An approach for Strategic Supply Chain Planning under Uncertainty based on Stochastic 0-1 Programming

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Abstract

We present a two-stage stochastic 0-1 modeling and a related algorithmic approach for Supply Chain Management under uncertainty, whose goal consists of determining the production topology, plant sizing, product selection, product allocation among plants and vendor selection for raw materials. The objective is the maximization of the expected benefit given by the product net profit over the time horizon minus the investment depreciation and operations costs. The main uncertain parameters are the product net price and demand, the raw material supply cost and the production cost. The first stage is included by the strategic decisions. The second stage is included by the tactical decisions. A tight 0-1 model for the deterministic version is presented. A splitting variable mathematical representation via scenario is presented for the stochastic version of the model. A two-stage version of a Branch and Fix Coordination (BFC) algorithmic approach is proposed for stochastic 0-1 program solving, and some computational experience is reported for cases with dozens of thousands of constraints and continuous variables and hundreds of 0-1 variables.

Key Words: Supply Chain, BoM, Plant Sizing, Vendor Selection, Strategic Planning, Two Stage Stochastic, Splitting Variable, Branch-and-Fix Coordination.

1 Introduction

Supply Chain Management (SCM) is concerned with determining supply, production and stock levels in raw materials, subassemblies at different levels of the given *Bills of Material (BoM)*, end products and information exchange through (possibly) a set of factories, depots and dealer centers of a given production and service network to meet fluctuating demand requirements, see Escudero et al. (1999b), MirHassani et al. (1999) and Hahn et al. (2000), among others. Four key aspects of the problem are identified, namely, *supply chain topology, time, uncertainty* and *cost*. The uncertainty aspect of the problem is due to the stochasticity inherent to some parameters for dynamic (multiperiod) planning problems; in our case, the main uncertain parameters are product demand and price, raw material supply cost and production cost.

In these circumstances, and following the classical taxonomy of planning/scheduling problems in strategic, tactical and operational problems proposed by Anthony (1965), the tactical supply

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chain planning problem consists of deciding on the best utilization of the available resources included by vendors, factories, depots and dealer centers along the time horizon, such that given targets are met at a minimum cost. The tactical planning problem assumes that the supply chain topology is given. The subject of the paper is the strategic planning for supply chains and, so, the problem consists of deciding on the production topology, plant sizing, product selection, product allocation among plants and vendor selection for raw materials. The objective is the maximization (in constant terms) of the expected benefit given by the product net profit over the time horizon minus the investment depreciation and operation costs.

There are important differences between strategic and tactical planning problems, see Bitran and Tirupati (1993), among others. Perhaps the most important difference is the own character of the problems. Tactical planning is devoted to better utilization of available resources and strategic planning is devoted to better acquisition of resources so that the tactical planning profit minus the resources depreciation cost is maximized subject to given strategic constraints. Two of the most crucial characteristics of the strategic planning is the potential high stochasticity of the parameters due to longer time horizon lengths, and the 0-1 character of the strategic decision variables. Very often there are thousands of constraints and variables for deterministic situations. Given today Optimization state-of-the-art tools, deterministic strategic and tactical planning should not present major difficulties for problem solving. However, it has long being recognized (Beale, 1955; Dantzig, 1955) that traditional deterministic optimization is not suitable for capturing the truly dynamic behaviour of most real-world applications and, certainly, strategic supply chain planning is one of them. The main reason is that such applications involve, as we said above, data uncertainties which arise because information that will be needed in subsequent decision stages is not available to the decision maker when the decision must be made.

There is an extensive literature on dynamic production/scheduling planning. See hierarchical approaches in Graves (1986) and Bitran and Tirupati (1993); single level based systems in Karmarkar (1989) and van Hoesel et al. (1989); multi-level based systems in Goyal and Gunasekeran (1990) and Escudero (1994); systems for line balancing in Pocket and Wolsey (1991); systems with lot size, inventory holding and setup considerations in Wagner and Within (1958), Shapiro (1993), Dillenberger et al. (1994), Constantino (1996) and Wolsey (1997); and transportation and inventory integration systems in Romero Morales (2000), among others. Billington et al. (1986), Cohen and Lee (1989) and Shapiro (1993), among others present models for global optimization of multi-level supply chains.

All the above references present models and algorithmic schemes for deterministic environments. So the uncertainty inherent to most of the important parameters is not dealt with. However, the treatment of the stochasticity is relatively recent in production planning. See Zipkin (1986), Modiano (1987), Eppen et al. (1989), Sethi and Zhang (1994), Wagner and Beman (1995), Cheung and Powell (1996), Baricelli et al. (1996), Mitra et al. (1997), Escudero et al. (1999b), MirHassani et al. (1999), Albornoz and Contesse (1999), Ahmed et al. (2000a) and Tommasgard and Høeg (2001), among others, for interesting approaches on production planning problem solving. Some of the above references are scenario-based approaches to deal with the uncertainty via the non-anticipativity principle, see Rockafeller and Wets (1991). It is very amenable for decomposition approaches, see Escudero (1998), among others.

Moreover, most of the stochastic approaches for supply chain management only consider tactical decisions (modeled by continuous variables) usually related to supply, production and market shipment of raw materials and products. There are very few schemes that we know, see e.g. MirHassani et al. (1999) and Ahmed et al. (2000a), that address the strategic planning in supply chain problems under uncertainty. One of the potential alternatives in this environment is based on two-stage scenario mixed 0-1 program schemes. The first stage decisions are the strategic decisions on the supply chain topology. There is not full information about the random events in the supply chain at the time when these decisions are being made. The second stage decisions include the tactical decisions and minor strategic decisions over the time horizon. These decisions are made after the realization of the random events is known (i.e., a given scenario occurs). The first stage decisions are modeled by using 0-1 variables and, in any case, its value is not subordinated to any scenario, but it must take into account all of them. The second stage decisions are modeled by using 0-1 variables as well as continuous variables for each given scenario.

In this paper, a 0-1 model is presented for the deterministic version of the *Strategic Supply Chain* (*SSCh*) planning problem, as well as a splitting variable 0-1 mixed deterministic equivalent model for the two-stage stochastic version of the problem. However, Stochastic 0-1 Programming is still in its infancy (Johnson et al., 2000), although it has a broad application field, see Laporte and Louveaux (1993), Carøe and Schultz (1996), Carøe and Tind (1998), Schultz et al. (1998), Alonso et al. (2000a,b), Takriti and Birge (2000) and Nürnberg and Römisch (2000), among others. In one way or another these approaches use some sort of Lagrangian and Benders Decomposition schemes, see MirHassani et al. (1999), for obtaining good lower and upper bounds for the *Strategic Supply Chain* (*SSCh*) planning problem. In this paper we also introduce a specialization of a *Branch-and-Fix Coordination* (*BFC*) algorithmic approach for multi-stage stochastic problems presented in Alonso et al. (2000a) to the two-stage problem under consideration. The splitting variable representation of the two-stage problem is very amenable for the proposed *BFC* approach to deal with the 0-1 character of the integer variables. Its main aim is to coordinate the selection of the branching nodes and branching variables in the scenario subproblems to be jointly optimized. The computational results are very interesting when applying *BFC* to the *SSCh* two-stage stochastic problem represented by the model that is proposed in this work. In any case the expected value of the objective function for the tested cases is not worse and actually better than the related value when the parameters variability is dealt with via the average scenario.

The rest of the paper is organized as follows. Section 2 presents the supply chain management problem to solve. Section 3 presents the 0-1 mixed model of our choice for the deterministic version. Section 4 gives the two-stage stochastic programming setting to deal with. The section also shows the splitting variable representation of the first stage variables; the *Deterministic Equivalent Model* (*DEM*) that results is included by the two-stage scenario-related models coupled with the first stage splitting variables equating constraints. Section 5 presents the *BFC* approach for problem solving. Section 6 reports on the computational results. And, finally, section 7 draws some conclusions from the work.

2 Problem Statement

A *time horizon* is a set of (consecutive and integer) time periods of non necessarily equal length where the operations planning will be considered. A *product* is any item whose production volume, location and scheduling is decided by the *Supply Chain Management* (*SCM*). An *end product* is the final output of the supply chain network. A *subassembly* is a product that is assembled by the supply chain and, together with other items, is used to produce other products. By the term *product* we will refer to both end products and subassemblies. Their own *BoM* is a concern of the *SCM*. Multiple external demand sources for a product (either an end product or a subassembly) are also allowed. We will name *raw material* to any storable item that is required in the products' BoM, but whose *BoM* is not a concern of the *SCM*, i.e., the supply is only from outside sources. Let us use the term *component* to describe any storable item that is required for the production. We may observe that a subassembly is a component in a given *BoM* of some other product. So, subassemblies and raw materials are *components*. The *stock* of an item (either a product or a raw material) is its available volume at the end of a given time period. Let us assume that the cycle time (i.e., lead time) of any unit product is smaller that the length of the given periods in the time horizon.

