An exact model for a slitting problem in the steel industry

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Abstract

From an economic point of view, the steel industry plays an important role and, when it comes to responding to new challenges, innovation is a crucial factor. This paper proposes a mathematical methodology to solve the slitting problem in a steel company located in Europe. The slitting problem occurs when large width steel coils are slit into narrower coils, known as strips, to meet the requirements of the customers. A major challenge here is defining a slitting plan to fulfil all these requirements, as well as ongoing operational constraints and customer demands. The company looks for a reduction of the leftovers generated in the entire process, while maximising the overall accuracy of the orders. These leftovers may be used in the future as part of new orders provided they are able to respond to specific requirements, or otherwise they are discarded and considered as scrap. This paper introduces a novel mixed integer linear optimisation model to respond to a specific slitting problem. The model is validated with real data and it outperforms the results obtained by the company in different ways: by adjusting the orders that are to be served, by reducing the amount of scrap and by using the retails for future orders. Furthermore, the model is solved in only a few minutes, while the company needs several hours to prepare the scheduling in the current operating process.

Keywords: Cutting, steel industry, mixed integer linear optimisation.

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1. Introduction

On a worldwide basis, the steel industry plays an important role. In 2019, steel mills produced 1,869 million tonnes of steel, 8.5% of which was produced within the European Union and 53.3% within China. In 2019, the largest producer in the world was ArcelorMittal with an annual tonnage of 97 million tonnes, followed by China Baowu Group with an annual tonnage of 95 million tonnes (World Steel Association (2020) [32]). The World Steel Association (2019) [33] reported that in 2017, the steel industry generated a total of US$500 billion value added and a further US$1.2 trillion through its global supply chain.

There is no doubt that innovation is of crucial importance in the steel industry. In 2017, 5.9% of its revenue was invested in capital investment projects, research and process improvement (World Steel Association (2019) [33]). The steel industry needs to respond to emerging challenges in terms of its processes, with optimisation methods been considered as an essential tool for the continuous improvement of the production processes and how they are managed (Mukherjee and Ray (2006) [23]). Applications of optimisation methods in the steel industry can be seen in Haessler (1978) [20], Ferreira et al. (1990) [14], Vasko et al. (1992) [30], Carvalho and Rodrigues (1995) [9], Dutta and Fourer (2001) [11] and Santos et al. (2018) [24]. Within this context, the paper proposes a mathematical methodology to solve the slitting problem in *Cortichapa*, a Spanish steel company which is part of *Comercial de Laminados* group.

One of the existing major problem in the steel industry is the slitting of material. The slitting problem arises when wide steel coils are slit into narrower coils, known as strips, to meet the requirements of the customers. This problem has been frequently investigated, such as in Coffield and Crisp (1976) [6], Haessler (1978) [20], Sarker (1988) [26], Sweeney and Haessler (1990) [27], Ferreira et al. (1990) [14], Vasko et al. (1992) [30] and Carvalho and Rodrigues (1995) [9].

The aforementioned problem leads to the well known family of Cutting Stock Problems (CSPs), which is broadly studied in the literature and is also known as Trim Loss Problems (TLP) (see Dyckoff (1985), [12]). Kantorovich (1960) [22] and Eisemann (1957) [13] are responsible for the fundamental works of the CSP. Kantorovich's work, first published in 1939, introduces the first mathematical formulation for the *one-dimensional* TLP. Eisemann's work defines the TLP for rolls of material and proposes a linear programming model to solve this problem. A few years later, Gilmore and Gomory (1961) [16] introduced a method based on duality with the aim of determining all possible cutting patterns, overcoming the difficulty of dealing with a large number of variables. This method used to solve the original CSP is considered as the classical approach. Consequently, these authors extended and adapted the method to the specific trim problem (Gilmore and Gomory (1963) [17]) and to the *two-dimensional* Cutting Stock Problem (2D-CSP) (Gilmore and Gomory (1965) [18]). Later, in Carvalho (1998) [7] a new formulation is introduced for the *one-dimensional Cutting Stock Problem (1D-CSP) and a exact solution is found using column generation and Branch-and-Bound techniques.* On the other hand, there have been a number of alternative approaches to solve the different variants of the problem (see Hinxman (1980) [21] and Delorme et al. (2016) [10]). Also an extensive revision of linear programming models for bin packing and cutting stock problems can be found in Carvalho (2002) [8].

The remaining part of this paper is organised as follows. Section 2 presents some literature review. Section 3 introduces the problem under study. Section 4 presents the mixed integer linear optimisation model proposed to solve the problem. In Section 5, an extensive computational experiment, based on a real-world situation, is introduced. Finally, Section 6 provides the conclusions and future research.

2. Literature review

An initial, simplified version of the problem presented in this work can be viewed more as a generalisation of the classical 1D-CSP. The coils held in stock have a specific width and external diameter, which are not necessarily the same for all of them, in other words, the stock is heterogeneous. These coils are then cut to produce strips to match the widths requested by the customers, taking into account that the number of knives used in the cutting process is limited. In the same way as in the 1D-CSP, the model should include constraints to ensure that the maximum number of slits and that the total width of the coil are not exceeded.

However, in the case of our problem, the orders are not based on the number of strips required, but rather by the total weight to be served for a specific strip width. Since not all coils share the same diameter, the weight of a strip will depend on the selected coil from which it is obtained. Therefore, it is impossible to know in advance the number of strips needed to meet the demand. It should be noted that if all coils share the same diameter, the transformation between the total weight and number of strips could be performed and we would be facing a 1D-CSP. CSPs with two relevant dimensions, where one is fixed (the width of the strips) and the other is variable (the weight of the strips) are named *one-and-a-half-dimensional* CSP (1.5D-CSP) (Hinxman (1980) [21]). This type of problem also differs from the 2D-CSP, where rectangles of a fixed length and width are cut from the rectangular stock.

Several authors have considered different versions of 1.5D-CSP. Haessler (1978) [20] dealt with the 1.5D-CSP in the metal industry. They assumed that each selected coil should be completely processed and proposed a heuristic procedure that would sequentially satisfy the requirements of each order, while controlling both trim losses and slitter changes. Saraç and Özdemir (2003) [25] proposed a genetic algorithm to solve a multi-objective mathematical model for the 1.5D assortment problem. Gasimov et al. (2007) [15] presented a 1.5D cutting stock and assortment problem which involved determining the number of different widths of the rolls in stock and the cutting patterns used. They propose a new multi-objective mixed integer linear programming model and an equivalent nonlinear version. A detailed survey of 1.5D-CSP's from 1965 to 1990 can be found in Sweeney and Paternoster (1992) [28].

The problem addressed in this paper also includes additional differences relative to the 1D-CSP. These include certain requirements, which should be fulfilled by the products that are ordered, such as the maximum diameter allowed for the strips being supplied. In many instances, the maximum diameters allowed for the orders are smaller than the diameter of the coils in stock. When this occurs, slitting the coils is not sufficient to meet these requirements so additional cross-cuts are necessary to reduce the diameter. These cross-cuts are guillotine cuts that cross the entire width of the coil from one edge to the other. This reduces the diameter of the coil and the strips obtained from it, to either by half or by a third and so forth, depending on the number of cross-cuts performed. Although the problem considers cross-cuts, it cannot be regarded as a 2D-CSP, since the length of the strips is not pre-set and it implies a continuous decision variable in the mathematical model.

Another important characteristic of our problem that differentiates it from the classic CSP, is the assortment of large objects. The coils held in stock are strongly heterogeneous. According to the extended typology of Wäscher et al. (2007) [31], our problem can be classified as a residual cutting stock problem (RSCP). Gradisar et al. (1999) [19] argue that a traditional pattern-oriented approach is possible only when the stock is of the same size or of several standard sizes, thus inappropriate for this type of problem. There is a need to use item-oriented approaches, which are characterised by treating each item to be cut, individually. Therefore, we propose an item-oriented solution based on a mixed integer programming model. In this regard, other item-oriented approaches have been studied in the literature. For example, Gradisar et al. (1999) [19] proposes a Sequential Heuristic Procedure to solve the problem of reducing trim losses in one-dimensional stock cutting, when all stock lengths are different.

