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Planning with Uncertain Feasible Decision Sets Supported by Pareto Frontier Visualization

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ABSTRACT. The paper describes the method for supporting the planning procedures in the case of uncertain feasible decision sets. The method is based on transformation of the decision problem into a multiobjective optimization problem and subsequent study of the multiobjective problem by visualizing the high-order Pareto frontier. In contrast to other Pareto frontier visualization techniques, we visualize the Pareto frontier by using the interactive decision maps which display the objective tradeoffs in a clear way. The decision maker learns information on the influence of uncertainty by studying objective tradeoffs between robustness and performance. This information helps to identify the preferred combination of robustness and performance and construct the associated decision.

KEYWORDS. Uncertain data; Multiobjective optimization; Pareto frontier visualization.

1. INTRODUCTION

The method described in this paper is aimed at the study of decision problems which contain uncertainty in the set of feasible decisions. The method is based on the generic idea to transform the decision problem associated with uncertainty and risk into a multiobjective optimization (MOO) problem and to base its study on the visual exploration of the Pareto frontier. The Pareto frontier is studied by using the Interactive Decision Maps (IDM) technique (Lotov et al, 2004), which turned out to be a powerful tool for interactive visualization of the objective tradeoffs in high-order MOO problems (i.e. problems with three to seven objectives). Here we apply this approach in the case of decision problems with the uncertain feasible sets. The original decision making problem under uncertainty can be formulated as a single objective optimization problem or a MOO problem. For the sake of generality, we formulate it as the MOO problem with *m* objectives.

The structure of the paper is as follows. Section 2 describes the procedure for transformation of the decision problem with the uncertain feasible set into the MOO problem and illustrates the procedure with the case of a linear multiobjective planning problem with uncertainties in the right-hand sides of the inequalities that describe the feasible decision set. Section 3 is devoted to visualization of the Pareto frontier by using the Interactive Decision Maps (IDM) technique. Section 4 provides an example of application of the method for water management in a river basin under uncertain precipitation.

2. GENERATING A MULTIOBJECTIVE PROBLEM

Let us introduce some notation. In decision problems without uncertainty, the set of feasible decisions X is usually given in advance and precisely. In this paper the set X is not given precisely. Namely, we assume that it is given by a system of compact sets X_s, $0 \le s \le 1$, which belong to the decision space R^n. The parameter s describes the level of certainty of the set X_s: the decrement of the value of s results in the increment of the uncertainty of the set. It is assumed that the less certain set $X_{s'}$ encloses the more certain set $X_{s''}$: $X_{s''} \subseteq X_{s'}$ while $0 \le s' < s'' \le 1$. The set X_1 contains the decision vectors $x \in R^n$, which are feasible for sure. The set X_0 is the largest feasible set, the decisions $x \in R^n$ outside of X_0 cannot be feasible in any case. To describe the robustness of a decision $x \in R^n$, it is possible to use the membership function $\mu(x) = \sup\{s : x \in X_s, 0 \le s \le 1\}$, which values vary between zero and one. Such formalization is used, for example, in fuzzy sets theory (Dubois and Prade 1988).

Let $f : R^n \to R^m$ be a mapping from R^n to the objective space R^m: performance of a decision $x \in R^n$ is described by the objective vector $y = f(x)$, with m objectives y_i, i=1,…,m. The set $Y = f(X)$ is denoted as the feasible objective set. Without loss of generality, we assume that the objectives must be maximized.

To transform the decision problem under uncertainty into a new MOO problem, which, however, does not contain uncertainty, we introduce a new objective $y_{m+1} = \mu(x)$, which describes the robustness of the decisions. Surely, one is interested in high robustness of the chosen decision. Therefore, the value of the objective $y_{m+1} = \mu(x)$ must be maximized. Thus, we have got the new multiobjective maximization problem with the objective vector $y \in R^{m+1}$, where $x \in X_0$ and

$$y = f(x) \text{ for } y \in R^m \text{ and } y_{m+1} = \sup\{s : x \in X_s, 0 \le s \le 1\}.$$