We may notice that the *BoM* of a product is the structuring of the set of components that are required for its manufacturing/assembly. The *BoM* can be described as a set of tiers, i.e., a set of levels in the supply chain. A so-called *first tier* component in a *BoM* of a given product is a component that is directly required for its manufacturing/assembly.

Let us term *vendor* to any external source for the supplying of raw materials. A warehouse within the supply chain can be associated to any item. A *plant* is a capacitated physical location where the products are processed. The plants may have different capacity production levels. The term *plant investment* will be used for the amount of a given currency that is needed for expanding a plant from, say, level $k-1$ to level k. We may observe that the expansion to level $k=1$ means that a plant will be open.

Note that single-level production requires that the components of a given *BoM* are assembled sequentially along the cycle time of the product. On the contrary, multilevel production, as it is in supply chain environments, allows the subsets of components to be assembled independently and, then, the production resources can be better utilized. See also Escudero et al. (1999b), among others.

Some parameters are deterministic by nature or the optimal solution may not be very sensitive to their variability. However, the product net profit and demand, as well as the raw material cost (and, with smaller intensity, the production cost) are uncertain parameters, mainly, for long time horizons as it is usually the case for strategic planning. The available information for the uncertain parameters can be structured in a set of scenarios (i.e., potential realizations of the parameters), see section 4.

The goal of the *SSCh* planning problem that is addressed in this work consists of determining the production topology, plant sizing, product selection, product allocation among plants and vendor selection for raw material. The objective is the maximization (in constant terms) of the expected benefit given by the product net profit minus the operation costs and the plant investment depreciation cost over the time horizon, by considering the set of given scenarios for the uncertain parameters.

Two *stages* are considered in the problem. The first stage is devoted to the *strategic* decisions about plants sizing, product allocation to plants and raw materials vendor selection. The second stage is devoted to the *tactical* decisions about the raw material volume to supply from vendors, product volume to be processed in plants, stock volume of product/raw material to be stored in plants/warehauses, component volume to be transported from origin plants/warehouses to destination plants and product volume to be shipped from plants to market sources at each time period along the time horizon, given the supply chain topology decided at the first stage. Obviously the strategic decisions, besides satisfying their related first stage constraints, will take into consideration the product net profit and operation cost related to the tactical environment besides the investment depreciation cost.

Let the following definition and notation of the elements for the deteministic version of the *SSCh* planning problem.

Sets:

$\mathcal{I}$, set of plants.

$\mathcal{J}$, set of products. (End products and subassemblies)

$\mathcal{R}$, set of raw materials.

$\mathcal{C}$, set of components. (Raw materials and subassemblies)

$\mathcal{L}$, set of subassemblies. ($\mathcal{L} = \mathcal{J} \bigcap \mathcal{C}$)

$\mathcal{E}$, set of items. ($\mathcal{E} = \mathcal{J} \bigcup \mathcal{R}$)

$\mathcal{V}$, set of vendors (or zones) for the raw material supplying.

$\mathcal{C}_j$, set of first tier components required by product j, $\forall j \in \mathcal{J}$.

$\mathcal{I}_j$, set of plants that are available to process product j, $\forall j \in \mathcal{J}$, ($\mathcal{I}_j \subseteq \mathcal{I}$), and set of candidate vendors (or zones) for raw material j, $\forall j \in \mathcal{R}$, ($\mathcal{I}_j \subseteq \mathcal{V}$).

$\mathcal{T}$, set of time periods along the time horizon (i.e., second stage).

$\mathcal{T}_i$, set of time periods where a capacity expansion for plant i is allowed, $\forall i \in \mathcal{I}(\mathcal{T}_i \subseteq \mathcal{T})$, besides time period $t = 0$ (i.e., first stage).

$\mathcal{K}_i$, set of capacity expansion levels for plant i, $\forall i \in \mathcal{I}$.

$\mathcal{M}_j$, set of market sources for product j, $\forall j \in \mathcal{J}$.

Parameters:

$\widetilde{N}$, maximum number of plants that can be open.

$\widehat{N}$, maximum number of end products that can be processed.

$\underline{N}_j, \overline{N}_j$, conditional minimum and maximum number of plants where product j can be processed, respectively, if any, $\forall j \in \mathcal{J}$, and
conditional minimum and maximum number of vendors for raw material j, respectively, if any, $\forall j \in \mathcal{R}$.

$\overline{N}^i$, maximum number of products to be processed in plant i at any time period, $\forall i \in \mathcal{I}$, and maximum number of raw materials to be supplied by vendor (or zone) i, $\forall i \in \mathcal{V}$.

P_t, available budget for plant capacity building/expansion at time period t, for $t \in \{0\} \cup \mathcal{T}$. Note: By convention, plant building (i.e., capacity expansion level $k = 1$) can only occur at time period $t = 0$.

$\underline{X}^i_j, \overline{X}^i_j$, conditional minimum and maximum volume of raw material j that can be supplied from vendor i at any time period, respectively, if any, $\forall i \in \mathcal{I}_j, j \in \mathcal{R}$, and
conditional minimum and maximum volume of product j that can be processed in plant i at any time period, respectively, if any, $\forall i \in \mathcal{I}_j, j \in \mathcal{J}$.

$\underline{S}^i_{jt}, \overline{S}^i_j$ conditional minimum and maximum volume of raw material j that can be in stock from vendor (or zone) i at the end of time period t and at any time period, respectively, if any, $\forall i \in \mathcal{I}_j, j \in \mathcal{R}, t \in \mathcal{T}$ and
conditional minimum and maximum volume of product j that can be in stock in plant i at the end of time period t and at any time period, respectively, if any, $\forall i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

o_j^i, unit capacity usage of plant i by product j, $\forall i \in \mathcal{I}_j, j \in \mathcal{J}$.

$\underline{p}_i$, minimum capacity usage of plant i at any time period, if any.

p_i^k, production capacity increment from level $k-1$ to level k in plant i, $\forall k \in \mathcal{K}_i, i \in \mathcal{I}$.

n_{gj}, volume of component g required by one unit of product j in its *BoM*, $\forall g \in \mathcal{C}_j, j \in \mathcal{J}$.

D_{jt}^m, demand of product j from market source m at time period t, $\forall m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

Cost estimations:

a_{it}^k : budget required for the capacity expansion from level $k-1$ to level k in plant i at time period t, $\forall k \in \mathcal{K}_i, t \in \{0\} \cup \mathcal{T}_i, i \in \mathcal{I}$.

q_{it}^k : depreciation cost (along the time horizon) of the investment a_{it}^k related to the k-th capacity expansion level in plant i at time period t , $\forall k \in \mathcal{K}_i, t \in \{0\} \cup \mathcal{T}_i, i \in \mathcal{I}$.

p_{jt}^{im} : net unit profit from selling product j from plant i to market source m at time period t, including local taxes, transport cost and others, $\forall i \in \mathcal{I}_j, m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

c_{jt}^i : processing unit cost of product j in plant i at time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$, and supplying unit cost of raw material j from vendor i at time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{R}, t \in \mathcal{T}$.

h_{jt}^i : holding unit cost of product/raw material j in plant/warehouse i at time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T}$.

b_g^{fi} : transport unit cost of component g from plant/ warehouse f to plant i at any time period, $\forall g \in \mathcal{C}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

The goal consists of determining the production topology (i.e., location of plants to open), plant sizing, end product selection, product allocation among plants and vendor selection for raw materials to maximize the total net revenue.

3 Strategic Supply Chain Planning Deterministic Model

This section is devoted to the deterministic version of the *SSCh* planning model and, so the goal is to obtain the optimal solution for a problem where all parameters are known. Several 0-1 equivalent models can be considered, in particular, the so-called *step variables* based model and the *impulse variables* based model. The basic idea for these two types of variable's representation is taken from Bertsimas and Stock (1998) for scheduling air traffic in a network of airports, see also Alonso et al. (2000b). The *step variables* model has been selected, since it is tighter that the other one; the extended version of this paper, see Alonso et al. (2001), presents the second model and gives the proof of its weakness.