As a consequence of the manner in which the coils are processed and cut, there is a wide variety of sizes held in stock. When the coils are cut into the exact number of strips required, this may result in a large number of strips that are not assigned to any specific order (these are the leftovers of the cutting process). As stated in Tomat and Gradi (2017) [29] the existence of leftovers is common during the cutting process. If they are greater than a certain threshold, they are considered as usable leftovers (UL) and are returned to the stock to be used for future orders. This problem is known as the Cutting Stock Problem with Usable Leftovers (CSPUL) (Cherri et al. (2009) [2]). In this respect, several papers have previously dealt with UL. In Coelho et al. (2017) [5] the possibility of generating standard pieces from the leftovers during the cutting process, is considered to reduce waste material. In Abuabara and Morabito (2008) [1] two mixed integer programming formulations are presented to deal with a one-dimensional cutting stock problem that arises in the manufacturing of agricultural light aircraft. Both models minimise the total trim loss considering the possibility of generating leftovers for future reuse. As in the CSP, the CSPUL can be classified as 1D, 1.5D and 2D. Cherri et al. (2012) [3] consider the 1D-CSPUL where the UL are kept for future use, prioritising the use of leftovers compared to the standard pieces held in stock. A survey on the 1D-CSPUL can be seen in Cherri et al. (2014) [4]. Therefore, we consider the leftovers of the process to be usable if specific conditions are met and are preferred to as opposed to obtaining scrap.

This paper studies the problem of the 1.5 dimensional residual cutting stock with usable leftovers (1.5D-RCSPUL), where cross-cuts are permitted to reduce the diameter of the resulting strips and knives constraints are considered. The stock consists of coils of different sizes containing leftovers from previous cutting processes. We developed a mixed integer linear programming model for this problem taking into account two main goals, which are related to the leftovers generated in the process, and the precision carried out when serving the customer's orders. Insofar as we are aware, no such problem has been previously discussed. Furthermore, our methodology has been validated by the company that proposed the problem, and it has significantly improved its current planning operation.

Table 1 presents a summary of the main characteristics and solution methods studied in the references that have been cited. In the table, the type of industry is specified only where the papers mention it, the rest could be applied to a wide range of them. The check mark "✓" refers to a defined problem characteristic. It should be noted that the problem characteristic is undefined for the cells marked with "-". Finally, in the Modelling approach and Solution approach columns, MILP/MINLP represent mixed-integer linear/nonlinear programming; ILP represents integer linear programming; LP represents linear programming; DP represents dynamic programming and "Survey" indicates that it is a survey type research.

3. Problem description

The production planning of steel strips is mainly based on the customer demand. Customers place orders for the strips, by specifying a certain width and a total weight. A combination of knives is set in the slitting machine (one knife more than the number of strips obtained). The strips are slit from the coils in stock which, besides new coils, also include leftovers from older cutting processes. Once a coil is selected to be processed, it is unwound in the slitting machine, which is equipped with a variable and limited number of knives to perform the corresponding cuts. While the slitting process is carried out, the coil is rewound, and a set of strips is obtained.

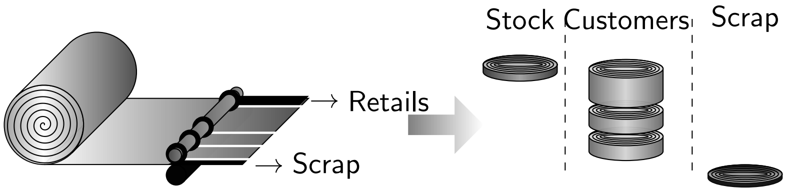
Figure 1a represents an unwound coil, where four knives have been set into the slitting machine to obtain three different strips, which is represented by the grey area. Knives at both extremes are necessary in order to keep the coil uniform, therefore, a minimum edge trim will always be required. The material on the outer sides of the extreme knives, which is illustrated by the black area in the figure, represents the leftovers of the process. These leftovers are usable and kept in stock when their width and weight are greater than a certain threshold. Whenever this occurs they are referred to as retails, otherwise, the leftovers are considered as scrap. When the width of the order perfectly matches the width of the coil, the coil is served as a complete strip so an edge trim is not required, since no slits are performed, as shown in Figure 1b.

Table 1: Literature review

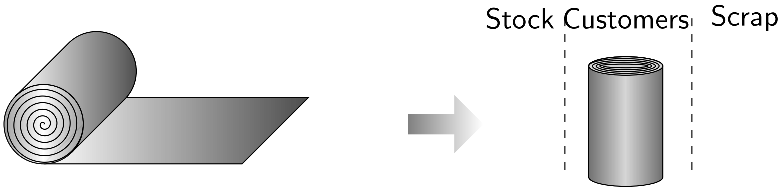
Literature	Dimension	Industry	Characteristics: Slitting	Two-stage	Usable leftovers	Residual stock	Modeling approach	Solution approach
Eisemann (1957)	1D	-	-	-	-	-	LP	General LP solver
Kantorovich (1960)	1D	-	-	-	-	-	LP	Method of resolving multipliers
Gilmore and Gomory (1961)	1D	-	-	-	-	-	ILP	Column generation
Gilmore and Gomory (1963)	1D	Paper	-	-	-	-	ILP	Column generation
Gilmore and Gomory (1965)	2D	-	-	-	-	-	ILP	Column generation
Coffield and Crisp (1976)	1D	-	✓	-	-	-	LP	LP solver (MPS/360)
Haessler (1978)	1.5D	Metal	✓	-	-	-	MILP	Heuristic (sequential generation of patterns)
Hinxman (1980)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Dyckhoff et al. (1985)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Sarker (1988)	1D	-	✓	-	-	-	DP	Dynamic progr.
Ferreira et al. (1990)	1.5D	Steel	✓	✓	-	-	MINLP	Heuristic
Sweeney and Haessler (1990)	1D	Paper	✓	-	-	-	ILP	Two-stage heuristic
Sweeney and Paternoster (1992)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Vasko et al. (1992)	1.5D	Steel	✓	-	-	-	MILP	Heuristic (MINSET)
Carvalho and Rodrigues(1995)	1.5D	Steel	✓	✓	-	-	LP	Column generation
Carvalho (1998)	1D	-	-	-	-	-	ILP	Column generation and Branch&Bound
Gradisar et al. (1999)	1D	-	-	-	-	✓	MILP	Sequential Heuristic (SHP)
Dutta and Fourer (2001)	-	Steel	-	-	-	-	Survey	Survey
Carvalho (2002)	1D	-	-	-	-	-	LP	Survey
Saraç and Özdemir (2003)	1.5D	-	-	-	-	-	MILP	Implicit enum. / genetic heuristic
Mukherjee and Ray (2006)	-	Metal	-	-	-	-	Survey	Survey
Wäscher et al. (2007)	1D,2D	-	-	-	-	✓	Survey	Survey
Gasimov et al. (2007)	1.5D	Corrugated box	✓	-	-	-	MILP, MINLP	Conic scalarization
Abuabara and Morabito (2008)	1D	Metal	-	-	✓	-	MILP	General LP solver
Cherri et al. (2009)	1D	-	-	-	✓	✓	-	Residual heuristics
Cherri et al. (2012)	1D	-	-	-	✓	✓	ILP	Heuristic (RGR)
Cheri et al. (2014)	1D	-	-	-	✓	-	Survey	Survey
Delorme et al. (2016)	1D	-	-	-	-	-	Survey	Survey
Coelho et al. (2017)	1D	-	-	-	✓	✓	ILP	Two residual heuristics
Tomat and Gradisar (2017)	1D	-	-	-	✓	✓	MILP	Heuristic search process
Santos et al. (2018)	1D	Steel	-	-	-	-	MILP	Two heuristics
This research	1.5D	Steel	✓	-	✓	✓	MILP	MIP solver (GUROBI 9.0.2)

Particular restrictions in the problem

In the case of this company, there are particular restrictions imposed by the customers that affect the way their orders need to be served. These restrictions include the maximum weight of the strip and the maximum external diameter allowed. It should be noted that given some properties such as width, thickness, density and internal diameter of the strip, the weight of a strip can be obtained from its external diameter. Hence, both requirements can be given in terms of



(a) Slit coil producing different strips



(b) No slits performed

Figure 1: Different possibilities to allocate orders to coils

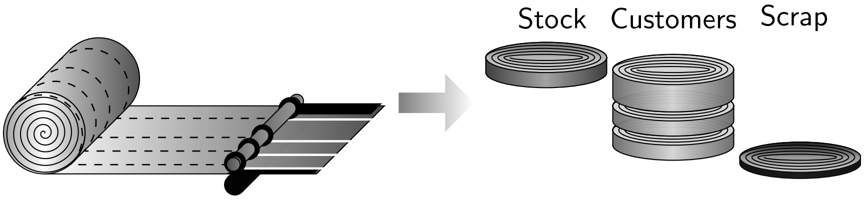
a maximum allowed weight for the strips where the most restrictive one is considered. For each order, the maximum allowed weight for the strips, as well as the size of the coil, would define the number of strips served.