We illustrate our approach with the linear problems with uncertainty in the right-hand sides of the inequalities $Ax \le b$, where $x \in R^n$, that describe the feasible set. Let us assume that $b \in [b^1, b^0]$, where b_i, $i = 1,\dots,I$, is described by the membership function $\phi_i(b_i)$:

$$\varphi_i(b_i) = \begin{cases} 1 & b_i \le b_i^1 \\ \dfrac{b_i^0 - b_i}{b_i^0 - b_i^1} & b_i^1 < b_i \le b_i^0 \\ 0 & b_i > b_i^0 \end{cases} \tag{1}$$

The robustness of a decision $x \in R^n$ is given by the membership function

$$\mu(x) = \max\{s: \ 0 \le s \le 1, < a_i, x > \le b_i^0 - (b_i^0 \text{-} b_i^1)s, \text{ where } i = 1,\dots,I;\ a_i \in R^n\}, \tag{2}$$

which can be used as the robustness objective.

3. HIGH-ORDER PARETO FRONTIER VISUALIZATION

The MOO method applied in this paper is based on visualization of the multiobjective Pareto frontier in the case of three to seven objectives. Informing the decision maker on the Pareto frontier is one of the approaches used in the framework of the modern multiobjective techniques (see, for example Branke et al., 2008). In this paper, visualization of the Pareto frontier based on approximating the Edgeworth-Pareto Hull (EPH) of the feasible objective set, which is the maximal set in the objective space that has the same Pareto frontier as the feasible objective set. Convenience for visualization is the main reason for approximating the EPH. The decision maker explores the Pareto frontier by displaying various collections of the tradeoff curves which are the frontiers of bi-objective slices of the EPH. At the computer display, collections of bi-objective slices of the EPH are provided in interactive mode in the graphical form of decision maps (see Example). This is the reason why such visualization of the Pareto frontier is known now as the Interactive Decision Maps (IDM) technique, which was proposed in (Lotov, 1984, 1989). A detailed description of the method and applications is given in (Lotov et al., 2004). In contrast to other methods, which apply computer graphics for visualization of objective points that belong to the Pareto frontier, see, for example, (Kollat and Reed, 2007), the IDM technique is concentrated at the display of the objective tradeoffs for all objectives.

Interactive exploration of the Pareto frontier, especially objective tradeoffs, helps the decision maker to identify the preferred point of the Pareto frontier (feasible goal). Then, an associated decision can be found by the computer automatically. Such an approach to the decision making, known as the Feasible

Goals Method (FGM), refines the goal programming approach and allows the user to identify the Pareto-optimal goal directly at a decision map by a simple click on it. An illustrative application of the FGM/IDM technique for solving a MOO problem under uncertainty is described in the next section. We illustrate our approach with an example from the water management field, see (Lotov et al. 2004).

4. EXAMPLE

Let us consider a hypothetical conflict between different users of a lake. The lake serves as the municipal water supply source. It is also used for irrigation purposes by farmers. Moreover, the lake is a recreational zone for nearby residents.

The crops production in the region has an impact on the environmental situation. It is due to the application of chemical fertilizers and water for irrigation, which can substantially increase crops output, but at the same time can result in negative environmental consequences – a part of the chemical fertilizers finds its way into the lake with the withdrawal of water. Moreover, a shortage of water may arise, since the irrigation is used during the dry season, when a natural drop of the water level of the lake may be high.

The mathematical model is based on the concept of the agriculture production technology. The crop production depends on technologies applied. Table 1 presents production and consumption of resources per acre by the technologies in the upper agricultural zone, see Figure 1. These characteristics in the lower agricultural zone are the same as in the upper zone, except for the crops production, which are 6, 12, 14, 20, 22, 24, 32, 34, 40 tones/acre for corresponding production technologies.

Table 1. Agriculture production technologies in the upper zone.

