed in (13). For the sake of simplicity, we have not included the Nash solution (12) in the graphics, as it virtually provided the same results than those of the utilitarian solution. In the same manner, we have not included the variance solution (14), as it led to very similar results compared to those of the egalitarian solution. Unless otherwise stated, we have plotted in the Y-axis the total pumping cost of the proposed solution in DA. With respect to the X-axis, rather than the values of the different objective functions, which are not easily comparable, we have plotted the value of the global water deficit resulting from the different solutions.

We have analyzed two scenarios of distribution network losses, which are actually taking place in the network, and are one of the main problems to solve. The first scenario, H_1, assumes that we can satisfy the demand at all the points of the network. The second scenario, H_2, more realistic, allows for unfulfillment of the demand at certain points of the network, due to several reasons, e.g., an insufficient flow capacity of a given pipe.

4.2 Scenario H_1

Figure 4 shows a graphic with some values of the Pareto frontier, displaying three different solutions: the egalitarian (light gray circles), the Smorodinsky-Kalai (dark gray squares), and the utilitarian one (black diamonds) to evaluate the water deficit.

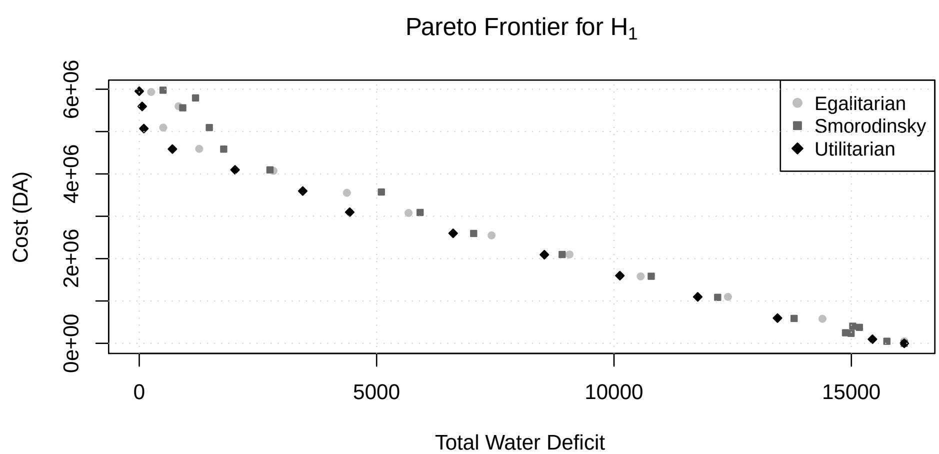

Figure 4: Pareto frontier for hypothesis H_1

We outline the most relevant features of the results obtained from each solution. We investigate how the different solutions redistribute the water when we spend less money in the system than what would be needed to completely satisfy the demand at all points.

- For the utilitarian solution (black diamonds), as the available budget is progressively reduced, the system will stop first supplying water to those points for which the pumping costs are higher. Usually, such points are those villages or houses placed on top of the hills or mountains. If there are several consumption points with the same (expensive) pumping costs, and the budget is further reduced, the system will first cut the water supply to those villages with larger population, because, in this manner, the terms L_k contributing to (13) will have smaller values, as they are divided by larger numbers — the population at the k-th consumption point. We should note that this solution leads to cutting the water supplying to the entire population of a certain consumption point at the same time. This water distribution policy is actually quite unfair, although from a purely economic point of view, it is the most advantageous one. As we have mentioned before, when we have implemented the Nash solution (12) in our experiments, we have observed the same trends in the manner in which restrictions of water supplies are managed. For the Tizi Ouzou water network, the first village which would eventually suffer water restrictions would be Bouhinoune Haut, which is placed high on the mountain, needs three intermediate pumping facilities (notably increasing the pumping cost), and has approximately 9,000 dwellings, when the average number in the nearby villages is around 1,200.

- For the egalitarian solution (light gray circles), the per capita water deficit is the same for all villages. Thus, when the pumping available budget is reduced, this water scarcity is equally distributed among all the inhabitants of all the villages, regardless of their population size or location. This would be the fairest solution, although it is more expensive than the utilitarian one. To gain insight into this phenomenon, we have zoomed the central area in Figure 4, through Figure 5. As we can observe, the costs in the utilitarian solution are, on average, around 15% less than those of the egalitarian solution (i.e., with the same available budget, the utilitarian solution supplies 15% more water, but at the cost that those villages with the most expensive pumping costs will not receive water at all). The fact that in Figure 5 some points apparently break the descending trend of the graphic is normal, as we have to keep in mind that what we are plotting in the Y-axis — the total water deficit — is not an objective itself, but, rather, a mean to compare the different objective functions proposed in Section 2.2.