The variables for the first model are as follows.

Strategic variables. They are 0-1 variables, such that

$$\alpha_j = \begin{cases} 1, & \text{if product/raw material } j \text{ is selected for processing/supplying} \\ 0, & \text{otherwise,} \end{cases} \qquad \forall j \in \mathcal{E}$$

$$\beta_j^i = \begin{cases} 1, & \text{if product/raw material } j \text{ is processed in plant } i\text{/supplied by vendor } i \\ 0, & \text{otherwise,} \end{cases} \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{E}$$

$$\gamma_{it}^k = \begin{cases} 1, & \text{if plant } i \text{ has capacity level } k \text{ \textit{at least} at period } t \\ 0, & \text{otherwise,} \end{cases} \qquad \forall k \in \mathcal{K}_i, i \in \mathcal{I}, t \in \{0\} \cup \mathcal{T}$$

Operation variables. They are continuous variables, such that

X_{jt}^i**:** volume of product j to be processed in plant i at time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$, and volume of raw material j to be supplied from vendor i at time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{R}$, $t \in \mathcal{T}$.

S_{jt}^i**:** stock volume of product/ raw material j in plant/ warehouse i at (the end of) time period t, $\forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T}$

E_{gt}^{fji}**:** volume of component g to be transported from plant/warehouse (origin) f to plant (destination) i at time period t for processing product j, $\forall f \in \mathcal{I}_g, g \in \mathcal{C}_j$, $i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

Y_{jt}^{im}**:** volume of product j to be shipped from plant i to market source m at time period t, $\forall i \in \mathcal{I}_j, m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}$.

Objective: maximize the total net revenue, given by $z_2 - z_1$, see below.

Stage 1 (Strategic) Submodel

$$z_1 = \min \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} q_{i0}^k \gamma_{i0}^k \tag{3.1}$$

subject to

$$\sum_{i \in \mathcal{I}} \gamma_{i0}^1 \leq \widetilde{N} \tag{3.2}$$

$$\gamma_{i0}^{k-1} \geq \gamma_{i0}^k \qquad \forall k \in \mathcal{K}_i \setminus \{1\},\ i \in \mathcal{I} \tag{3.3}$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} a_{i0}^k \gamma_{i0}^k \leq P_0 \tag{3.4}$$

$$\sum_{j \in \mathcal{J} \setminus \mathcal{L}} \alpha_j \leq \widehat{N} \tag{3.5}$$

$$\alpha_j \leq \alpha_g \qquad \forall g \in \mathcal{C}_j, j \in \mathcal{J} \tag{3.6}$$

$$\underline{N}_j \alpha_j \leq \sum_{i \in \mathcal{I}_j} \beta_j^i \leq \overline{N}_j \alpha_j \qquad \forall j \in \mathcal{E} \tag{3.7}$$

$$\beta_j^i \leq \gamma_{i0}^1 \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{J} \tag{3.8}$$

$$\sum_{j \in \mathcal{J} / i \in \mathcal{I}_j} \beta_j^i \leq \overline{N}^i \gamma_{i0}^1 \qquad \forall i \in \mathcal{I} \tag{3.9}$$

$$\sum_{j \in \mathcal{R} / i \in \mathcal{I}_j} \beta_j^i \leq \overline{N}^i \qquad \forall i \in \mathcal{V} \tag{3.10}$$

$$\alpha_j \in \{0,1\} \qquad \forall j \in \mathcal{E} \tag{3.11}$$

$$\beta_j^i \in \{0,1\} \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{E} \tag{3.12}$$

$$\gamma_{i0}^k \in \{0,1\} \qquad \forall k \in \mathcal{K}_i, i \in \mathcal{I} \tag{3.13}$$

Constraints (3.2) ensure that the number of plants in the supply chain will not exceed the allowed maximum. Constraints (3.3) ensure that the γ-variables are well defined. Constraints (3.4) take into account the investment budget. Constraints (3.5) bound the number of end products for processing. Constraints (3.6) force the production/supplying of the first tier components of any product selected. By considering the *BoM* requirements in the operation submodel, see below specifically constraints (3.24), it is easy to see the redundancy of (3.6). However, this type of cut reduces the linear programming (LP) solution space and, then, helps to tighten the model. Constraints (3.7) conditionally lower and upper bound the number of plants/vendors for each product/raw material. Constraints (3.8) restrict the processing of products to those plants that are in operation. Constraints (3.9) and (3.10) ensure that the number of products/raw materials for processing in plant/supplying from vendor i will not exceed the allowed maximum.

We may observe that the right-hand-side (*rhs*) of (3.9) has been reinforced by multiplying it by γ_{io}^1. On the other hand, enlarging the model by appending the variable upper bound $\beta_j^i \leq \alpha_j$, $i \in \mathcal{I}_j, j \in \mathcal{E}$ results in a 0-1 equivalent LP stronger model as well. However, given the potentially high number of β-variables, the appending should only be performed for violated cuts by the current LP solution.

Stage 2 (Operation) Submodel

(3.14) subject to (3.15)-(3.27)

Stage 2 submodel. Time period indexed profit function to maximise

$$z_2 = \max \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_j} \sum_{m \in \mathcal{M}_j} p_{jt}^{im} Y_{jt}^{im} - \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{E}} \sum_{i \in \mathcal{I}_j} (c_{jt}^i X_{jt}^i + h_{jt}^i S_{jt}^i) -$$
$$- \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \sum_{g \in \mathcal{C}_j} \sum_{f \in \mathcal{I}_g} \sum_{i \in \mathcal{I}_j} b_g^{fi} E_{gt}^{fji} - \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i \setminus \{1\}} \sum_{t \in \mathcal{T}_i} q_{it}^k (\gamma_{it}^k - \gamma_{i,t-1}^k) \tag{3.14}$$

Stage 2 submodel. Time period indexed capacity expansion constraints

$$\gamma_{i,t-1}^1 = \gamma_{it}^1 \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3.15}$$

$$\gamma_{i,t-1}^k = \gamma_{it}^k \qquad \forall k \in \mathcal{K}_i \setminus \{1\}, t \in \mathcal{T} \setminus \mathcal{T}_i, i \in \mathcal{I} \tag{3.16}$$

$$\gamma_{i,t-1}^k \leq \gamma_{it}^k \qquad \forall k \in \mathcal{K}_i \setminus \{1\}, t \in \mathcal{T}_i, i \in \mathcal{I} \tag{3.17}$$

$$\gamma_{it}^{k-1} \geq \gamma_{it}^k \qquad \forall k \in \mathcal{K}_i \setminus \{1\}, i \in \mathcal{I}, t \in \mathcal{T} \tag{3.18}$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i \setminus \{1\}} a_{it}^k (\gamma_{it}^k - \gamma_{i,t-1}^k) \leq P_t \qquad \forall t \in \mathcal{T} \tag{3.19}$$

$$\underline{p}_i \gamma_{i0}^1 \leq \sum_{j \in \mathcal{J} / i \in \mathcal{I}_j} o_j^i X_{jt}^i \leq \sum_{k \in \mathcal{K}_i} p_i^k \gamma_{it}^k \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3.20}$$

Stage 2 submodel. Time period indexed operation constraints

$$S^i_{j,t-1} + X^i_{jt} = \rho^i_{jt} + \sigma^i_{jt} + S^i_{jt} \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T} \tag{3.21}$$

$$\underline{X}^i_j \beta^i_j \leq X^i_{jt} \leq \overline{X}^i_j \beta^i_j \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T} \tag{3.22}$$

$$\underline{S}^i_{jt} \beta^i_j \leq S^i_{jt} \leq \overline{S}^i_j \beta^i_j \qquad \forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T} \tag{3.23}$$

$$\sum_{f \in \mathcal{I}_g} E^{fji}_{gt} = n_{gj} X^i_{jt} \qquad \forall g \in \mathcal{C}_j, i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{3.24}$$

$$\sum_{i \in \mathcal{I}_j} Y^{im}_{jt} \leq D^m_{jt} \qquad \forall m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{3.25}$$

$$Y^{im}_{jt} \geq 0 \qquad \forall i \in \mathcal{I}_j, m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{3.26}$$

$$E^{fji}_{gt} \geq 0 \qquad \forall f \in \mathcal{I}_g, g \in \mathcal{C}_j, i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{3.27}$$

where

$$\rho^i_{jt} = \begin{cases} \sum_{\ell \in \mathcal{J}/j \in \mathcal{C}_\ell} \sum_{f \in \mathcal{I}_\ell} E^{i\ell f}_{jt}, & \text{for } j \in \mathcal{C} \\ 0, & \text{for } j \in \mathcal{J} \setminus \mathcal{L} \end{cases}$$

and

$$\sigma^i_{jt} = \begin{cases} \sum_{m \in \mathcal{M}_j} Y^{im}_{jt}, & \text{for } j \in \mathcal{J} \\ 0, & \text{for } j \in \mathcal{R} \end{cases}$$