Tech. no.	1	2	3	4	5	6	7	8	9
Crops production (a_{i1}^{1}), tones/acre	6	12	16	20	24	24	32	36	44
Water consumption (a_{i1}^{2}), m^3/acre	0	120	240	120	240	120	240	240	360
Fertilizer consumption (a_{i1}^{3}), kg/acre	0	0	0	40	40	80	80	120	120
Water withdrawal (a_{i1}^{4}), m^3/acre	0	24	48	24	48	24	48	48	72
Fertilizer withdrawal (a_{i1}^{5}), kg/acre	0	0	0	6	6	12	12	18	18

The mathematical model is, thereby, a linear model with uncertainty, as described in section 2. Let x_{ij} be the area of the j-th zone where the i-th technology is applied. The areas x_{ij} are non-negative

$$x_{ij} \geq 0,\ i=1,2...,9,\ \ j=1,2$$

and restricted by the total areas of zones

$$\sum_{i=1}^{N} x_{ij} = b_j,\ j=1,2,$$

where $b_j, j=1,2,$ is the area of zone j. The i-th production technology in the j-th zone is described by the parameters a_{ij}^{k}, $k=1,2,3,4,5,$ given per unit area, where a_{ij}^{1} is production, a_{ij}^{2} is water application during the dry period, a_{ij}^{3} is fertilizers application during the dry period, a_{ij}^{4} is volume of the withdrawal (return) flow during the dry period, a_{ij}^{5} is amount of fertilizers brought to the river with the return flow during the dry period. By using the allocation of the area among the technologies, one obtains the values of performance indicators per a zone

$$z_j^{k} = \sum_{i=1}^{N} a_{ij}^{k}\, x_{ij},\ \ k=1,2,3,4,5,\ j=1,2$$

The water balances are fairly simple. They include changes in water flow and water volumes during the irrigation period. The deficit of the inflow into the lake due to the irrigation equals to $z_2^{1} - z_4^{1}$. The additional water release through the dam during the dry period is denoted by d. Let T be the length of the dry period. The level of the lake at the end of the dry period $L(T)$ is approximately given by

$$L(T) = L - (z_2^1 - z_4^1 + d)/\alpha$$

where L is the normal level, i.e. the level without irrigation and additional release, and α is a given parameter. It is assumed that the release d and water applications are constant during the dry season. Then, the flow in the mouth of the river near monitoring point $\boldsymbol{A}$ denoted by v_A equals to

$$v_A = v_A^0 + (d - z_2^2 + z_4^2)/T$$

where v_A^0 is the normal flow at point $\boldsymbol{A}$, see Figure 1. The constraint is imposed on the value of the flow

$$v_A \geq v_A^*$$

where the value v_A^* is given. So, the following constraint is included into the model

$$v_A^0 + (d - z_2^2 - z_4^2)/T \geq v_A^* \,.$$

The increment in pollution concentration in the lake denoted by w_L is supposed to be equal to

$$w_L = z_5^1/\beta$$

where β is a given parameter. This means that we neglect the change of the volume of the lake in comparison with the normal volume in the formula for pollution concentration. Along with the constraint on the flow at point $\boldsymbol{A}$, the constraint

$$w_A \leq w_A^*$$

is imposed on pollution concentration at this point where the value w_A^* is given. The pollution flow (per day) at the monitoring point $\boldsymbol{A}$ is given by

$$z_5^2/T + q_A^0$$

where q_A^0 is the normal pollution flow. This means that we neglect the influence of fertilizers application in the upper zone on pollution concentration in the mouth. Then, the concentration of pollution at point $\boldsymbol{A}$ denoted by w_A equals to

$$w_A = (z_5^2/T + q_A^0) / v_A \,.$$

Taking into account the above expression for v_A, we obtain

$$w_A = (z_5^2 / T + q_A^0) / (v_A^0 + (d - z_2^2 - z_4^2) / T).$$

Thus, the following constraint is included into the model

$$(z_5^2/T + q_A^0) \leq w_A^* (v_A^0 + (d - z_2^2 - z_4^2) / T).$$

Note that all expressions of the model are linear. The conflict is described by three objectives: *agricultural productivity* ($y_1 = z_1^1 + z_2^1$), lake *level drop* (L-$L(T)$) and *lake pollution* (w_L). *Agricultural productivity* is measured in tons of crops by an acre of land, lake *level drop* is measured in meters of the level drop, *lake pollution* is measured in grams of the pollutant in 1 cubic meter of water.