In order to reduce the diameter, and consequently the weight of the strips to meet the customers' requirements, it is possible to make one or more guillotine cross-cuts with a shear blade. These cuts affect all the strips assigned to the coil and must ensure that each resulting strip has a certain weight which lies within the limits set by the customer. These limits may vary from customer to customer. For this reason, it is worth noting that the knives cannot be redirected after a cross-cut remaining the cutting pattern the same. Figure 2 presents an example of how a cross-cut reduces the external diameter or weight of the finished strips. In Figure 2a the external diameter of the finished strips is greater than those in Figure 2b where a cross-cut has been performed.

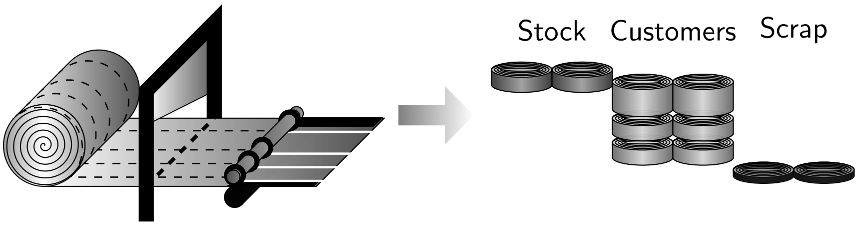
Each time a cross-cut is carried out, a decision must be made on whether to continue cutting or stop the cutting process. In the latter case, the remaining part of the coil is rewound and kept in stock for future cutting processes. It should be noted that this partial rewound process produces coils in stock with a variety of external diameters (see Figure 2c).

Compatibility of stock and orders

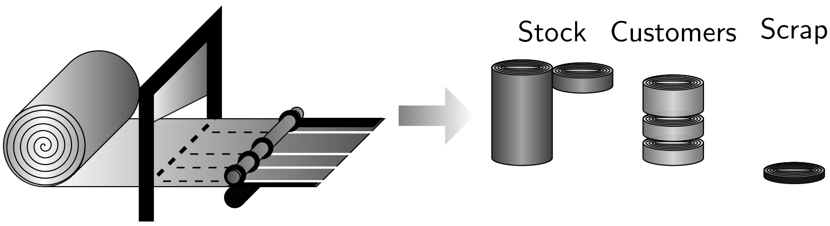
An additional feature that complicates the problem lies in the quality requirements requested by the customer, regarding the product being served. The company handles more than 20 parameters that define the technical characteristics of each coil, the type of material, its thickness, etc. For



(a) Strips obtained without cross cuts



(b) Strips obtained when a cross cut is made when slitting process continues



(c) Strips obtained when a cross cut is made stopping the slitting process

Figure 2: Example of strips obtained

each of these parameters, the customer requests specific values and, in some cases, small tolerances are allowed. For example, the type of material has to match both the order and stock. However, if the order requires a thickness of 2 mm with a tolerance of 0.1 mm, the thickness of the stock could range between 1.9 mm and 2.1 mm. These requirements should be considered in order to determine the set of compatible coils for each order. Nevertheless, since the tolerances are so small, the costs involved for the company are fairly insignificant.

These compatibility requirements create a problem that is more difficult to solve, as the same coil can be used to serve orders with different characteristics. As a result, the problem cannot be easily separated into several sub-problems. An illustrative example is shown in Figure 3, where any R type coil is compatible with order 1, but not with order 2. However, the M-coil is compatible with both orders 1 and 2. This situation can arise, for example, when customers allow their orders

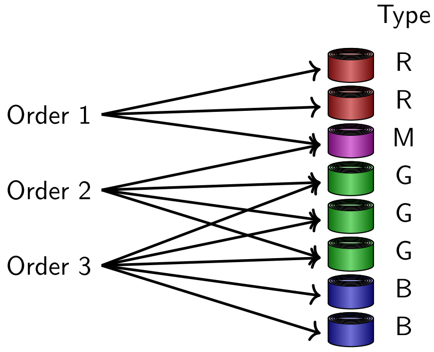


Figure 3: Illustrative example of compatibility between orders and stock

to be served with better quality products than requested, or with a certain thickness tolerance. It should be noted that this characteristic introduces a new difficulty: an order can be served with coils of different densities or weights per linear metre. Therefore, converting weight to length on a general basis for all coils is not possible and has to be carried out for each coil individually.

Cutting patterns

A number of cutting scenarios may arise, resulting in various types of cutting patterns, which are shown in Figure 4. With regard to slitting the coil, two situations may occur: (A) the strip width matches the coil width and, (B) the strip width is narrower than the coil width. In case A, the coil is not slit while in case B, the coil is slit and at least two knives are needed for the slitting machine to perform the cuts. In both cases, A and B, the coil can be fully used lengthwise (cases A1, A2, B1 and B2 in Figure 4) or partially used (cases A3 and B3 in Figure 4). In cases A3 and B3, the remaining part without slits is rewound and restocked. One or more guillotine cross-cuts may be performed in order to meet the maximum strip weight requirements (cases A2, A3, B2 and B3).

Objective and problem hypotheses

The company pursues the following goals, namely:

- Maximise the utilisation of each coil.

  The company currently uses about 50-60% of the weight of the coils to meet demand. The leftovers of the cutting process are 1) discarded if their weight or width are not large enough; 2) stocked for future use; or 3) used to prepare strips for expected future orders. Based on

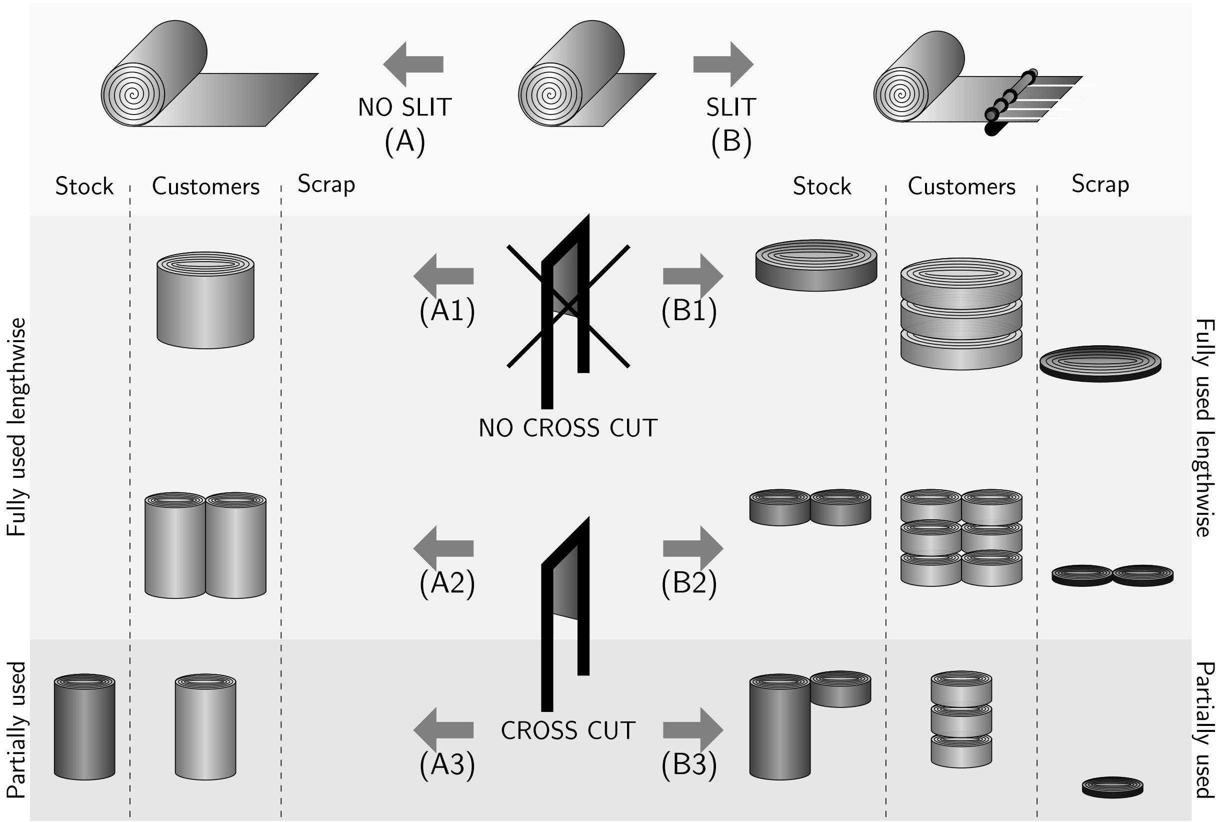


Figure 4: Different cutting patterns considered

past record data, the company can forecast future demand and then seek to make efficient use of the unused pieces by anticipating future demand. These three situations imply economic costs to the company that include the generation of scrap, inventory costs and the risk of producing non-demanded strips that would cause an increase in the stock.