The constraints have been divided in two blocks, namely, capacity expansion related constraints (3.15)-(3.20) and operation related constraints (3.21)-(3.27). Constraints (3.15) ensure that the plants are only open at time period $t = 0$. Constraints (3.16) ensure that the capacity expansion of the plants will only occur at allowed time periods. Constraints (3.17) and (3.18) assure that the γ-variables are well defined. Constraints (3.19) take into account the capacity expansion budget. Constraints (3.20) limit the production from each plant to a conditional minimum, as well as to the maximum capacity given by the expansion plan. Constraints (3.21) are the typical stock balance equations for products and raw materials. Constraints (3.22) and (3.23) define the semi-continuous character of the production and stock variables. These constraints imply the non-negativity of the variables X^i_{jt} and S^i_{jt}, $\forall i \in \mathcal{I}_j, j \in \mathcal{E}, t \in \mathcal{T}$. Constraints (3.24) force the *BoM* requirements for the products. Constraints (3.25) ensure that the product shipment to the market sources will not exceed the related demand.

4 Modeling the Strategic Supply Chain Planning under Uncertainty

Let the following representation of the model (3.1)-(3.27)

$$\begin{array}{ll} \max & ax + by + cz \\ \text{s.t.} & A^1 x = q \\ & A^2 x + By + Cz = p \\ & x, y \in \{0,1\}, z \geq 0 \end{array} \tag{4.1}$$

where a, b and c are the vectors of the objective function coefficients; x, y and z are the vectors of the variables, such that x represents the 0-1 first stage variables, in our case, the strategic α-β- and γ- variables, y represents the strategic 0-1 second stage variables, in our case, γ_t, $t \in \mathcal{T}$, and z represents the continuous second stage variables, in our case, the tactical X-, S-, E- and Y- variables; A^1 and A^2 are the first stage and second stage constraint matrices related to the x-variables, respectively, B and C are the second stage constraint matrices related to the y- and z- variables, respectively; and q and p are the *rhs* vectors for the first stage and second stage, respectively; all parameters with conformable dimensions.

The model must be extended to deal properly with the uncertainty on the product net profit and demand, and raw material and production costs. We employ the so-called *scenario analysis* approach, where the uncertainty on the stochastic parameters is modeled via a set of scenarios, say, Ω. We also introduce the weight, say w^ω, representing the likelihood that the modeler associates with scenario ω, for $\omega \in \Omega$.

One way to deal with the uncertainty is to obtain the solution x, y, z that best tracks each of the scenarios, while satisfying the constraints for each scenario (or, for the matter, minimizing their infeasibility). This can be achieved in different ways, all of them known as *Simple Recourse* (for short, *SR*) but, in our case, *SSCh* planning, it implies a *non-necessary integration* of the strategic and tactical decisions (i.e., in a simple recourse mode the production and market decisions are to be made in advance to the realization of the scenarios); as a consequence an increment of cost stock values would happen, see Escudero et al. (1993) and section 6.

However, when only first stage decisions (i.e., the strategic decisions, in our case) are to be made, obviously, by considering all given scenarios but without subordinating to any of them (an approach so-called *full recourse*), then the *hierarchical* consideration of the decisions does not anticipate the tactical decisions but subordinates them to the occurrence of the scenarios. In order to introduce the modeling of this approach, let us use the following notation: y^ω and z^ω, the strategic and tactical decisions to be made under scenario ω (i.e., once the realization of the scenario occurs), respectively, for $\omega \in \Omega$; c^ω, the objective function coefficients of the z-variables under scenario ω (where the net unit profit and processing cost for the products and the supplying unit cost for the raw materials are the uncertain parameters); and p^ω, the *rhs* parameter vector for the second-stage constraints under scenario ω (where the product demand is the uncertain parameter).

The *compact representation* of the full recourse two stage stochastic version of the deterministic model (4.1) can be represented by the *Deterministic Equivalent Model* (DEM),

$$
\begin{aligned}
Z_{IP} = \max \quad & ax + \sum_{\omega \in \Omega} w^\omega (by^\omega + c^\omega z^\omega) \\
\text{s.t.} \quad & A^1 x = q \\
& A^2 x + By^\omega + Cz^\omega = p^\omega \quad \forall \omega \in \Omega \\
& x \in \{0,1\}, y^\omega \in \{0,1\}, z^\omega \geq 0 \quad \forall \omega \in \Omega
\end{aligned} \tag{4.2}
$$

Different types of decomposition approaches can be used for solving model (4.2) with continuous variables. We favour Lagrangian and Benders Decomposition schemes. Benders (1962) decomposition methods can be applied to exploit the structure of the *DEM* (4.2). The first application to two-stage stochastic LP is due to van Slyke and Wets (1969). See also in Birge and Louveaux (1997), among others, some schemes for dealing with the integer version of the model.

On the other hand, we can also consider some other types of mathematical representations, specifically, the so-called *splitting variable representation*, since it is very amenable for our approach to deal with 0-1 variables. Given the large-scale instances of the model for *SSCh* planning, decomposition in smaller models is a key for success. One type, so-called *node-based* representation, requires to produce sibblings of some of the x- variables (in particular, the variables with non-zero elements in the second stage constraints, in our case, the β- and γ- variables). Another type so-called *scenario-based* representation requires sibblings, say x^ω, of the full set of x-variables for each scenario $\omega \in \Omega$ and adding explicitly the non anticipativity contraints, see Alonso et al. (2000a) and below. Mulvey and Ruzczynski (1992) and Escudero et al. (1999a), among others present detailed algorithms for solving the LP relaxation of this type of models, by using an augmented Lagrangian function. In a different context see in Nürnberg and Römisch (2000) a Lagrangian based stochastic dynamic programming approach, and see in Ahmed et al. (2000b) a finite branch-and-cut approach for a two-stage stochastic mixed integer program.

In spite of the good performance of the above approaches for the LP version of model (4.2), Benders and Lagrangian Decomposition schemes may have a better performance for smaller instances than the cases discussed below; see Laporte and Louveaux (1993), Carøe and Tind (1998), and Carøe and Schultz (1996), among others.

The *splitting variable representation via scenario* of model (4.2) is as follows

$$\begin{aligned} Z_{IP} = \max \sum_{\omega\in\Omega} & w^\omega (ax^\omega + by^\omega + c^\omega z^\omega) \\ \text{s.t.} \quad & A^1 x^\omega = q \quad \forall \omega \in \Omega \\ & A^2 x^\omega + By^\omega + Cz^\omega = p^\omega \quad \forall \omega \in \Omega \\ & x^\omega - x^{\omega+1} = 0 \quad \forall \omega = 1,2,\ldots,|\Omega|-1 \\ & x^\omega, y^\omega \in \{0,1\}, z^\omega \geq 0 \quad \forall \omega \in \Omega \end{aligned} \tag{4.3}$$

This model includes the so-called *non-anticipativity constraints* (4.4).

$$x^\omega - x^{\omega+1} = 0, \quad \forall \omega = 1,2,\ldots,|\Omega|-1. \tag{4.4}$$

Carøe and Schultz (1996) use a similar decomposition approach. However, that approach focuses more on using Lagrangian relaxation to obtain good lower bounds, and less on branching and variable fixing. See also Takriti and Birge (2000). The methodology to be presented below focuses on branching and variable fixing and, in any case, Lagrangean relaxation schemes can be added on top.

Given the structure of the *Strategic Supply Chain (SSCh)* planning deterministic model (3.1)-(3.27), the stochastic version as represented in (4.3) consists of the same deterministic representation but it should be replicated for each scenario from Ω plus appending the non-anticipativity constraints for the (first-stage) 0-1 α-, β-, γ_0- variables. The scenario related element of the *rhs* vector p^ω for $\omega \in \Omega$ is the demand of the products at each time period. The related elements of the objective function vector c^ω are the product market price and production cost and the raw material supply cost for each scenario ω, for $\omega \in \Omega$.