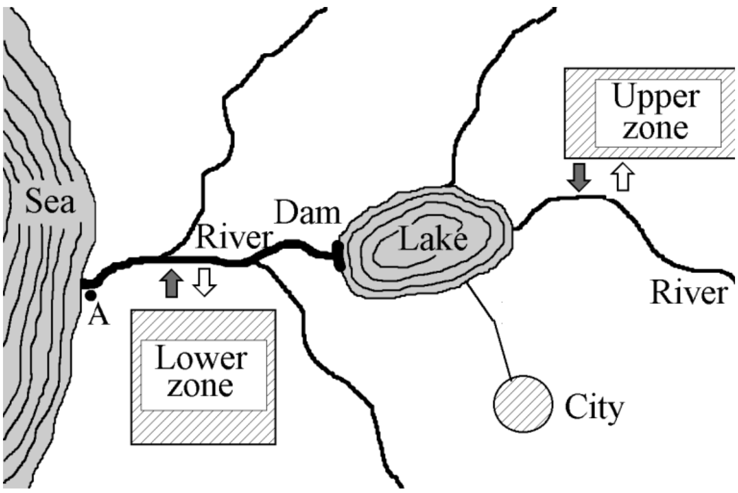

Figure 1. Map of the region.

In (Lotov et al., 2004), the lateral inflow into the lake was considered to be constant. Here we study a modified model, where the inflow is given by the membership function of the form (1), where b^1 and b^0 are numbers and correspond to the lower and the higher inflow respectively. The variations in the lateral inflow will affect both lake *level drop* and *lake pollution*, since, given some absolute value of the contaminant, its relative concentration is lesser when the inflow is greater due to the dilution. After modifying the initial deterministic model in compliance with stated in Section 1, we have applied FGM/IDM technique to analyze the model. We shall illustrate the analysis with explaining first the conflict between *lake pollution*, *agricultural productivity* and *robustness*.

Figure 2 shows the decision map where *agricultural productivity* and *lake pollution* are displayed on axes, while *robustness* is given by shades of gray. One can easily see the tradeoffs between *productivity* and *lake pollution* for different values of *robustness*. The tradeoffs between performance objectives and *robustness* is visible, too. It is clear that the smaller is *lake pollution*, the narrower are the bands corresponding to different values of *robustness* (see arrows in Figure 2), that is, the smaller will be changes in *productivity* with the changes in *robustness*. Therefore, the smaller is *lake pollution*, the more stable are solutions in the sense of *productivity* with respect to the varying lateral inflows. Moreover, would it be necessary to meet some certain requirements on *productivity*, the higher are these requirements, the lesser will be *robustness* of the corresponding solutions. In Figure 2, this fact is clearly represented by the number of strips of different shades of gray, enclosed between the boundary of the feasible objective set, which corresponds to zero *robustness*, and the dotted line, which identifies the limits of effective increase in *lake pollution*, for different values of *robustness*. Obviously, the fewer shades of gray crosses a vertical line (see the vertical line in Figure 2), the less reliable value of *productivity* has been chosen. Given this, it is senseless to choose any solution above the dotted line. One can summarize that the high demands on *robustness* of solutions will automatically provide high water quality standards.

