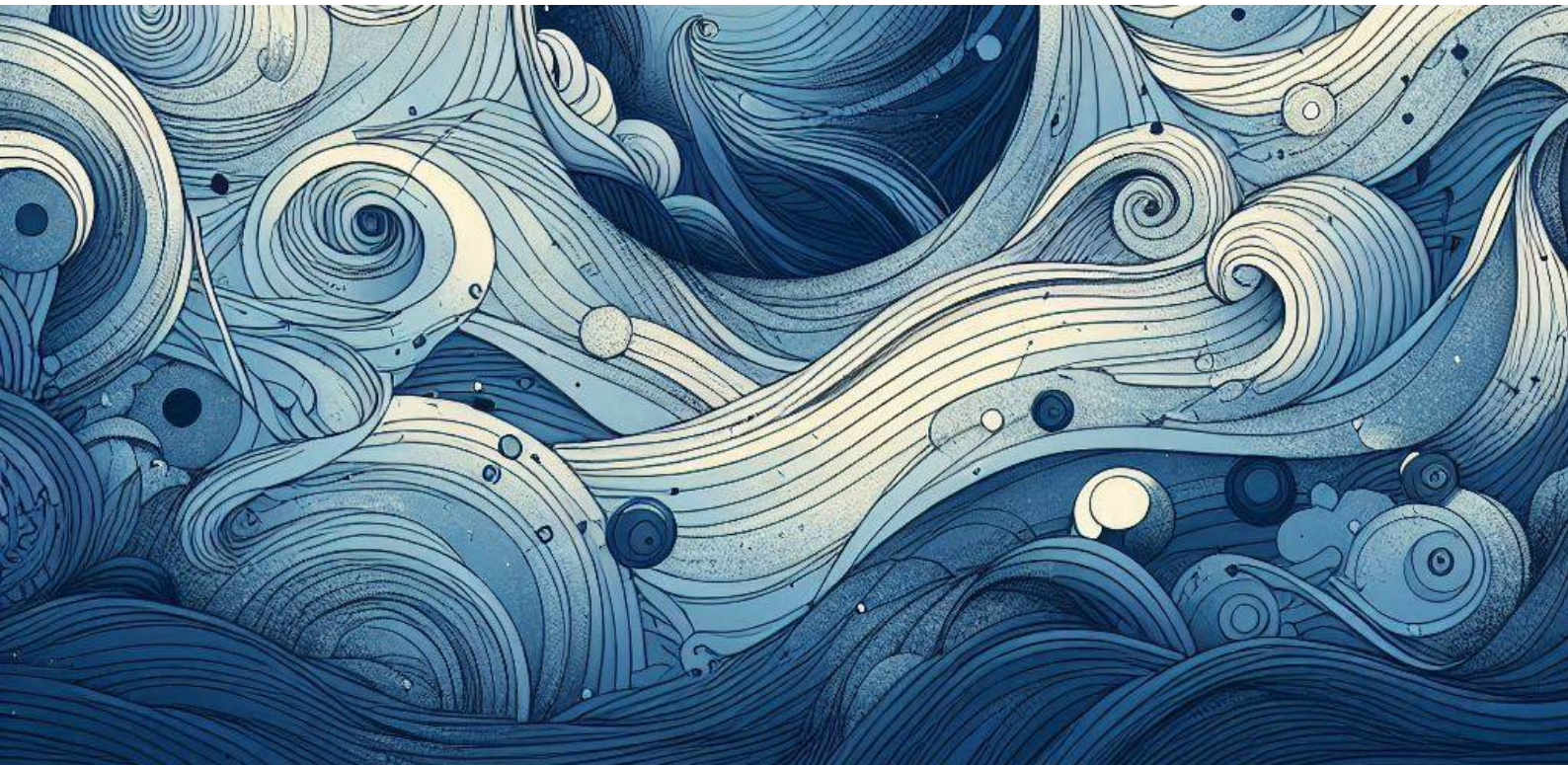
Mathematical Methods for Bioengineering

Study Guide

Open Educational Material Universidad Rey Juan Carlos
Degree on Biomedical Engineering – 2024 – 2025 v1.02

Javier Quintero



1. Introduction

This study guide serves as a comprehensive resource for students enrolled in the **Biomedical Engineering Degree** program, specifically for the course **Mathematical Methods for Bioengineering**. It includes a detailed **schedule** for each topic, a curated list of **resources**, clear learning objectives, relevant activities, and an exhaustive **list of topics** covered throughout the course.

2. Course Structure

The course is divided into four parts, each focusing on a key area of multivariable calculus. Each topic aims to achieve specific learning outcomes that help students build a solid foundation in multivariable calculus.

1. Geometry in the Euclidean Space
2. Differentiation in Several Variables
3. Integration in Several Variables
4. Vector Calculus

By the end of the course, students should be able to:

- Understand the geometry of Euclidean spaces in multiple dimensions.
- Differentiate and integrate functions of several variables.
- Apply integration techniques to solve real-world problems in engineering and science.
- Use vector calculus theorems such as Green's, Stokes', and Gauss' to analyze physical fields.

3. Learning Structure

For participants in **remote mode**, it is recommended to study the topics in the order outlined in the proposed schedule. Afterward, proceed to solve the problems corresponding to each topic, which are available in the *Exercise Collection*. At the end of the second and fourth parts, assess your understanding by reviewing the learning objectives and working through the sample solved exams.

For **in-class participants**, the first session of each week will be a **lecture** where the theoretical concepts are introduced. These concepts will then be applied during the second session, which focuses on **problem-solving**. To prepare effectively for each exam, it is highly recommended to use the available sample exams as practice.