In order to improve the current operation process, the model includes a penalisation for the weight of the leftovers. This penalisation is greater for scrap than for retails. It should be noted that the third option, that of anticipating possible future demand, is only carried out by the company as a last option to reduce scrap, and it is not contemplated in the model.

- Adjust the weight served to the real demand.

  Owing to the way the company operates, it is very difficult to serve the exact amount of product that each customer has requested. Currently, the company works with a $\pm 20\%$ tolerance; this seems reasonable for the company since most of the customers are regular ones

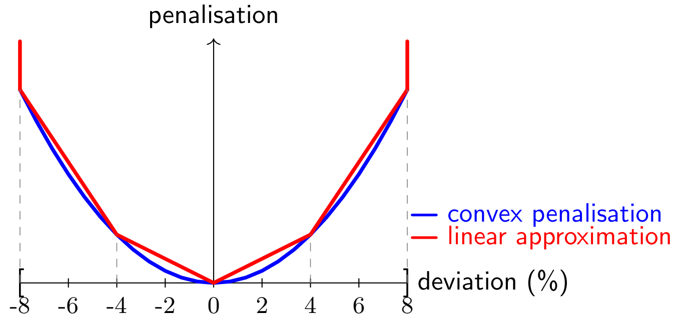


Figure 5: Penalisation of deviation from actual weight for each order

and, therefore, part of the demand can be transferred to other days, usually at a cost in the form of a discount. To reduce this deviation, the objective function includes a penalisation. A non-linear (convex) function is used which results in larger deviations being more penalised than smaller ones. This convex function can be approximated by a piece-wise linear function without the need of binary variables. More specifically, we have considered two intervals for the approximation: up to a certain level, deviations are acceptable as it is rather difficult to meet the exact weight ordered for all of the customers, but for deviations over the maximum *desired* deviation, a greater penalisation is imposed. Either way, a maximum *allowed* deviation is established. Figure 5 shows an example, where a deviation up to 4% of the weight ordered is less penalised than a deviation from 4% to 8%. The latter is the maximum deviation allowed.

In summary, the problem under study considers the following hypotheses:

- Customers place their orders by specifying a certain width and total weight. Neither the number of strips nor their weight are predefined.
- All strips supplied should meet the customer's requirements in terms of a maximum permitted weight.
- The existing stock consists of coils with different sizes, and includes the retails from former cutting processes.
- Compatibility requirements imply that different types of coils are considered for a given order.

- Both slits and guillotine cross-cuts are considered.
- After a cross-cut, the reconfiguration of knives is not allowed. Either the rest of the coil is cut with the same configuration of the knives or it is rewound and kept in stock.
- A minimum edge trim is required when slitting the coil.
- Leftovers are divided into retails (usable leftovers) and scrap.
- Customer's orders have to be fulfilled. A maximum allowed deviation on the weight served with respect to the requested weight is considered.

4. Mixed Integer Linear optimisation model

Below, a Mixed Integer Linear optimisation model to solve the problem is presented.

4.1. Notation

- $\mathcal{O}$, set of customer's orders. The following information is given for each order $o \in \mathcal{O}$:

  a_o, width (m) of strips.

  b_o, required weight (kg).

  $\overline{b}_o$, maximum weight (kg) allowed for each single strip of the order.

  $\overline{u}_o$, $\overline{u}_o^{\mathbf{d}}$, maximum absolute deviation on the required weight (kg) that is permitted and desired, respectively.

- $\mathcal{C}$, set of coils in stock. For each coil $c \in \mathcal{C}$ the following data are known:

  A_c, width (m).

  B_c, weight (kg).

  L_c, length (m).

  $\underline{L}_c, \overline{L}_c$, for partially used coils, minimum and maximum length (m) used from the coil, respectively. $\overline{L}_c$ is such that it guarantees that the rest of the coil can be rewound and kept in stock.

  D_c, weight (kg) per m^2. It depends on the density and thickness of the coil.

J_c, maximum number of knives.

$\mathcal{J}_c = \{1, \dots, J_c - 1\}$, index set for the slits performed.

$\mathcal{O}_c$, set of orders that are compatible with the coil.

- Parameters for operation configuration:

  r, minimum edge trim required for quality purposes.

  $\underline{a}, \underline{b}$, minimum width and weight required for a trim waste to be reusable, respectively.

- Objective function:

  q, q^{d} penalisation for the allowed and desired deviation of the weight served with respect to the required weight, respectively. Note: $q > q^{\mathrm{d}}$.

Decision variables

$\alpha_c = 1$, if coil c is used, 0 otherwise, $c \in \mathcal{C}$.

$\delta_c^{\mathrm{T}} = 1$, if coil c is fully used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\delta_c^{\mathrm{P}} = 1$, if coil c is partially used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_{c0} = 1$, if coil c is used without slitting, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_{cj} = 1$, if the j-th slit is performed in coil c, 0 otherwise, $c \in \mathcal{C}, j \in \mathcal{J}_c : j > 0$.

$\mu_{ocj} = 1$, if order o is assigned to the j-th slit of coil c, 0 otherwise, $c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c$.

$\theta_c = 1$ if trim waste of coil c is reusable, 0 otherwise, $c \in \mathcal{C}$.

β_c, number of guillotine cross-cuts made in the used part of coil c, $c \in \mathcal{C}$. It should be noted that this variable does not count the last cross-cut made if the coil is partially used, in other words, it only counts the cross-cuts made to assure that the weights of the strips are less than the maximum allowed. Observe Figure 2b where $\beta = 1$ and Figure 2c where $\beta = 0$.

x_c, used length of coil c, $c \in \mathcal{C}$.

v_{ocj}, length of the strip of order o obtained from the j-th slit made in coil c, $c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c$.

u_o^+, u_o^-, excess and lack of weight served for order o, $o \in \mathcal{O}$.

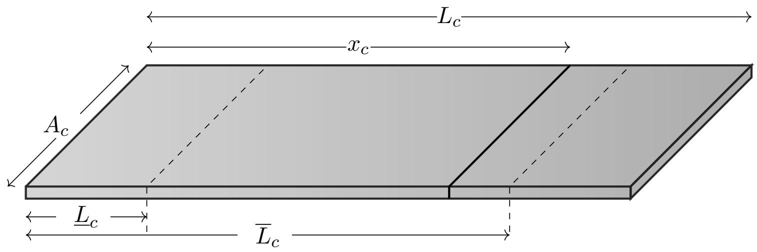


Figure 6: Parameters referred to a given coil

$u_o^{\mathrm{d}+}, u_o^{\mathrm{d}-}$, excess and lack of weight served for order o within the maximum desired deviation, $o \in \mathcal{O}$.

y_c, width of the leftovers of coil $c \in \mathcal{C}$ which is broken down in the sum of y_c^{r}, for retails and y_c^{s}, for scrap, $c \in \mathcal{C}$.

$z_c^{\mathrm{r}}, z_c^{\mathrm{s}}$, weight of the retails and scrap of coil c, $c \in \mathcal{C}$.

Figure 6 shows some elements of the notation on an unwound coil and Table 2 indicates how the values of the variables determine the different cutting patterns described in Figure 4.

	Fully used lengthwise $\delta_c^{\mathrm{T}} = 1, x_c = L_c$		Partially used $\delta_c^{\mathrm{P}} = 1, \underline{L}_c \le x_c \le \overline{L}_c$
	(1) No cross-cut	(2) Cross-cut	(3) Cross-cut
(A) No slit	$\gamma_{c1} = 0,\ \beta_c = 0$	$\gamma_{c1} = 0,\ \beta_c > 0$	$\gamma_{c1} = 0,\ \beta_c \ge 0$
(B) Slit	$\gamma_{c1} = 1,\ \beta_c = 0$	$\gamma_{c1} = 1,\ \beta_c > 0$	$\gamma_{c1} = 1,\ \beta_c \ge 0$

Table 2: Relationship between variables and the different cutting patterns for a coil c

4.2. Mathematical formulation

Objective function

The objective function is a weighted sum of three elements:

$$\min\Big\{w_1\Big(\sum_{c\in\mathcal{C}}\big(B_c\alpha_c - D_cA_cx_c\big) + z_c^{\mathrm{r}}\Big) + w_2\sum_{c\in\mathcal{C}} z_c^{\mathrm{s}} + w_3\sum_{o\in\mathcal{O}}\big(q(u_o^+ + u_o^-) + (q^{\mathrm{d}} - q)(u_o^{\mathrm{d}+} + u_o^{\mathrm{d}-})\big)\Big\} \quad (1)$$

where w_1, w_2 and w_3 represent the weights assigned to each component:

1. Usable leftovers or retails. There are two kinds of usable leftovers: the rewound coils obtained from the partially used coils and retails that appear when a coil is not fully used widthwise.