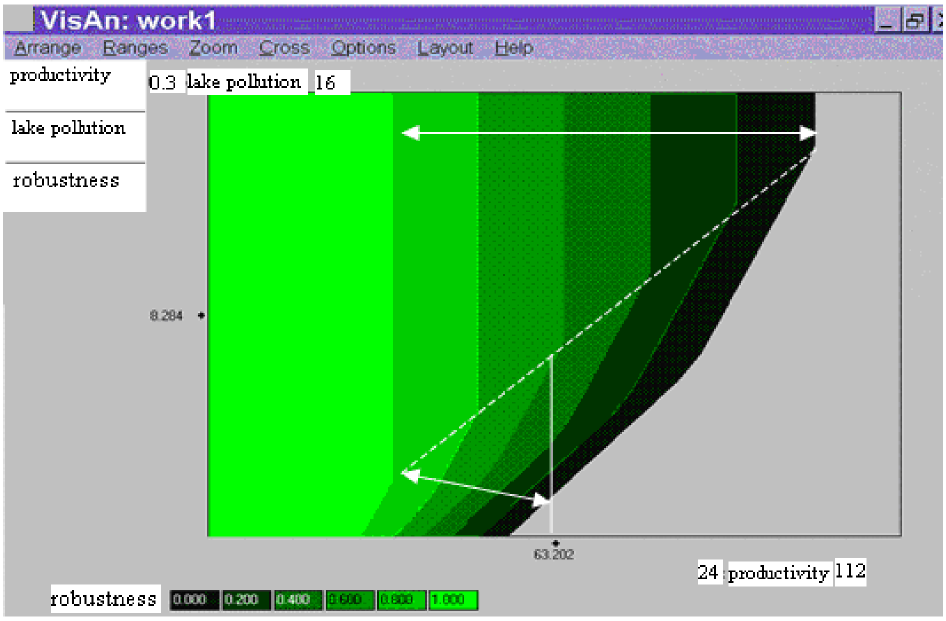

Figure 2. Dependence of *lake pollution* on *productivity* for different levels of *robustness*, shown by the shades of gray

Let us choose another perspective and visualize the conflict between the four objectives by using a *matrix* of decision maps, where each decision map represents the tradeoffs between the criteria *level drop* and *productivity*, while some restrictions are imposed on the objectives *robustness* and *lake pollution.* A decision map which corresponds to *robustness* $\geq a$ and *lake pollution* $\leq b$ will appear in the intersection of the row named a and the column named b.

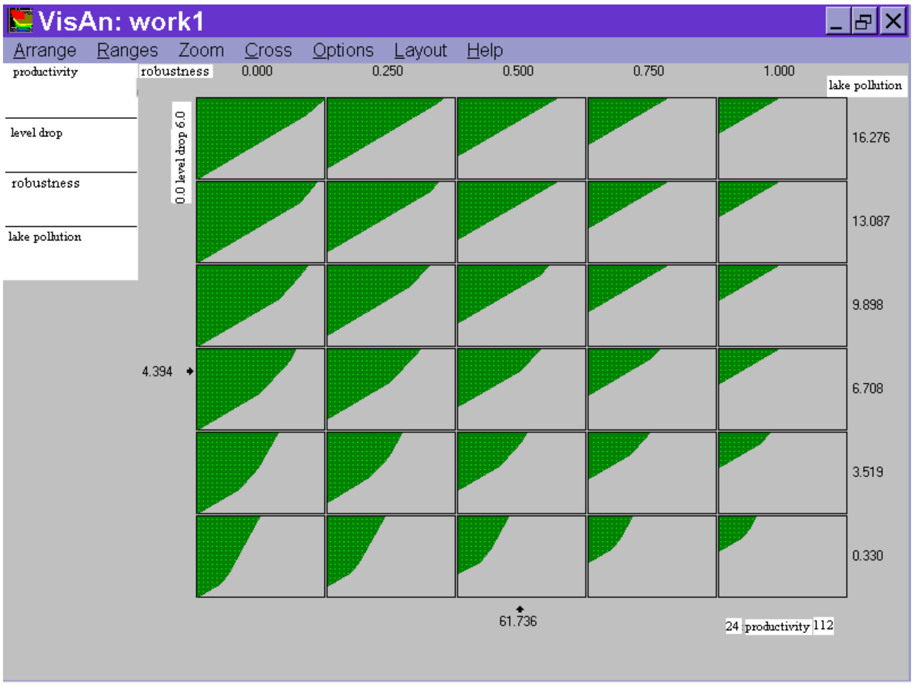

Figure 3. Tradeoff curves between *productivity* and lake *level drop* for different *robustness* and lake *pollution*, represented as the matrix of decision maps

Figure 3 shows that *level drop* is an essential factor of increase in *productivity* for any feasible values of the other two objectives. Indeed, intensive agricultural technologies require extensive use of water. Furthermore, maximal substitution of *productivity* by *level drop* can be observed for the low values of *lake pollution*, see the decision maps located at the lower left corner of the matrix.