- For the Smorodinsky-Kalai solution (dark gray squares), the results obtained are similar to those of the egalitarian solution. However, there are subtle differences in the way in which restrictions are managed as the budget is progressively reduced. In the egalitarian solution, if in a given village the demands are satisfied up to, say, a 95%, the

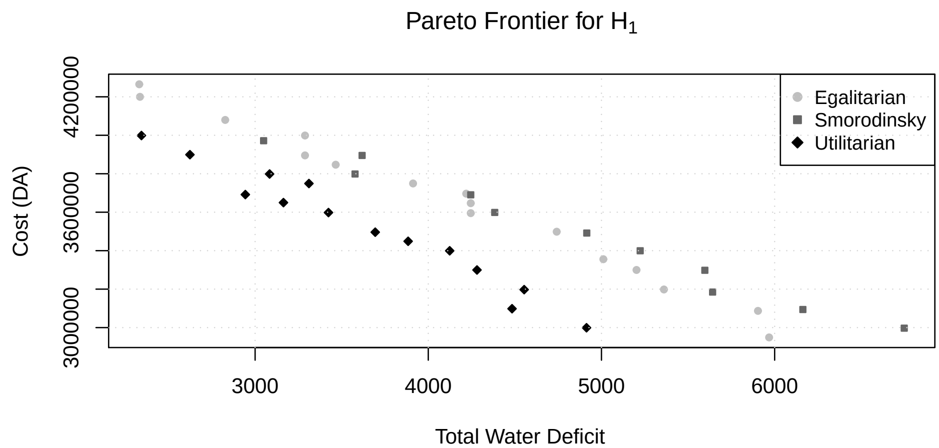

Figure 5: Pareto frontier for hypothesis H_1 (zoomed)

same fulfillment is accomplished for the rest of the villages. If this percentage continues decreasing, it will do so in the same way for all the villages. On the contrary, for the Smorodinsky-Kalai solution, the villages with worse supplying conditions are identified and penalized first with water restrictions, but the rest of villages are not influenced by them. In our network, it happens that the pipes with the highest pumping cost will be the first ones to be cut, stopping the supply to all the villages which depend on those pipes.

If instead of plotting the cost *vs* the total water deficit, we plot each objective function in the Y-axis *vs* the total cost in the X-axis, we can analyze further interesting features. As we can see in Figure 6a, the objective function of the egalitarian solution is approximately a linear function of the cost, something which is logical, as this solution stops supplying the same amount of water from all the consumption points simultaneously. On the other hand, in Figures 6b and 6c we can observe that the dependence between the objective function and the cost is not linear, due to the manner in which water restrictions are distributed among the different villages. As we mentioned before, the Smorodinsky-Kalai solution (6b) stops supplying first the most expensive pumping facilities. This is a nonlinear behavior. Regarding the utilitarian solution (6c), it first cuts the water supply in those villages which have more population and more expensive pumping costs. Thus, the reductions in the costs are larger at the beginning of the budget reduction, and become smaller as less budget is available.

4.3 Scenario H_2

We discuss now the results we have obtained when we assume that the demand at certain points of the network may not be fulfilled. We have plotted in Figure 7 the Pareto frontiers for the the egalitarian (light gray circles), the Smorodinsky-Kalai (dark gray squares), and the utilitarian (black diamonds) solutions. In this case, when there are some limitations in the network (e.g. a pipe with an insufficient flow capacity), the egalitarian or the Smorodinsky-Kalai objectives may imply solutions that are not desirable. The problem lies in the fact that if it is impossible to fulfill the demand of a certain village, and we keep the egalitarian criterion, all the villages will suffer similar restrictions.