2. Scrap: Leftovers whose weight and/or width are below a certain threshold.
3. Difference between the weight served and the weight that is actually ordered by each customer. Two different penalisations are considered: q^{d} for deviation within the desirable limits and q for the excess deviation over these desirable limits.

Constraints

1. Cutting patterns: Constraint (2) states, for each used coil, whether it is served with or without slitting. Constraint (3) forces not to make a slit if the previous slit has not been performed, in other words, introduce an order in the slits. Constraint (4) states, for used coils, whether they are completely or partially slit. Constraint (5) guarantees that no guillotine cross-cuts are performed on unused coils.

$$\gamma_{c0} + \gamma_{c1} = \alpha_c \quad \forall c \in \mathcal{C} \tag{2}$$

$$\gamma_{cj} \leq \gamma_{cj-1} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c : j > 1 \tag{3}$$

$$\delta_c^{\mathrm{T}} + \delta_c^{\mathrm{P}} = \alpha_c \quad \forall c \in \mathcal{C} \tag{4}$$

$$\beta_c \leq G_c \alpha_c \quad \forall c \in \mathcal{C}, \tag{5}$$

   Where G_c is an upper bound of the number of guillotine cross-cuts in coil c.

2. Assignment of orders to strips: Constraint (6) assigns exactly one order to each strip (it should be noted that an order can be assigned to one or more slits). Constraint (7) ensures that the width of each coil is not exceeded. Constraint (8) ensures a minimum edge trim in slit coils.

$$\sum_{o \in \mathcal{O}_c} \mu_{ocj} = \gamma_{cj} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c \tag{6}$$

$$\sum_{j \in \mathcal{J}_c} \sum_{o \in \mathcal{O}_c} a_o \mu_{ocj} + y_c = A_c \alpha_c \quad \forall c \in \mathcal{C} \tag{7}$$

$$2r\gamma_{c1} \leq y_c \leq A_c \gamma_{c1} \quad \forall c \in \mathcal{C} \tag{8}$$

3. Bounds on the used length of the coils: Constraint (9) forces the used length of the coils to be equal to the total length of the coil when it is completely slit and between the given bounds when it is partially slit.

$$L_c \delta_c^{\mathrm{T}} + \underline{L}_c \delta_c^{\mathrm{P}} \leq x_c \leq L_c \delta_c^{\mathrm{T}} + \overline{L}_c \delta_c^{\mathrm{P}} \quad \forall c \in \mathcal{C} \tag{9}$$

4. Length of the strips ordered: Constraint (10) allows to assign a length only to the slits assigned to the orders. Constraint (11) guarantees the length of all the strips in a coil to be equal to the used length of the coil.

$$\underline{L}_c\mu_{ocj} \leq v_{ocj} \leq L_c\mu_{ocj} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{10}$$

$$0 \leq x_c - v_{ocj} \leq L_c(1 - \mu_{ocj}) \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{11}$$

5. Guillotine cross-cuts: Constraint (12) computes the number of guillotine cross-cuts needed in coil c to keep the weight of the strips lower than the maximum allowed for each order assigned to this coil.

$$(D_c a_o)v_{ocj} \leq \overline{b}_o(\beta_c + 1) \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{12}$$

6. Demand: Constraints (13) –(14) compute the lack or excess weight served to each order.

$$\sum_{c\in\mathcal{C}:o\in\mathcal{O}_c} \sum_{j\in\mathcal{J}_c} (D_c a_o)v_{ocj} - u_o^+ + u_o^- = b_o \quad \forall o \in \mathcal{O} \tag{13}$$

$$u_o^{\mathrm{d}+} \leq u_o^+,\ u_o^{\mathrm{d}-} \leq u_o^- \quad \forall o \in \mathcal{O} \tag{14}$$

7. Leftovers: When leftovers are rewound for further use, they are considered as usable. Alternatively, the material is discarded and considered as scrap. When leftovers are usable, there is always a weight that represents the minimum edge trim required which is considered as scrap. Thus, the width of the leftovers can be divided into usable (retails) and non-usable (scrap) parts (15). In an analogous manner, the weight of the leftovers can also be divided into usable and non-usable parts (16). Constraints (17)-(20) assure that if leftovers are usable, their width and weight should be at least the minimum established. Failure to comply with at least one of these conditions will result in the leftovers as being considered scrap. Finally, constraint (21) states that leftovers only appear when the coil is slit.

$$y_c = y_c^{\mathrm{r}} + y_c^{\mathrm{s}} \quad \forall c \in \mathcal{C} \tag{15}$$

$$(D_c A_c)x_c - \sum_{o\in\mathcal{O}_c} \sum_{j\in\mathcal{J}_c} (D_c a_o)v_{ocj} = z_c^{\mathrm{r}} + z_c^{\mathrm{s}} \quad \forall c \in \mathcal{C} \tag{16}$$

$$\underline{a}\theta_c \leq y_c^{\mathrm{r}} \leq A_c\theta_c \quad \forall c \in \mathcal{C} \tag{17}$$

$$\underline{b}\theta_c \leq z_c^{\mathrm{r}} \leq B_c\theta_c \quad \forall c \in \mathcal{C} \tag{18}$$

$$r\theta_c \le y_c^{\mathrm{s}} \le r\theta_c + \overline{s}_c(1-\theta_c) \quad \forall c \in \mathcal{C} \tag{19}$$

$$(D_c r \underline{L}_c)\theta_c \le z_c^{\mathrm{s}} \le (D_c r \overline{L}_c)\theta_c + B_c(1-\theta_c) \quad \forall c \in \mathcal{C} \tag{20}$$

$$\theta_c \le \gamma_{c1} \quad \forall c \in \mathcal{C} \tag{21}$$

Constant $\overline{s}_c$ in constraint (19) is an upper bound of the width of the leftovers of coil c to be considered as scrap. It can be computed as the maximum between $\underline{a}$ and the width corresponding to the leftovers of weight $\underline{b}$:

$$\overline{s}_c = \max\left\{\underline{a}, \frac{\underline{b}}{D_c \underline{L}_c}\right\}$$

8. Variables' domain:

$$\alpha_c, \delta_c^{\mathrm{T}}, \delta_c^{\mathrm{P}}, \theta_c \in \{0,1\} \quad \forall c \in \mathcal{C} \tag{22}$$

$$x_c, y_c, y_c^{\mathrm{r}}, y_c^{\mathrm{s}}, z_c^{\mathrm{r}}, z_c^{\mathrm{s}} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C} \tag{23}$$

$$\beta_c \in \mathbb{Z}_0^+ \quad \forall c \in \mathcal{C} \tag{24}$$

$$\gamma_{cj} \in \{0,1\} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c \tag{25}$$

$$u_o^+, u_o^- \in [0, \overline{u}_o] \quad \forall o \in \mathcal{O} \tag{26}$$

$$u_o^{\mathrm{d}+}, u_o^{\mathrm{d}-} \in [0, \overline{u}_o^{\mathrm{d}}] \quad \forall o \in \mathcal{O} \tag{27}$$

$$\mu_{ocj} \in \{0,1\} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{28}$$

$$v_{ocj} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{29}$$

5. Computational experience

The model presented above has been tested using real data provided by Cortichapa, a Spanish steel company interested in improving the planning of steel strip production to meet their customers' orders, and also reducing the leftovers generated during the cutting process. The company holds a permanent stock level of 30,000 tonnes with more than 400 different types of products, and a service and delivery capacity of up to 1,000 tonnes per day, for more than 100 customers.