Let us consider the relaxation of condition (4.5), which includes the 0-1 character of the x- and y- variables in model (4.3) as well as the non-anticipativity constraints (4.4),

$$x^\omega - x^{\omega+1} = 0, \quad x^\omega, y^\omega \in \{0,1\}, \quad \forall \omega \in \Omega, \tag{4.5}$$

such that the new problem can be expressed

$$\begin{aligned} Z_{IP} = \max \sum_{\omega\in\Omega} & w^\omega (ax^\omega + by^\omega + c^\omega z^\omega) \\ \text{s.t.} \quad & A^1 x^\omega = q \quad \forall \omega \in \Omega \\ & A^2 x^\omega + By^\omega + Cz^\omega = p^\omega \quad \forall \omega \in \Omega \\ & x^\omega, y^\omega \in [0,1], z^\omega \geq 0 \quad \forall \omega \in \Omega \end{aligned} \tag{4.6}$$

Note that the LP model (4.6) consists of a set of $|\Omega|$ independent models; we can execute, say, a *Branch-and-Fix (BF)* procedure for each scenario related model in order to ensure the integrality condition. Instead of obtaining independently the optimal solution for each one, we propose an ad-hoc approach specifically designed to coordinate the node and variable branching for each scenario-related *BF* tree, such that the relaxed constraints (4.4) are satisfied when fixing the appropriate variables to either one or zero. The proposed approach also coordinates the prunning of the required scenario-related *BF* nodes.

5 Algorithmic approach

Let I denote the set of indices of the first stage variables (i.e., the x-variables) such that x_i^ω for $i \in I$ is an element of vector x^ω. Let also G^ω denote the *BF* tree associated with scenario ω, and H^ω the set of active nodes in G^ω. Any two active nodes, say, $h \in H^\omega$ and $h' \in H^{\omega'}$ with $\omega \neq \omega'$ are said *twin* nodes if the path from the *root* node to each of them in their own *BF* trees, say, G^ω and $G^{\omega'}$ has branched or fixed on the same values of the x-variables. A family of *twin* nodes, say $\mathcal{T}^f$, is a set of nodes, such that any node is a *twin* node to all the other nodes in the familly. Let $\mathcal{F}$ denote the set of families of *twin* nodes, such that it is said that the nodes h and h' are *twins* if $h, h' \in \mathcal{T}^f$, $f \in \mathcal{F}$. Note that in order to satisfy the non-anticipativity constraint (4.4) the branching and fixing of the x-variables must be with the same value $k \in \{0, 1\}$ for the *twin* nodes. See figure 1.

As an illustration, let us consider the active node h in *BF* tree G^ω and assume a branching is required on the variable x_i^ω; in that case, two new subproblems are created in the *BF* trees associated with the scenarios ω' from set Ω, such that the new branches from each node h' from set $\mathcal{T}^f$ where $h \in \mathcal{T}^f$, $f \in \mathcal{F}$ are as follows:

$$x_i^\omega = x_i^{\omega'} = 1 \text{ on one descendant node from each node in set } \mathcal{T}^f, \text{ and}$$
$$x_i^\omega = x_i^{\omega'} = 0 \text{ on the other descendant node.}$$

So, the proposal is to execute in a coordinated way $|\Omega|$ *BF* phases (one per scenario). For this purpose, consider a *Master Program*, say, *MP* whose mission is to make decisions about the selection of the branching node and branching variable as well as the cut identification and appending to the LP subproblem attached to the node under consideration for each scenario. The algorithmic framework is as follows, see also our approach (Alonso et al., 2000a) for the multi-stage case.

BFC Algorithm

Step 1: Solve the LP problem (4.6), by solving the $|\Omega|$ scenario-related LP models. Each model will be the *root* node problem of G^ω $\forall \omega \in \Omega$. If the conditions (4.5) are satisfied, then stop; the optimal solution to the original stochastic 0-1 problem (4.2) has been obtained. Otherwise, go to step 2.

Step 2: The following parameters are saved into MP: The fractional values of the variables and the solution value (i.e., the optimal objective function value) of each scenario-related LP model as well as the appropriate information for fixing any variable to 0 or 1 in the set of families $\mathcal{T}^f$ of the active nodes H^ω for G^ω $\forall \omega \in \Omega$, $f \in \mathcal{F}$. A decision in MP is made for the selection of the branching node and the branching variable as well as for the variables fixing. See below. The decision is made available for the execution of each scenario-related *BF* phase.

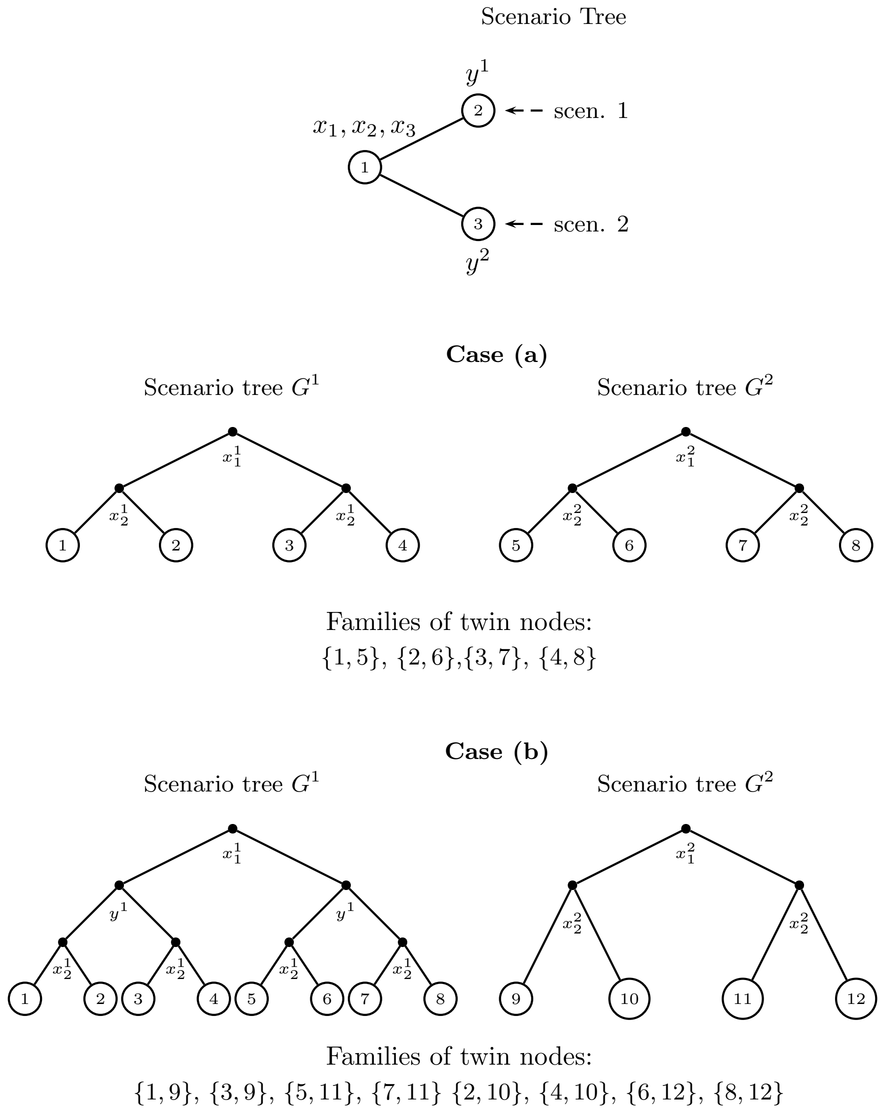

Figure 1: Branch-and-Fix Coordination scheme

Step 3: The same branching and fixing variables are to be used at the *twin* active nodes, i.e., the active nodes whose paths from the *root* node in their related BF trees have already branched or fixed on the same values of the first stage variables. $|\Omega|$ LP subproblems are optimized at each iteration (one per *twin* node).

Step 4: In case that the solution for the LP subproblem that has been optimized in Step 3 satisfies the relaxed constraints (4.5), a new feasible solution has been obtained for the original problem (4.2). The *incumbent* solution can be updated and the *twin* nodes can be jointly *prunned* at the BF trees, if any. Additionally, the updating of the set H^ω of *active* nodes at G^ω $\forall\omega \in \Omega$ is also performed. If the set of active nodes in the BF trees is empty then stop, since the optimality of the *incumbent* solution has been proved, if any. Otherwise, go to Step 2 and start a new iteration.