To better understand this phenomenon, we have plotted in Figure 8 each objective function in the Y-axis *vs* the total cost in the X-axis. As we can observe, the egalitarian solution (8a) cannot satisfy the demand in all the villages. Specifically, we cannot obtain values of the objective function under, approximately, 0.07, which means that around a 50% of the overall demand is not satisfied. The situation for the Smorodinsky-Kalai solution (8b) is even worse, with a lower bound on the objective function of, approximately, 0.08, amounting to a non-fulfillment of around 66% of the overall demand.

On the other hand, as we can see in Figure 8c, the utilitarian criterion will not suffer from these drawbacks, although it will be still an unfair solution. By noting the slight differences between Figures 6c and 8c, it becomes clear that if a specific problem happens in a given village it barely affects the rest of the villages.

5 Discussion

We have described here a complex bi-criteria water distribution problem with equity and cost criteria. We have discussed several equity related criteria and showed the relevance of approximating the Pareto frontier as a powerful tool for managers to find out the equity of the distribution schedule, given a certain distribution budget. The study originated from a consulting problem in the region of Kabylia, Algeria, supported by the Spanish Agency of Cooperation and Development. In this particular case, we have been able to produce distribution schedules much better than the current one, both in terms of equity and cost, which have been adopted by ADE.

Note that we have not modeled uncertainty in demand, which we have assumed fixed in this paper. This uncertainty would be reflected in some of our right-hand terms and may be dealt with stochastic programming methods, see e.g. Birge and Louveaux (1997). However, note that estimating such demand is specially difficult, as we may lack some of such data. As we cannot always satisfy demand, we do not actually know such demand. Note also that we have not imposed any condition on the availability of water within the aquifers. This might be specially relevant in case some of the source points are reservoirs being affected by uncertainty in available water. The development of a decision support system would be also of interest for this problem.

Acknowledgements

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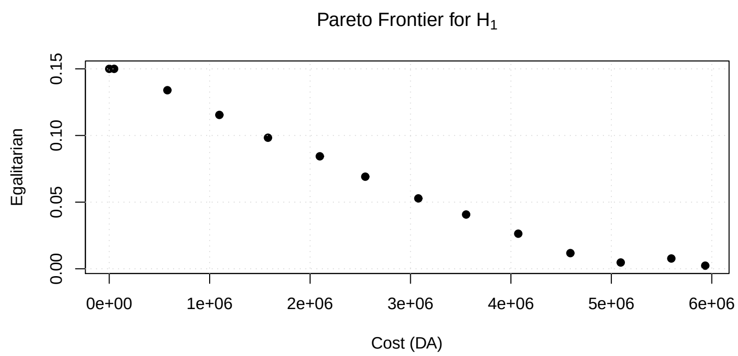

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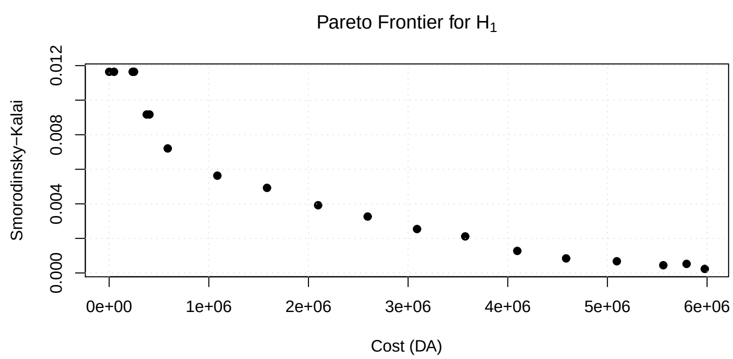

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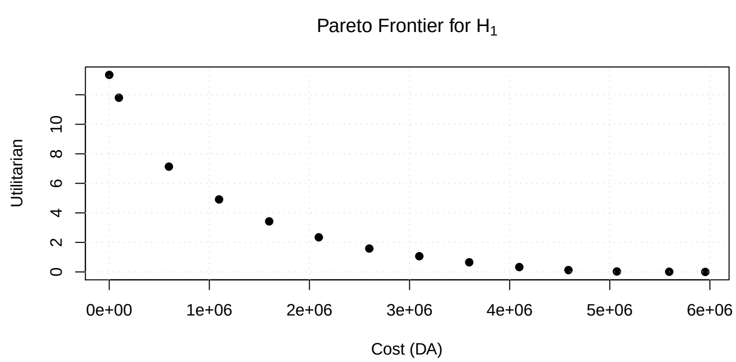


(c)