Let *productivity* be set fixed at 63 tones/acre and consider another decision map in the coordinates of *lake pollution*, *level drop* (both on the axis) and *robustness* (shades of gray), see Figure 4. One can see that *lake pollution* substitutes *level drop* if its value is less than 6.93 kg/m3, for any *robustness*. For this pollution value, *level drop* increases linearly with the increase in *robustness*: the lack of lateral inflow results in the drop of lake level.

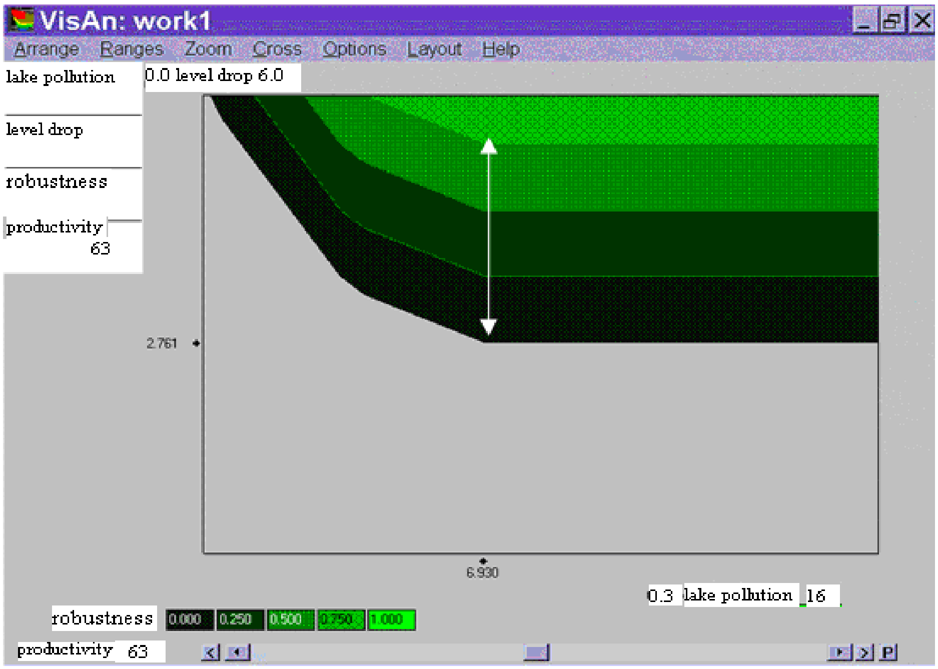

Figure 4. Dependence of lake *level drop* on *lake pollution* for a given *productivity* and several levels of *robustness*, shown by the shades of gray

If one is going to “pay” by *lake level* in order to achieve a reliable solution, he can find all available values of robustness on the white vertical line on Figure 4. Further decrease in *lake pollution* may not be rational since it will reduce both *level drop* and the availability of reliable solutions. In addition, one can find that the solution strategy does not change along the vertical line: it uses the 1st, 4th and 6th technology. From Table 1 one can see that these strategies are the most economical, with respect to water consumption, among the technologies with the same fertilizer consumptions levels.

5. CONCLUSIONS

We propose an approach that uses a MOO model to represent a decision problem with uncertainty. In the framework of MOO, the uncertainty is modelled by adding a new objective to the model description, which corresponds to the robustness of solutions. Due to it, the robustness can be considered as any other objective, given an appropriate MOO method. We use IDM to visualize the Pareto frontier of MOO models. One can do it in the case of three to seven objectives. We present an example which provides some insights on how to interpret the robustness objective on decision maps.

The authors anticipate extending in the future the applications of the approach towards probabilistic models. This, however, could imply computational difficulties, since the description of the solution set, X, in this case will be essentially non-linear. Indeed, the analogue to the membership function (2) in probabilistic case will be the product of the probabilities corresponding to the consequent uncertainties. Nevertheless, the non-linear models can be studied, too, by using the visualization of the Pareto frontier, see Chapter 5 of the book (Lotov et al. 2004).

Several different examples of application of the Pareto frontier visualization for exploration of non-linear finite decision problems with uncertainty is given in Chapter 4 of the book (Lotov et al. 2004).

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