We may notice that the above sequence must be executed even if all x- and y- variables get 0-1 values, but any of the constraints (4.4) is not satisfied.

Fixing variables and prunning nodes

To present the main ingredients of our proposal to solve the stochastic 0-1 mixed problem (4.3) let us consider the model for scenario $\omega \in \Omega$,

$$\begin{array}{lll} \max & ax^{\omega} + by^{\omega} + c^{\omega}z^{\omega} & \\ \text{s.t.} & A^1x^{\omega} = q & \\ & A^2x^{\omega} + By^{\omega} + Cz^{\omega} = p^{\omega} & \\ & x^{\omega}, y^{\omega} \in \{0,1\}, z^{\omega} \geq 0. & \end{array} \tag{5.1}$$

Let also the following concepts and notation.

$Z_{LP}^{\omega,h}$**,** solution value of the LP relaxation of the problem (5.1) attached to the active node h in the BF tree G^{ω}, for $h \in H^{\omega}$, $\omega \in \Omega$.

$J \subseteq I$**,** set of x-variables whose related constraint (4.5) is not satisfied (i.e., either they have fractional values or the non-anticipativity constraints are violated) at a given node of a BF tree. Note that the variables from $I \setminus J$ have already the same value $k \in \{0,1\}$ in all *twin* nodes h for $h \in \mathcal{T}^f$, $f \in \mathcal{F}$.

$\mathcal{N}_{ik}^{\omega,h,k'}$**,** set of x- and y- variables whose value must be fixed to k' in any feasible solution to the problem (5.1) attached to the active node h from the set H^{ω} in the BF tree G^{ω} for $\omega \in \Omega$, if x_i^{ω} is fixed to k, for $k, k' \in \{0,1\}$. It can be obtained by using *probing*, see Guignard and Spielberg (1981) and *conflict graph analysis*, see Savelbergh (1994) and Atamtürk et al. (2000), among others. Note that these two types of schemes do only work on single problems (5.1).

$\mathcal{M}_{ik}^{\mathcal{T}^f,k'}$**,** set of x- and y- variables that include the $\mathcal{N}$-related sets of variable fixings for the *twin* nodes from set $\mathcal{T}^f$, $f \in \mathcal{F}$ and the related k-fixing in the problem (5.1) attached to the nodes. Note. The two-way cascade of fixing iterations that is required to generate the set should not be over until no more fixing implications are detected, in order to satisfy the constraints (4.5).

$\underline{\Delta}_{LP_j}^{\omega,h,k}$**,** lower bound of the solution value $Z_{LP}^{\omega,h}$ deterioration if the variable x_j^{ω} is fixed to k, for $k \in \{0,1\}$, $j \in J$, $h \in H^{\omega}$, $\omega \in \Omega$. We present elsewhere, see Alonso et al. (2000a), two procedures for obtaining the lower bound by using sensitivity analysis and taking advantage of the $\mathcal{N}$-related set of variable fixings.

$\underline{Z}_{IP}$**,** objective function value of the incumbent solution.

Proposition 5.1. *If the branching variable to select in active node, say, h from set H^{ω} at the BF tree G^{ω}, $\omega \in \Omega$ is the variable x_i^{ω}, $i \in J$, then an upper bound, say, $\overline{Z}_{IP_{ik}}^{\mathcal{T}^f}$, of the solution value of the enlarged problem for $x_i^{\omega} = k$, where $k \in \{0,1\}$ and $f \in \mathcal{F}$ so that $h \in \mathcal{T}^f$, can be expressed*

$$\overline{Z}_{IP_{ik}}^{\mathcal{T}^f} = \sum_{\omega' \in \Omega} w^{\omega}(Z_{LP}^{\omega',h'} - \underline{\Delta}_{LP_i}^{\omega',h',k}) \tag{5.2}$$

where $h' \in \mathcal{T}^f$ for $h' \in H^{\omega'}$, $\omega' \in \Omega$.

Proof. Problem (4.6) consists of $|\Omega|$ independent LP subproblems (5.1). A lower bound of the solution value deterioration due to fixing $x_i^{\omega} = k$, $i \in J$ in active node h is $\underline{\Delta}_{LP_i}^{\omega,h,k}$. The related lower bound for the solution value deterioration due to the satisfaction of the constraints (4.5) in the subproblem (5.1) attached to the node h' from the same family of *twin* nodes $\mathcal{T}^f$, $f \in \mathcal{F}$ is $\underline{\Delta}_{LP_i}^{\omega',h',k}$, $\omega' \in \Omega - \{\omega\}$. □

Corollary 5.1. *Consider a family of twin nodes, say, $\mathcal{T}^f$, $f \in \mathcal{F}$. Then,*

1. *The set $\mathcal{M}_{i\overline{k}}^{\mathcal{T}^f,k'}$ of variables fixings to $k' \in \{0,1\}$ in the nodes from the set $\mathcal{T}^f$, $f \in \mathcal{F}$ implied by the potential fixing $x_i^\omega = \overline{k}, i \in J$ can be considered permanent at the given set $\mathcal{T}^f$ of branching nodes if the following condition holds for $k = 1 - \overline{k} \in \{0,1\}$.*

$$\overline{Z}_{IP_{ik}}^{\mathcal{T}^f} \leq \underline{Z}_{IP} \tag{5.3}$$

2. *The whole set of twin nodes $\mathcal{T}^f$, $f \in \mathcal{F}$ can be prunned if the following condition holds.*

$$\max_{k\in\{0,1\}} \{\overline{Z}_{IP_{ik}}^{\mathcal{T}^f}\} \leq \underline{Z}_{IP}. \tag{5.4}$$

Branching criteria

Let us consider the set $\mathcal{F}$ of families of *twin* nodes. Notice that each active node must belong to a family, at least, provided that the x-variables do not satisfy yet the constraints (4.5) and, so, $|\mathcal{T}^f| \geq 1$, $\forall f \in \mathcal{F}$. On the other hand, the branching must be performed jointly for all members of a given family, if any.

Given a branching family of *twin* nodes and a branching first-stage variable, the branching must be performed on the same value for all node members of the family. The branching should be replicated in cascade for all *twin* nodes where the same non-anticipativity constraint must be satisfied. As an illustration, the nodes 1 and 9 in figure 1, case (b) should branch on the same value for the variables x_3^1 and x_3^2, respectively, given that both nodes belong to the same family of twins nodes. Moreover, since also the nodes 3 and 9 belong to a family, it results that the nodes 1 and 3 should branch on the same value for the variable x_3^1. Similarly, a common branching should be performed for the nodes in the sets $\{5,7,11\}$ and $\{2,4,10\}$.

See in Linderoth and Savelsbergh (1999) a performance comparison for single trees' branch-and-cut strategies. Among the different criteria to select the next node family to branch and the related branching variable, we use the following criteria:

Family selection criterion: Depth first strategy

The branching family, say, $f_{k'}$, is the set of *twin* nodes with the *smallest lower bound deterioration of the solution value* among the two families, say, $f_0, f_1 \in \mathcal{F}$, that have just been created by fixing the last selected variable, say, x_i^ω to 0 or 1, respectively, such that

$$k' = \mathrm{argmin}_{k\in\{0,1\}}\Big\{ \sum_{\omega:h\in H^\omega:h\in\mathcal{T}^f} w^\omega \underline{\Delta}_{LP_i}^{\omega,h,k} \Big\}. \tag{5.5}$$

where $f \equiv f_k$.