Figure 6: Objective functions *vs* total cost for H_1

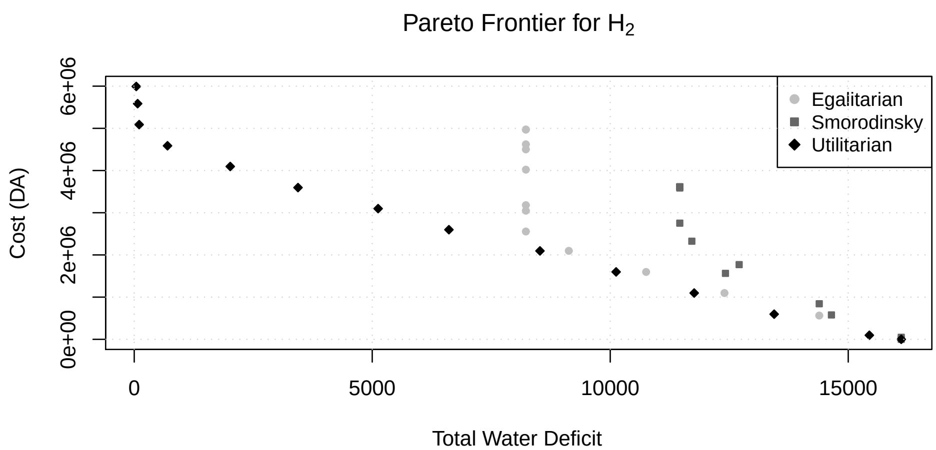
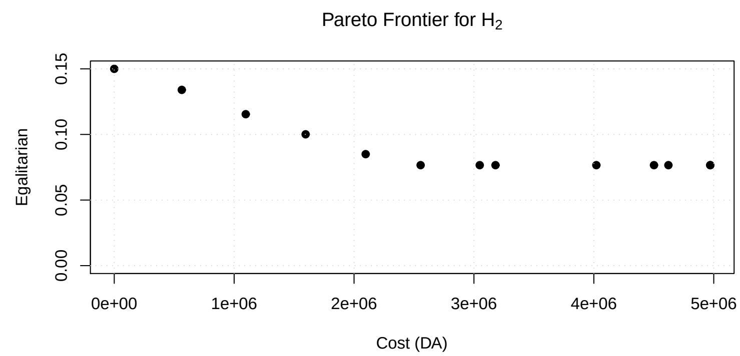


Figure 7: Pareto frontier for hypothesis H_2



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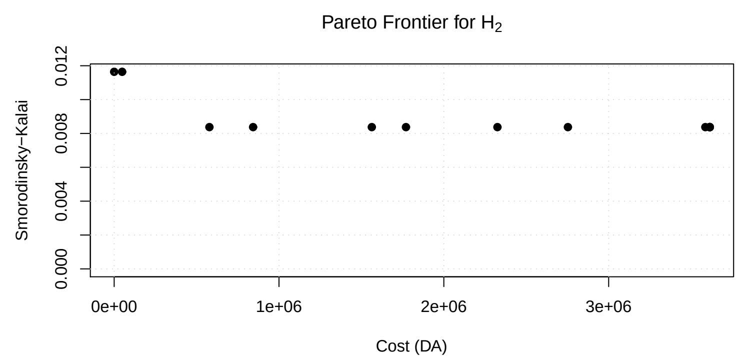

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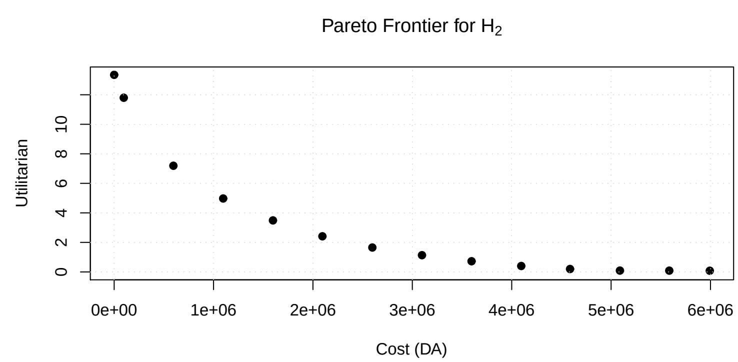

(c)

Figure 8: Objective functions *vs* total cost for H_2