4. Resources

The following materials are typically used throughout the course:

- **Textbook:** Marsden, J. and Tromba, A. *Vector Calculus*, 6th edition.
- **Lecture Notes:** A comprehensive guide of the topics of the subject organized with the class structure, available at Aula Virtual.
- **Slides:** The slides used in lectures, avaible at Aula Virtual.
- **Problem Set:** A huge problem set containing the problems to solve in each practice class, extra problems for individual learning and some examples of exams. Avaible at Aula Virtual.
- **Numerical tool:** Python, see
- **Numerical tool:** Geogebra 3-d Graphing tool.
- **Office hours:** Contact Professor at javier.quintero@urjc.es
- **Teaching guide** See Teaching Guide of the subject

5. Proposed Schedule

Table 1: Weekly Schedule

Week	Subject	Theory
1	Introduction. Vectors. Distance. Vector product. **Problems:** 1.1 – 1.2	1.1 –1.2
2	Geometry and Equations. **Problems:** 1.2	1.2
3	New System of Coordinates. Open and closed subsets **Problems:** 1.3-1.4	1.3 – 1.4
4	Functions of Several Variables. Limits. **Problems:** 1.3 – 1.4	2.1– 2.2
5	Continuity. Partial and Higher order Partial Derivatives **Problems:** 2.1 – 2.3	2.3 – 3.1
6	Chain Rule. Directional and higher order derivatives **Problems:** 2.4 – 2.5	3.2 – 3.4

Continued on next page

Table 1: Weekly Schedule (Continued)

7	Differentials and Taylor's Theorem. Extrema of Func. **Problems:** 2.7 – 2.8	4.1– 4.4
8	General Review of First Part This week corresponds to the First Partial	Exam Ch 1 – 4
9	Areas and Volumes. Iterated integrals **Problems:** 2.1 – 3.2	5.1
10	Double and Triple Integrals. Changing order **Problems:** 3.2 – 3.3	5.2 – 5.5
11	Paths, arc length, and Vector fields **Problems:** 2.6, 4.1	6.1 – 6.2
12	Line Integrals. Conservative Vector Fields **Problems:** 4.1	7.1 – 7.2
13	Green's Theorem. Surface Integrals. **Problems:** 4.3	7.3 – 7.4
14	Stokes' and Gauss' Theorems **Problems:** 4.3	7.4 – 7.5

6. Farewell

I hope this study guide serves as a valuable resource in deepening your understanding of the subject and developing the key skills outlined as its objectives. Wishing you great success in your academic journey,

Javier Quintero
javier.quintero@urjc.es
January 2025

7. Comprehensive list of topics

1. Geometry in the Euclidean Space
   (a) Geometry in the Euclidean Space
      i. Vectors in several dimensions
      ii. Equations for Planes, Distances, and Coordinate Systems
         A. Normal Vector
      iii. New Coordinate Systems
      iv. Neighbourhoods. Open and Closed Sets
2. Differentiation in Several Variables
   (a) Functions of Several Variables
      i. Domain and Range of Functions of Several Variables
      ii. Limits of Functions of Several Variables

   iii. Continuity of Functions of Several Variables
   iv. Level Curves and Surfaces
(b) Differentiation in Several Variables
   i. Partial Derivatives and Differentiability
      A. Conditions for Differentiability
   ii. Higher-Order Partial Derivatives
      A. Mixed Partial Derivatives and Symmetry of Second-Order Derivatives
      B. Higher-Order Derivatives in Multivariable Calculus
   iii. The Chain Rule for Multivariable Functions
   iv. Differentiation of Functions from $\mathbb{R}^n$ to $\mathbb{R}^m$
      A. Differentiability of Functions from $\mathbb{R}^n$ to $\mathbb{R}^m$
      B. The Chain Rule for Functions from $\mathbb{R}^n$ to $\mathbb{R}^m$
      C. Properties of Differentiation for Functions from $\mathbb{R}^n$ to $\mathbb{R}^m$
   v. Directional Derivatives and the Gradient
      A. Directional Derivative
(c) Extrema of Functions and Optimization
   i. Differentials and Taylor's Theorem
      A. Taylor's Theorem
   ii. Extrema of Functions
      A. Local Minima and Maxima for Functions of Several Variables
      B. Critical Points
      C. Second Derivative Test
   iii. Lagrange Multipliers: Finding Constrained Extrema
      A. Multiple Constraints
      B. Lagrange Multiplier Strategy for Finding Absolute Maxima and Minima on Regions with Boundary
   iv. Applications of Extrema: Optimization Problems in Engineering
      A. Applications in Engineering

3. Integration in Several Variables
(a) Multiple Integrals
   i. Double Integrals
      A. Double Integrals over Rectangular Regions
      B. Double Integrals over General Regions
   ii. Changing the Order of Integration: Fubini's Theorem
   iii. Triple Integrals
      A. Other Types of Regions
   iv. Jacobians. Transformation of Integrals
   v. Applications of Integration
      A. Areas and Volumes
      B. Mass, Center of Mass, and Moments of Inertia

4. Vector Calculus

(a) Introduction to Differential Geometry and Vector Calculus
    i. Parametrized Paths. Arclength
        A. Arclength of a Parametrized Curve
        B. Reparametrizations
    ii. Differential Geometry. Operators
        A. Introduction to Vector Fields
        B. Gradient, Divergence, Curl, and Del Operator

(b) Integral Theorems of Vector Calculus
    i. Line Integrals
    ii. Conservative Vector Field
    iii. Green's Theorem
    iv. Parametric Surfaces. Surface Integrals
    v. Stokes' and Gauss' Theorems