Branching variable selection criterion: Most deterioration strategy

The branching x-variable must be the same for all nodes from the selected set, say, $\mathcal{T}^f, f \in \mathcal{F}$, in order to satisfy the non-anticipativity constraint (4.4). The variable x_i^ω to branch on $\forall h \in \mathcal{T}^f$, $\omega : h \in H^\omega$, is such that

$$i = \mathrm{argmax}_{j\in J}\Big\{ \min_{k\in\{0,1\}} \{ \sum_{\omega:h\in H^\omega:h\in\mathcal{T}^f} w^\omega \underline{\Delta}_{LP_j}^{\omega,h,k} \} \Big\}. \tag{5.6}$$

Table 1: Test cases dimensions

Case	Deterministic model				Stochastic compact representation			
	m	nc	$n01$	dens(%)	m	nc	$n01$	dens(%)
c1	3388	2937	107	0.103	76318	65989	899	0.005
c2	3458	3068	108	0.100	77928	68980	900	0.004
c3	3145	2663	103	0.112	70795	59775	895	0.005
c4	3405	3065	105	0.099	76775	68977	897	0.004
c5	3933	3654	114	0.086	88743	82326	906	0.004
c6	3145	2663	103	0.112	70795	59775	895	0.005
c7	3081	2543	103	0.116	69411	57015	895	0.005
c8	3894	3634	114	0.087	87824	81866	906	0.004
c9	3388	2937	107	0.103	76318	65989	899	0.005
c10	3101	2533	103	0.114	69871	56785	895	0.005

Note 1. In case of tie in (5.6), the branching order of the variables for the *SSCh* planning model is as follows: α-, γ_0^1-, β- and γ_0^k- variables for $k = 2, 3, ...$

Note 2. We also use the above branching criteria for the y-variables in the problem (5.1) attached to the active nodes. See that this type of variables does not generate any family of *twin* nodes, by construction. In any case, the second stage variables have lower branching priority.

6 Computational Results

We report the computational experience obtained while optimizing the stochastic *SSCh* planning model for a set of instances by using the *BFC* approach. The instances have the following dimensions: $|\mathcal{I}| = 6$ plants/warehouses, $|\mathcal{K}_i| = 3$ capacity levels per plant, $|\mathcal{J}| = 12$ products, where $|\mathcal{J} \bigcap \mathcal{C}| = 8$ are subassemblies, $|\mathcal{R}| = 12$ raw materials, $|\mathcal{V}| = 24$ vendors, $|\mathcal{M}_j| = 2$ markets per product, $|\mathcal{T}| = 10$ time periods and $|\Omega| = 23$ scenarios. There is a variety of plants in the sense that some plants can only assemble given end products, others are dedicated to one product, some are flexible manufacturing plants and, finally, some others are specialized on the manufacturing of given subassemblies. The decisions about capacity expansion have been restricted to intermediate periods besides the decisions that can be made in the first stage.

To build the scenario tree, different levels of demand (high and low demand levels) and different levels of prices for raw materials have been combined. There are two schemes, namely, five demand levels and five price levels, and nine demand levels and three price levels. In both schemes the extreme cases (low demand and prices, and high demand and prices) have been eliminated. It seems that for a long term planning, 23 scenarios could be a good approach (notice that Tomargard and Høeg (2001) report experience with only 10 scenarios).

Our *BFC* algorithmic approach has been implemented in a FORTRAN experimental code. It uses the optimization engine IBM OSL v2.1 for solving the LP problems at the active nodes in the *BF* trees. The computational experiments were conducted on a 800 MHz Pentium III Processor with 512 Mb of RAM.

Table 1 gives the dimensions of the scenario-related deterministic model (4.1). It also gives the dimensions of the deterministic equivalent model to the two-stage stochastic version, compact representation (4.2). The headings are as follows: m, number of constraints; nc, number of continuous variables; $n01$, number of 0-1 variables; and *dens*, constraint matrix density.

Table 2: Stochastic Solution

Case	Z_{LP}	$\underline{Z}_{IP}$	GAP	nn	T_{LP}	T_{IP}	T
c1	238471.13	178366.79	25.20	654	1213.53	1800.00	3013.53
c2	64128.62	0.00(∗)	100.00	7	337.03	62.78	399.81
c3	286773.63	224564.20	21.69	2286	548.82	1800.00	2348.82
c4	255419.80	197487.36	22.68	2201	535.76	1800.00	2335.76
c5	53297.06	0.00(∗)	100.00	17	825.53	443.85	1269.38
c6	285728.66	226578.02	20.70	2224	585.88	1800.00	2385.88
c7	180256.99	144181.28(∗)	20.01	641	293.02	771.26	1064.28
c8	140115.70	89607.39	36.05	269	2104.70	1800.00	3904.70
c9	237866.97	174250.56	26.74	208	1286.03	1800.00	3086.03
c10	173404.62	139738.36(∗)	19.41	877	274.15	1439.70	1713.85

(*) Optimality has been proved.

Table 3: The value of the stochastic solution (1)

Case	VSS	EEV	$\underline{Z}_{IP}$	WS	EV
c1	3253.00	175113.79	178366.79	202582.05	208670.34
c2	12220.54	−12220.54	0.00	19447.83	13812.86
c3	1903.97	222660.23	224564.20	251195.44	259863.19
c4	1787.75	195699.61	197487.36	217078.93	218827.75
c5	0.00	0.00	0.00	6525.56	0.00
c6	8655.94	217922.07	226578.02	249449.34	271117.43
c7	7006.66	137174.62	144181.28	157942.74	169135.22
c8	0.00	89607.39	89607.39	107655.30	103102.67
c9	5727.55	168523.01	174250.56	201494.28	219875.02
c10	0.00	139738.36	139738.36	145404.96	157558.52

Table 2 shows the main results of our computational experimentation for solving the *SSCh* planning problem by using the scenario-based splitting variable representation (4.3). The headings are as follows: Z_{LP}, solution value of the LP relaxation (4.6); $\underline{Z}_{IP}$, value of the incumbent solution for problem (4.3); GAP, optimality gap (%) defined as $(Z_{LP} - \underline{Z}_{IP})/Z_{LP} \times 100$; nn, number of branching nodes for the whole set of $|\Omega| = 23$ *BF* trees; T_{LP} and T_{IP}, the elapsed time (secs.) to obtain the LP solution and the additional time to obtain the integer solution, respectively; T, total time.

Given the relaxation of the constraints (4.5), i.e., the non-anticipativity constraints and the 0-1 character of the variables, it is not a surprise that GAP is very big. This fact together with the extremely high dimensions of the problem makes unrealistic to pretend to prove solution optimality. However see below a smaller optimality gap. In 6 out of 10 cases the 30 minutes time limit for branching activity was reached. Moreover, table 3 shows some parameters for analysing the goodness of the stochastic approach, see, e.g., Birge and Louveaux (1997) for more details. The headings are as follows: *WS* (*Wait-and-See*) can be expressed as

$$WS = \sum_{\omega \in \Omega} w^{\omega} Z_{IP}^{\omega},$$

where Z_{IP}^{ω} is the solution value for scenario ω; EV is the solution value for the average scenario

Table 4: Wait and See Solution

Case	Z_{LP}^{WS}	Z_{IP}^{WS}	GAP	nn	T_{LP}^{WS}	T_{IP}^{WS}	T^{WS}
c1	238471.13	202582.05	15.05	2973	93.77	1818.96	1912.73
c2	64128.62	19447.83	69.67	461	46.91	93.65	140.56
c3	286773.63	251195.44	12.41	2357	40.87	905.11	945.98
c4	255419.80	217078.93	15.01	4203	46.98	1353.18	1400.16
c5	53297.06	6525.56	87.76	1589	186.29	1463.73	1650.02
c6	285728.66	249449.34	12.70	2386	37.67	886.17	923.84
c7	180256.99	157942.74	12.38	2277	23.55	405.53	429.08
c8	140115.70	107655.30	23.17	5296	215.94	4977.65	5193.59
c9	237866.97	201494.28	15.29	3175	102.17	1682.08	1784.25
c10	173404.62	145404.96	16.15	3483	22.71	989.46	1012.17

(i.e., the expected value); EEV is the expected result of the expected value that can be expressed as

$$EEV = \sum_{\omega \in \Omega} w^{\omega} Z^{\omega},$$

where Z^{ω} is the solution value for the scenario ω related deterministic model, where the solution for the first stage has been fixed to the optimal solution for the average scenario deterministic model; and VSS is the value of the stochastic solution that can be expressed as

$$VSS = \underline{Z}_{IP} - EEV.$$

Note that EEV and WS are lower and upper bounds of the solution value Z_{IP}, respectively. See that VSS is positive in 7 out of 10 cases, i.e., it always pays the effort to use the stochastic approach instead of obtaining the strategic decisions based on the average scenario parameters. For example, observe that the stochastic solution does not recommend to start business for case 2 (by analyzing the uncertainty of the parameters given by the set of scenarios), but the average scenario solution does it. As a consequence there is an EEV expected loss derived from the wrong decision based on the average scenario.

Tables 4 and 5 give the computational results of the experimentation for obtaining the WS, EV and EEV parameters. Some headings have the same meaning as in table 2. Note: The headings in table 4 give the total value for the set of $|\Omega| = 23$ scenarios. On the other hand, see that $Z_{LP}^{WS} = Z_{LP}$, $Z_{IP}^{WS} = WS$, and $Z_{IP}^{EEV} = EEV$.