5.1. Case study and current operation

In order to carry out the computational research, the model was tested using real data (available stock and orders) for 11 working days in 2019. Additionally, we were provided with the solution

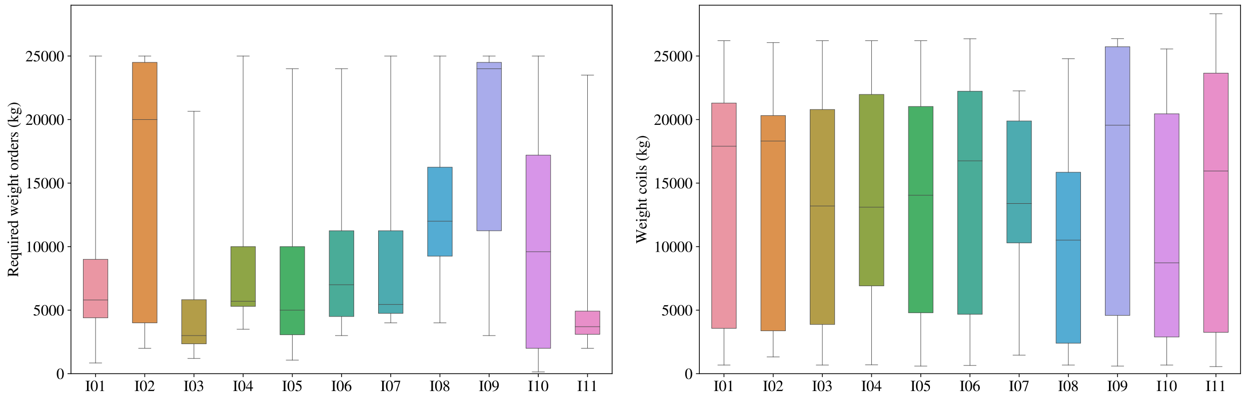
Table 3: Main characteristics of instances

Instance	I01	I02	I03	I04	I05	I06	I07	I08	I09	I10	I11
# orders	15	7	14	9	14	11	8	7	7	13	10
Total required weight (t)	115	104	83	90	112	97	74	92	123	123	61
# compatible coils	380	76	222	175	193	271	65	207	154	180	68
Weight of compatible coils (t)	5,192	1,085	2,972	2,486	2,740	4,024	954	2,027	2,442	1,941	999

applied by the company which was compared with the one provided by the model. The actual planning of the cutting process takes several hours and very often, there are situations where the company cannot find a way to complete and fulfil all its orders in one day. In other words, the weight served in one day is not within the permitted deviations from the total weight required. Consequently, those orders remain open for the next day. In addition, as mentioned before, the anticipation of future demand is practised by the company (a posteriori) by planning make-to-stock orders to complete the coils used. In order to validate the model, and at the company's request, the records only include confirmed orders that have been completely fulfilled, thereby allowing there to be a comparison between both solutions.

Table 3 reports the main characteristics of the instances tested: number of orders, total weight required in tonnes, number of compatible coils in stock and their total weight in tonnes. Additionally, for each instance, Figure 7 shows the distribution of the weights of the orders (7a) and the distribution of the weights of the compatible coils in the stock (7b). It should be noted that instances $I03$ and $I11$ have mainly small orders and little variability, while the variability in the weight of the order is greater for instances $I02$, $I09$ and $I10$. Regarding the weight of the coil, the instances are more homogeneous.

At the beginning of the day, the company has around 5000-6000 coils available in stock, with thicknesses that vary from 0.48 mm to 6 mm. The thickness is directly related to the maximum number of knives that can be configured in the slitting machine (varying from 4 to 18 knives). Some customers allow a certain degree of tolerance in the thickness; this tolerance can range from 0.05 mm to 0.21 mm. The coils held in stock have an external diameter that may reach 5500 mm, providing that the maximum diameter established by the customer does not exceed 2000 mm, it would be necessary to include cross-cuts to ensure that the strips served do not exceed this limit. Another aspect that may imply cross-cuts is the maximum weight per strip that varies from 140 to



(a) Distribution of weight required (tonnes)

(b) Distribution of coil's weight (tonnes)

Figure 7: Distribution of order and compatible sizes

16000 kg. Finally, the widths of the coils vary from 77 mm to 1500 mm, while customers demand strips that range from 36 mm to 1500 mm.

Table 4 presents the main performance indicators of the solution currently implemented in the company. In each instance, the number of coils used in the solution and their total weight in kg are indicated, together with the coils served, the retail weights and the weights of the scrap. In addition, the percentage that each weight represents over the total weight used is also indicated. Leftovers are considered retail when their widths and weights are greater than 100 mm and 500 kg respectively. The *Slit* and *Cross* columns provide the number of slits performed on average per coil, and the total number of guillotine cross-cuts made in the solution, respectively. The *Rewound* column provides the number of coils that are rewound. Deviations up to ±20% from the required weight are allowed and, in order to measure how the solution behaves in this matter, we define the order accuracy as the ratio between the required weight and the served weight. In this way, the order accuracy values over 1 indicate that the served weight is more than the required weight. Finally, the last three columns of Table 4 report the minimum, average and maximum values of the order accuracy obtained using the current solution established by the company. As an illustrative example of this indicator, in instance 1, at least 84% (resp. 117% maximum) of the required weight was served for all of the orders, the average being 101%.

Table 4: Real operation performance indicators

	Used coils		Served		Retail		Scrap		Cuts		Rewound	Order accuracy		
Instance	#	Weight	Weight	%	Weight	%	Weight	%	Slit	Cross	#	Min	Mean	Max
I01	14	251340	113680	45.23	136438	54.28	1222	0.49	6	4	3	0.84	1.01	1.17
I02	8	126780	104032	82.06	22086	17.42	662	0.52	7	2	2	0.99	1.01	1.04
I03	12	217140	86416	39.80	129177	59.49	1547	0.71	4	2	0	0.98	1.04	1.18
I04	12	207130	88942	42.94	117168	56.57	1020	0.49	4	1	0	0.89	1.01	1.11
I05	14	186293	110688	59.42	73127	39.25	2478	1.33	6	8	2	0.88	1.00	1.08
I06	15	251175	98899	39.37	150746	60.02	1530	0.61	4	3	3	0.92	1.04	1.13
I07	7	147920	73395	49.62	73577	49.74	948	0.64	9	0	0	0.93	1.02	1.14
I08	17	174800	90505	51.78	83050	47.51	1245	0.71	2	0	0	0.83	1.01	1.12
I09	9	186490	114793	61.55	70562	37.84	1135	0.61	7	0	2	0.88	0.95	1.00
I10	18	227102	120210	52.93	104852	46.17	2040	0.90	4	2	7	0.81	0.98	1.06
I11	11	125652	62994	50.13	61362	48.84	1296	1.03	5	0	1	0.95	1.03	1.20
Average	12.4	191075	96778	50.65	92922	48.63	1375	0.72	5.3	2.0	1.8	0.90	1.01	1.11

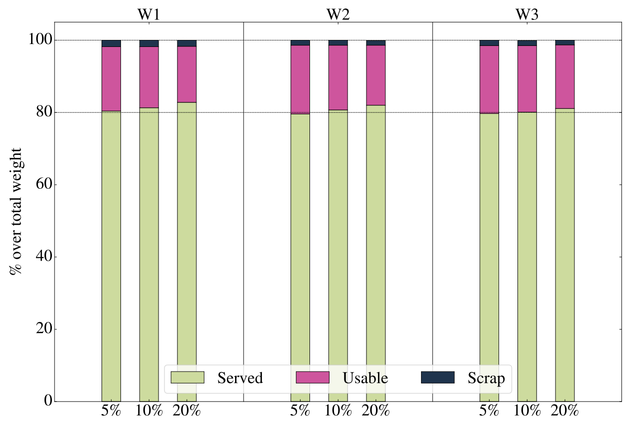
5.2. Experiments

The model has been implemented using the algebraic modeling language AMPL and solved using the MIP Gurobi v.9.0.2 optimizer in a virtual machine managed by OpenStack with 4 CPUs, 8GB RAM, Ubuntu 18.04 OS. The default Gurobi settings are used, except for the time limit, which is set to 10 minutes. In addition, different experiments have been performed varying the weights of the objective function: minimizing both retail sales and scrap, as well as the deviation from the weight required for each order.

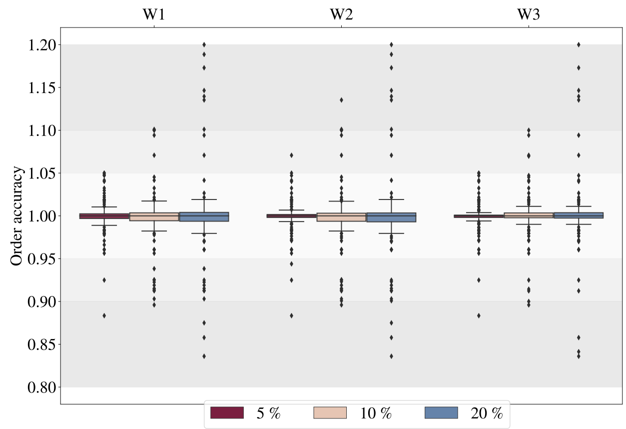
Regarding the first goal, a higher penalisation has been assigned to the scrap than to the retails, while for the second goal; the maximum deviation allowed has been set equal to $\pm 20\%$, which is the one currently used by the company. In addition to this value, different limits have been tested for the desired deviation: $\pm 20\%$, $\pm 10\%$ and $\pm 5\%$. In order to achieve solutions that are adjusted as close as possible to the required limits, deviations above the required limit have been further penalised. In particular, the values used for q and q^{d} are 10 and 1, respectively.

Furthermore, three different weight combinations (w_1, w_2, w_3) have been tested and are used to penalise retails, scrap and deviation from the weight ordered, respectively, $W1 = (1, 3, 2)$, $W2 = (1, 4, 2)$ and $W3 = (1, 4, 3)$.

Each of the combinations has been tested in the set of instances presented above. Figure 8a



(a) Utilisation of coils



(b) Order accuracy variability

Figure 8: Performance of the model for different combinations of weights and desired deviations

shows the performance of the model: the percentage of the weight served to customers over the weight of the used coils, the percentage of the weight of the retails held in stock and the percentage of scrap that is discarded. These data correspond to the aggregation of all the orders included in all instances. Moreover, it can be observed that in almost all cases, more than 80% of the total weight is served to customers, and the higher the maximum desired deviation, the more efficient the coil usage, although the differences are not really significant. The performance of the model for both, $W2$ and $W3$ weights, is very similar. It is worth noting that the penalisation set $W1$ provides an increase of 1% in the weight served together with an increase of 1% in scrap.

The distribution of the order accuracy is represented in Figure 8b. Each box-plot shows the distribution of the accuracy obtained in all orders of the instances. As would be expected, one can observe that in the three penalisation sets used, the variability of the deviations is reduced as is the maximum desired deviation. Although the differences are not very relevant, we have decided to discard the penalisation set $W1$ since it increases the scrap, which is contrary to one of the goals of the company. It is worth pointing out that in all cases, very few orders are outside the desired limits, although it can be observed that the penalisation set $W3$ is slightly better adjusted than $W2$.

Therefore, taking into account the information in Figure 8 and although the differences are not very distinguishable, we will use penalisation set $W3$ and a maximum desired deviation of $\pm5\%$ for the remaining analysis.

Table 5: Computational statistics: Model dimensions and solution

Instance	# cons.	# vars.	# int.vars.	# 0-1 vars.	time (s)	Z_{IP}	gap (%)
I01	57590	30077	380	17510	264.39	34501.3	0.00
I02	8233	4378	76	2560	1.51	17484.1	0.00
I03	39154	20404	222	11388	600.19	27811.7	10.24
I04	28045	14650	175	8242	275.53	15310.5	0.00
I05	23984	12644	193	7489	10.28	39692.5	0.00
I06	39503	20669	271	11918	125.63	40634.8	0.00
I07	12215	6383	65	3482	600.21	20654.3	9.28
I08	40573	21058	207	11688	600.23	7457.3	23.63
I09	23733	12402	154	7114	126.60	20167.0	0.00
I10	20090	10659	180	6208	3.20	80280.3	0.00
I11	8760	4632	68	2736	1.74	68563.4	0.00

Table 5 shows a number of statistics on the computational performance of the model. The following information is presented for each instance: the number of constraints, the variables, the integer and binary variables, the value of the objective function within the best feasible solution provided, and the optimality gap for this solution. It is possible to observe that in eight of the instances, the model provides an optimal solution, and in only three instances, I03, I07 and I08, the optimality gap is greater than 9%. It is worth noting that, as we will observe below, in instance I08, there is an optimality gap greater than 23%, 98% of the weight of the used coils is served to the customers and the order accuracy varies between 0.99 and 1.01. Therefore, it appears that the optimality gap is quite large because of the quality of the lower bound, while the proposed solution is close to the optimal one. The model was run for two hours for each instance, and obtained a gap of 0% for instances I03 and I07 with the same objective value, whereas the gap of instance I08 was reduced to 7% and the best known solution improved 5%.

5.3. Interpretation and analysis of the obtained results

Table 6 provides a number of statistics on the quality of the solution after solving the proposed mathematical model in the same terms as in Table 4. It also indicates the number and weight of coils used, the coils served, the retails and the scrap, including the percentage that each of these

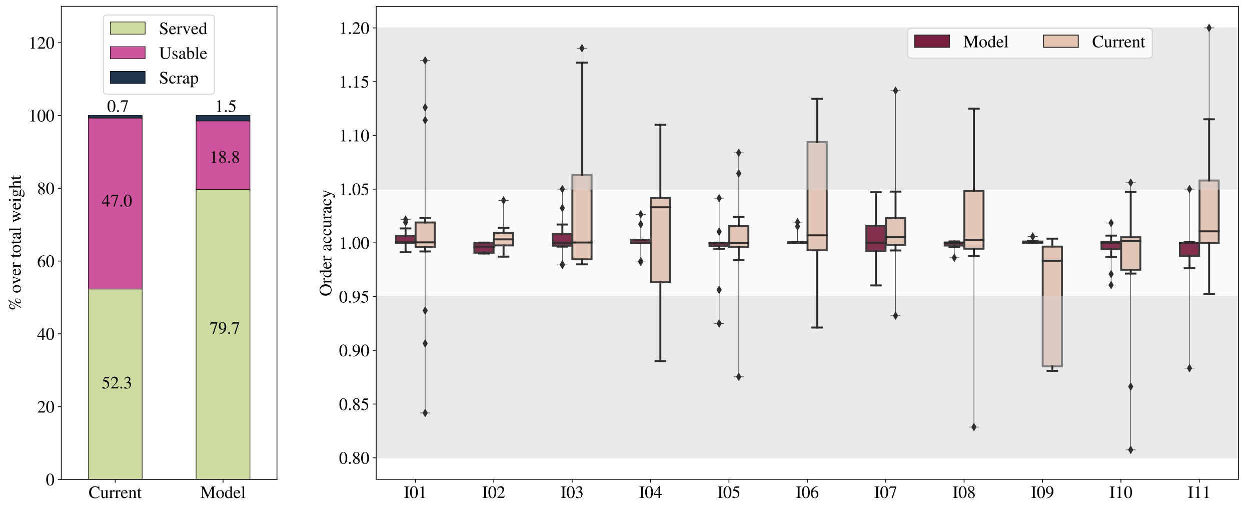
Table 6: Model performance indicators

	Used coils		Served		Retail		Scrap		Cuts		Rewound	Order accuracy		
Instance	#	Weight	Weight	%	Weight	%	Weight	%	Slit	Cross	#	min	mean	max
I01	18	140732	115587	82.13	22562	16.03	2583	1.84	8	1	2	0.99	1.00	1.02
I02	10	118173	103445	87.54	14143	11.97	585	0.49	6	3	1	0.99	1.00	1.00
I03	10	102744	83330	81.10	17119	16.66	2295	2.23	6	4	4	0.98	1.00	1.05
I04	11	101970	89874	88.14	11329	11.11	767	0.75	9	0	2	0.98	1.00	1.03
I05	20	141963	111580	78.60	28280	19.92	2103	1.48	6	7	3	0.92	0.99	1.04
I06	17	130054	97289	74.81	29586	22.75	3178	2.44	4	3	5	1.00	1.00	1.02
I07	7	91032	73982	81.27	16645	18.28	406	0.45	8	1	2	0.96	1.00	1.05
I08	13	93922	91794	97.73	578	0.62	1550	1.65	9	0	0	0.99	1.00	1.00
I09	13	138591	123634	89.21	13159	9.49	1798	1.30	6	1	1	1.00	1.00	1.01
I10	19	194924	123098	63.15	69631	35.72	2195	1.13	5	1	3	0.96	1.00	1.02
I11	16	113373	60308	53.19	50016	44.12	3049	2.69	5	5	6	0.88	0.99	1.05
Average	14.0	124316	97629	78.53	24823	19.97	1864	1.50	6.0	2.4	2.6	0.97	1.00	1.03

weights represents over the total weight used, the number of slits and cross-cuts performed and, finally, the minimum, the average and the maximum order accuracy achieved.