Comparing the column nn in tables 2 and 4 we can realize that the effort for obtaining the optimal solution independently for the scenarios requires more branching nodes and, usually, smaller time than obtaining the stochastic solution. In any case the GAP for WS is also very big. The optimality of WS has been proved for all cases.

Table 5 reports on the computational effort for obtaining the expected solution value for the stochastic problem based on the average scenario. The two values for each entrance in column nn give the number of branching nodes for obtaining EV and EEV, respectively. The time for obtaining both parameters and the total time is also reported. We can observe that the computational time is very small.

Table 6 shows the deviation (in percentage) of the WS, EV and EEV with respect to the stochastic solution $\underline{Z}_{IP}$. It is also reported the deviation of *SR* (*Simple Recourse*) with respect to $\underline{Z}_{IP}$, see section 4. Since the *SR* solution anticipates both the strategic and operation decisions

Table 5: Expected Result of the Expected Value Solution

Case	Z_{LP}^{EEV}	Z_{IP}^{EEV}	GAP	nn	T^{EV}	T_{LP}^{EEV}	T_{IP}^{EEV}	T^{EEV}
c1	189814.19	175113.79	7.74	73+43	71.89	7.07	6.23	85.19
c2	−170.94	−12220.54	7049.01	38+30	9.72	4.83	5.00	19.55
c3	238961.76	222660.23	6.82	57+55	24.77	5.57	6.34	36.68
c4	197055.68	195699.61	0.69	183+0	74.54	3.62	1.44	79.60
c5	0.00	0.00	0.00	29+0	38.83	2.69	1.27	42.79
c6	237053.87	217922.07	8.07	178+53	75.36	4.05	5.95	85.36
c7	142780.49	137174.62	3.93	85+9	17.63	2.57	1.72	21.92
c8	90872.73	89606.97	1.39	444+64	378.60	13.49	11.72	403.81
c9	179903.69	168523.01	6.33	147+75	100.18	5.41	9.09	114.68
c10	156168.81	139738.36	10.52	109+164	35.26	4.17	17.96	57.39

Table 6: The value of the stochastic solution (2)

Case	SR	EEV	$\underline{Z}_{IP}$	WS	EV
c1	−47.47	−1.82	178366.79	13.58	16.99
c2	0.00	−−	0.00	−−	−−
c3	−45.15	−0.85	224564.20	11.86	15.72
c4	−46.36	−0.91	197487.36	9.92	10.81
c5	0.00	0.00	0.00	−−	0.00
c6	−59.99	−3.82	226578.02	10.09	19.66
c7	−100.00	−4.86	144181.28	9.54	17.31
c8	−66.52	0.00	89607.39	20.14	15.06
c9	−92.04	−3.29	174250.56	15.63	26.18
c10	−62.10	0.00	139738.36	4.06	12.75

for the whole planning horizon, the solution value is more costly than $\underline{Z}_{IP}$. However, both approaches coincide on the same recommendation to cancel the project in cases $c2$ and $c5$.

Table 7 shows some relevant statistics. For each case and solution type it gives the solution value and weight for the best and worst scenarios. Note: In case of more than one scenario with the same profit (e.g., zero profit), the heading *weight* gives the total weight of the scenarios in which that profit is attained. It also shows the probability P^- that a scenario can occur with negative profit; for example, there is a probability of 0.51 that a scenario with losses can occur for the strategy based on the average scenario in case $c2$. Finally, $\varphi = \sigma/\mu$ gives the variation coefficient as a ratio of the standard deviation σ and the expected value μ. (Note: $\varphi = 0$ for $\sigma = \mu = 0$, since there is no variation). A conclusion that can be drawn from the table and above is that, although in most of the cases the expected values RP and EEV do not differ too much, the stochastic solution is more robust. See that the probability P^- is smaller, if any, the variation coefficient is smaller, and, although the solution value for the best scenarios is also smaller, it is greater for the worst scenario.

Table 7: Statistics

Case		Best scenario		Worst scenario		P^-	φ
		Value	Weight	Value	Weight		
c1	WS	334703.27	0.04	74007.78	0.03	−−	0.35
	RP	296564.41	0.04	56007.97	0.03	−−	0.36
	EEV	281475.87	0.04	8127.54	0.03	−−	0.44
c2	WS	75064.91	0.04	0.00	0.51	−−	1.29
	RP	0.00	1.00	0.00	1.00	−−	0.00
	EEV	75064.91	0.04	−154334.66	0.04	0.51	5.18
c3	WS	395080.74	0.04	103283.07	0.03	−−	0.32
	RP	353390.73	0.04	85283.24	0.03	−−	0.33
	EEV	339941.73	0.04	38893.85	0.03	−−	0.38
c4	WS	357502.10	0.04	86502.07	0.03	−−	0.34
	RP	318943.45	0.04	67939.53	0.03	−−	0.35
	EEV	337643.64	0.04	−3141.92	0.03	0.03	0.46
c5	WS	42069.48	0.04	0.00	0.69	−−	1.81
	RP	0.00	1.00	0.00	1.00	−−	0.00
	EEV	0.00	1.00	0.00	1.00	−−	0.00
c6	WS	407105.91	0.04	90036.00	0.04	−−	0.37
	RP	366613.34	0.04	90036.00	0.04	−−	0.36
	EEV	407105.91	0.04	−10600.16	0.04	0.04	0.58
c7	WS	295560.60	0.02	0.00	0.02	−−	0.48
	RP	247111.23	0.02	−41758.86	0.02	0.03	0.48
	EEV	291080.47	0.02	−71683.92	0.02	0.09	0.69
c8	WS	218630.99	0.04	14793.19	0.03	−−	0.53
	RP	176772.50	0.04	−36508.63	0.03	0.07	0.61
	EEV	176772.50	0.04	−36508.63	0.03	0.07	0.61
c9	WS	361098.46	0.02	13636.49	0.02	−−	0.43
	RP	319388.36	0.02	13636.49	0.02	−−	0.48
	EEV	361098.46	0.02	−97836.53	0.02	0.13	0.72
c10	WS	232767.10	0.04	34996.23	0.03	−−	0.36
	RP	232767.10	0.04	16650.09	0.03	−−	0.42
	EEV	232767.10	0.04	16650.09	0.03	−−	0.42

7 Conclusions

In this paper we have presented a modeling framework for *Strategic Supply Chain (SSCh)* planning under uncertainty in the main parameters. The approach splits the problem in two stages. The decisions to be made in the first stage are the strategic decisions related to the chain topology, product and vendor selection, and plant location, sizing and assignment; the subproblem is modeled by a pure 0-1 program. The second stage decisions are related to the tactical decisions for a better utilization of the supply chain along a time horizon with uncertainty in the product demand and price, and production and raw material costs; the subproblem is modeled by a mixed 0-1 program for each given scenario. Both models are coupled by some of the first stage 0-1 variables. The proposed framework allows to accommodate a great variety of strategic and tactical problems. A mathematical representation has been selected, so called *step variables* based model, for the deterministic version. In any case, the related *Deterministic Equivalent Model* (*DEM*) for the stochastic version that results is extremely large with dozens of thousands of constraints and continuous variables and hundreds of 0-1 variables. The modeling framework allows to decompose the *DEM* in different ways. An splitting variable representation by scenario is used, such that the problem is converted in a set of scenario related mixed 0-1 programs with *non-anticipativity constraints* for the first stage variables. A two-stage specialization of a *Branch-and-Fix Coordination* (*BFC*) approach that we describe elsewhere is introduced to coordinate the *BF* phase execution for each scenario based model, such that the *non-anticipativity constraints* are also satisfied. For this purpose the concept of families of *twin nodes* among the different branching trees is used. Computational results for very large instances are reported. Given the problem dimensions is unrealistic to pretend to prove the solution's optimality in affordable time. Although more computational testing is required, the new approach for *SSCh* planning problem solving seems very promising. We have compared the proposed approach via scenario analysis for obtaining the *full recourse stochastic solution* with the more traditional approach based on the average scenario mixed 0-1 program solving. Although we have obtained the optimal solution for the second type of problems in all instances, the stochastic solution never has worse expected performance (i.e., net profit) and it is always more robust (i.e., the variation coefficient is smaller) than the average scenario based solution. On the other hand both solutions are always much better that the simple recourse stochastic solution.

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