Figure 9 provides a graphical comparison between the performance of the solution proposed by the mathematical model and the solution which is currently implemented in the company. On the other hand, Figure 9a shows the utilisation of the coils for both solutions. On average, under the solution that is currently applied, 52.3% of the total weight is used to serve the orders, 47% of it is retails and the remaining 0.7% is considered as scrap. On the other hand, under the solution proposed by the model, 79.7% of the total weight is served, 18.8% is stocked as retails and the remaining 1.5% is considered as scrap. Although the amount of scrap increases from 0.7% to 1.5%, there is a more efficient use of the coils, since the model manages to reduce the weight of the retails to one third. This implies a direct reduction in the management costs of these retails, and, consequently, a reduction in the management costs in the warehouse. In general terms, the mathematical model aims at increasing the use of the coils by using the available stock, while at the same time, reducing it since the amount of retails is also reduced.

Figure 9b shows the distribution of the order accuracy for both solutions. The horizontal band with a white background corresponds to the maximum desired deviation of 5%. It is evident how clearly the model provides solutions as most of the orders are within these limits (except for



(a) Utilisation of coils

(b) Order accuracy variability

Figure 9: Comparison between model solution and the solution currently implemented

instances I05 and I11 which have an order accuracy below 95%). In addition, it can be observed that in the boxplot boxes (representing the distance between the first and third quartiles), the variability of the model solution is far lower since the boxes are much narrower and the whiskers considerably shorter. In other words, using the mathematical model, is possible to provide solutions that are better suited to the weight required by the customers with a consequent saving in material.

Finally, Figure 10 shows a scatter plot indicating the number of used coils along with their weight, for both, the currently used and the model solutions. Each instance is represented by two points connected by an arrow. One point for the current solution and the other for the model solution. A trend line is also shown, representing the relationship between the number of coils and their weight. It can be observed that in all cases, the weight of the used coils is lower in the solution proposed by the model (downward arrows) and there is also a clear tendency to use more coils in this solution (rightward arrows): there are only three cases where the current solution uses more coils. In addition, the dotted arrow shows the average values in terms of used weight and number of coils for the current and model solutions. On average, the number of coils increases while the weight used decreases. Therefore, it can be determined that the model proposes solutions where the coils are smaller. These small coils often correspond to retails from previous days which are more difficult to allocate, since the company tends to use large coils on a daily basis, which provides more flexibility in the planning. It is worth noting that this strategy will imply an increase in the

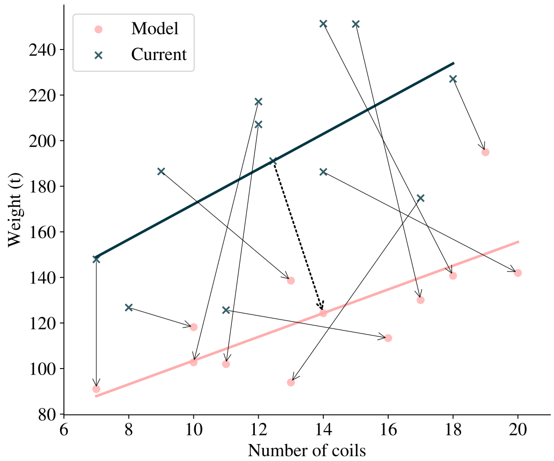


Figure 10: Number and weight of the used coils in the current and model solutions

stock, contrary to the solutions provided by the model which optimises the management of the available stock more efficiently.

5.4. Extended computational experiment

In order to assess the limits of the model, larger instances have been generated and solved by increasing the number of orders and coils in stock. Instances with 15, 30, 60, 90 and 120 orders are created and for each orders' size two different sets of stock are generated resulting in a total of ten different scenarios. For each scenario, 5 different instances are created by randomly selecting orders from a list of orders of the company and coils from the available stock, taking into account stock-orders compatibility. Therefore, the computational experiment includes 50 instances. The computational time limit for solving each instance has been set to 20 minutes.

Table 7 reports the average values obtained for each scenario. The following data are reported: the number of orders and coils, the dimensions of the model (number of constraints, variables, integer and binary variables), the computational time in seconds (time), the value of the objective function for the best known feasible solution (Z_{IP}) and the optimality gap given in % (gap). Last columns present the proportion of the weight of the coils used to serve orders, retails and scrap (minimum, mean and maximum values of the instances solved for each scenario are reported).

Regarding the performance of the model, it may be observed that as the size of the instances increases so does the optimality gap. Furthermore, in scenarios 9 and 10 only 4 and 3 out of 5 instances, respectively, were solved within 20 minutes. These results suggest that for larger

Table 7: Computational statistics: Average values for each scenario

Scenarios			Model dimensions				Performance			Served (%)			Retail (%)			Scrap (%)		
n	orders	coils	cons.	vars.	int.vars.	0-1 vars.	time	Z_{IP}	gap	min	mean	max	min	mean	max	min	mean	max
1	15	250	36732	19241	250	11025	641.1	66880	1.4	65.5	73.6	81.2	17.4	25.3	33.4	0.7	1.1	1.4
2	15	500	65137	34198	500	19958	561.1	43894	0.6	78.8	80.6	83.3	15.0	18.1	20.1	0.8	1.3	2.3
3	30	500	74906	39224	500	22338	968.9	145884	2.1	72.4	75.7	81.0	17.5	23.2	26.9	0.8	1.1	1.5
4	30	750	109974	57589	750	32655	985.5	118786	5.6	66.3	73.5	84.4	14.6	25.5	32.7	0.8	1.0	1.2
5	60	750	128897	67334	750	37320	1200.0[c]	266420	6.6	71.0	73.6	76.5	22.6	25.4	28.0	0.9	1.0	1.1
6	60	1000	167540	87518	1000	48553	1200.0[c]	239925	7.6	70.4	76.4	81.5	17.1	22.4	28.4	0.7	1.2	1.4
7	90	1000	205745	107011	1000	58059	1200.0[c]	323209	15.6	75.8	78.4	80.9	18.1	20.4	23.0	1.0	1.2	1.5
8	90	1500	311683	161911	1500	87799	1200.0[c]	296803	17.3	76.6	78.9	80.8	18.4	20.1	22.4	0.8	1.0	1.3
9[a]	120	1500	371608	192450	1500	102540	1200.0[c]	457106	28.5	76.5	78.4	79.5	19.2	20.5	22.6	0.9	1.1	1.3
10[b]	120	2000	464941	240958	2000	129066	1200.0[c]	512079	42.1	72.2	76.9	80.1	18.7	21.7	26.1	1.2	1.4	1.8
Average											76.6			22.3			1.1	

[a] 4 out of 5 instances were solved within 20 minutes. [c] No instance was solved to optimality within 20 minutes.

[b] 3 out of 5 instances were solved within 20 minutes.

problems the model will need more time to obtain a feasible solution and other strategies would have to be investigated to obtain good feasible solutions in reasonable time, such as heuristic or metaheuristic approaches.

However, if we look at the quality of the solution, we may highlight that the utilisation of the coils is similar in all scenarios, even for those with a large optimality gap. On average, 76.6% of the coils is used to serve orders, 22.3% is intended for retails and only 1.1% is considered scrap. This distribution is similar to the one obtained in the previous instances (see Table 6).

6. Conclusions and future work

A mixed integer linear optimisation model has been presented to address the specific cutting stock problem in a European Steel Company. The model has been validated with real data provided by the company, and has succeeded in surpassing its current performance. One of the main benefits of this approach is the reduction of the response time.

Mathematical optimisation is able to provide solutions which are difficult to analyse manually. Moreover, mathematical optimisation avoids mistakes caused by human errors, such as scheduling wrong quantities, which imply higher costs for the company. In technical terms, the model provides us with solutions that increase the size of each used coil that is effectively sent to the customers, and decrease the retails accordingly. This fact represents an improvement in the stock management such as saving time in locating the coils, as well as reducing the management costs of the raw materials. Besides these issues, the model is able to respond more efficiently to customers' orders by delivering a weight that is much closer to the one ordered.

As it has been studied in the extended computational experience, as the number of orders and available stock increase, so does the difficulty in solving the problem. Due to the characteristics of the problem, a heuristic strategy that decomposes the orders according to a rule based in the compatibility matrix between orders and stock could be investigated in order to deal with bigger instances.

As far as the future investigation is concerned, the company is interested in carrying out a planning process over several days simultaneously. Therefore, in our particular case, this new model needs to include more orders with their corresponding deadlines and a sequencing of the workload over different days, indicating which coils should be cut each day to meet the due dates. Set up costs due to the adjustments of the knives will need to be considered as well. At the same time, this new model will be far more complex and it will require more computational effort to solve full-size instances. At this phase, we therefore plan to develop approximate methods.

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